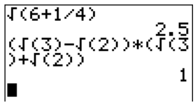
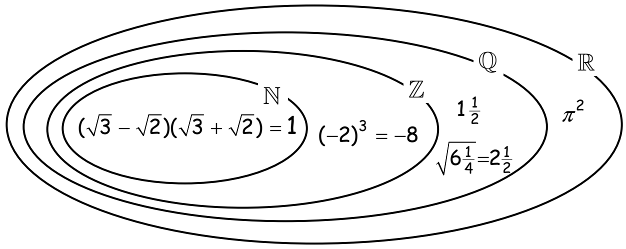
1a
I $2x+5=13$, $2x=8$, $x=4$.
II $2x+5=3$, $2x=-2$, $x=-1$.
III $2x+5=8$, $2x=3$, $x=1\frac{1}{2}$.
IV $x^2+5=8$, $x^2=3$, $x=\sqrt{3} \vee x=-\sqrt{3}$.

1b Bij III en IV vind je geen oplossingen.

2
$\sqrt{6\frac{1}{4}}=\sqrt{\frac{25}{4}}=\frac{5}{2}$.
$(-2)^3=-2\cdot -2\cdot -2=-8$.
$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3+\sqrt{6}-\sqrt{6}-2=3-2=1$.
(zie het Venn-diagram hiernaast)





3a
$x^2+x=6$
$x^2+x-6=0$
$(x+3)(x-2)=0$
$x=-3 \vee x=2$.

3b
$x^2+x=4$
$x^2+x-4=0$
$D=1^2-4\cdot 1\cdot -4=17$
geen oplossingen in $\mathbb{Q}$.

3c
$x^3-x=0$
$x\cdot(x^2-1)=0$
$x=0 \vee x^2=1$
$x=0 \vee x=1 \vee x=-1$.

3d
$x^3-2x=0$
$x\cdot(x^2-2)=0$
$x=0 \vee x^2=2$
$x=0 \vee$ geen oploss. in $\mathbb{Q}$
$x=0$.

3e
$x^4-9x=0$
$x\cdot(x^3-9)=0$
$x=0 \vee x^3=9$
$x=0 \vee$ geen oploss. in $\mathbb{Q}$
$x=0$.

3f
$x^4-9x^2=0$
$x^2\cdot(x^2-9)=0$
$x^2=0 \vee x^2=9$
$x=0 \vee x=3 \vee x=-3$.

4a
$(x+3)^2=7$
$x+3=\sqrt{7} \vee x+3=-\sqrt{7}$
$x=-3+\sqrt{7} \vee x=-3-\sqrt{7}$.

4b $(x+3)^2=-7$ heeft geen oplossingen in $\mathbb{R}$
omdat $(x+3)^2$ (een kwadraat) niet negatief kan zijn.

5a
$3x+5i+3=2i-x$
$4x=-3-3i$
$x=-\frac{3}{4}-\frac{3}{4}i$.

5b
$2x^2+10=0$
$2x^2=-10$
$x^2=-5$
$x^2=5i^2$
$x=i\sqrt{5} \vee x=-i\sqrt{5}$.

5c
$(x+2)^2+10=0$
$(x+2)^2=-10$
$(x+2)^2=10i^2$
$x+2=i\sqrt{10} \vee x+2=-i\sqrt{10}$
$x=-2+i\sqrt{10} \vee x=-2-i\sqrt{10}$.

5d
$\boxed{x^2-10x}+40=0$
$\boxed{(x-5)^2-25}+40=0$
$(x-5)^2+15=0$
$(x-5)^2=-15$
$(x-5)^2=15i^2$
$x-5=i\sqrt{15} \vee x-5=-i\sqrt{15}$
$x=5+i\sqrt{15} \vee x=5-i\sqrt{15}$.

5e
$\boxed{x^2+8x}+14=0$
$\boxed{(x+4)^2-16}+14=0$
$(x+4)^2-2=0$
$(x+4)^2=2$
$x+4=\sqrt{2} \vee x+4=-\sqrt{2}$
$x=-4+\sqrt{2} \vee x=-4-\sqrt{2}$.

5f
$(x+3)^2=-16$
$(x+3)^2=16i^2$
$x+3=4i \vee x+3=-4i$
$x=-3+4i \vee x=-3-4i$.

6a
$(x-3)^2+x=0$
$x^2-6x+9+x=0$
$\boxed{x^2-5x}+9=0$
$\boxed{(x-\frac{5}{2})^2-\frac{25}{4}}+9=0$
$(x-\frac{5}{2})^2+\frac{11}{4}=0$
$(x-\frac{5}{2})^2=-\frac{11}{4}$
$(x-\frac{5}{2})^2=\frac{11}{4}i^2$
$x-\frac{5}{2}=\frac{1}{2}i\sqrt{11} \vee x-\frac{5}{2}=-\frac{1}{2}i\sqrt{11}$
$x=\frac{5}{2}+\frac{1}{2}i\sqrt{11} \vee x=\frac{5}{2}-\frac{1}{2}i\sqrt{11}$.

6b
$(2x+3)^2+10=0$
$(2x+3)^2=-10$
$(2x+3)^2=10i^2$
$2x+3=i\sqrt{10} \vee 2x+3=-i\sqrt{10}$
$2x=-3+i\sqrt{10} \vee 2x=-3-i\sqrt{10}$
$x=-\frac{3}{2}+\frac{1}{2}i\sqrt{10} \vee x=-\frac{3}{2}-\frac{1}{2}i\sqrt{10}$.

6c
$\frac{1}{3}x+10+2i=\frac{1}{4}x+12-5i$
$\frac{1}{12}x=2-7i$ (keer 12)
$x=24-84i$.

6d
$\boxed{4x^2+4x}+7=0$
$\boxed{(2x+1)^2-1}+7=0$
$(2x+1)^2+6=0$
$(2x+1)^2=-6$
$(2x+1)^2=6i^2$
$2x+1=i\sqrt{6} \vee 2x+1=-i\sqrt{6}$
$2x=-1+i\sqrt{6} \vee 2x=-1-i\sqrt{6}$
$x=-\frac{1}{2}+\frac{1}{2}i\sqrt{6} \vee x=-\frac{1}{2}-\frac{1}{2}i\sqrt{6}$.

7a $(2+i)\cdot(10-5i) = 20 - 10i + 10i - 5i^2 = 20 - 5i^2 = 20 - 5\cdot -1 = 20 + 5 = 25.$

7b $(a+bi)\cdot(c+di) = ac + adi + bci + bdi^2 = ac + adi + bci - bd = (ac - bd) + (ad + bc)i.$

8a $(2+5i) + (4-6i) = 6 - i.$

8b $(5-i)\cdot(5+i) = 25 + 5i - 5i - i^2 = 25 + 1 = 26.$

8c $(2+i)^2 = (2+i)\cdot(2+i) = 4 + 2i + 2i + i^2 = 4 + 4i - 1 = 3 + 4i.$

8d $i\cdot(6+7i) = 6i + 7i^2 = 6i - 7 = -7 + 6i.$

8e $i^5 = i^2\cdot i^2\cdot i = -1\cdot -1\cdot i = i.$

8f $(1+i)\cdot(6-i) + (3-i)\cdot(3+2i) = 6 - i + 6i - i^2 + 9 + 6i - 3i - 2i^2 = 15 + 8i - 3i^2 = 18 + 8i.$

9a $\frac{2}{1+i} = \frac{2}{1+i}\cdot\frac{1-i}{1-i} = \frac{2\cdot(1-i)}{1^2 - i^2} = \frac{2\cdot(1-i)}{1+1} = \frac{2\cdot(1-i)}{2} = 1 - i.$

9b $\frac{2+3i}{i} = \frac{2+3i}{i}\cdot\frac{-i}{-i} = \frac{-2i+3i^2}{-i^2} = \frac{-2i-3}{1} = -3 - 2i.$

9c $\frac{2+i}{2-i} = \frac{2+i}{2-i}\cdot\frac{2+i}{2+i} = \frac{4+4i+i^2}{2^2 - i^2} = \frac{3+4i}{4+1} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i.$

9d $\frac{3+5i}{12+5i} = \frac{3+5i}{12+5i}\cdot\frac{12-5i}{12-5i} = \frac{36-15i+60i-25i^2}{12^2-(5i)^2} = \frac{36+45i+25}{144+25} = \frac{61+45i}{169} = \frac{61}{169} + \frac{45}{169}i.$

9e $\frac{2i}{1+3i} = \frac{2i}{1+3i}\cdot\frac{1-3i}{1-3i} = \frac{2i-6i^2}{1^2 - 9i^2} = \frac{-6+2i}{1+9} = \frac{-6+2i}{10} = -\frac{3}{5} + \frac{1}{5}i.$

9f $(2+3i)\cdot\frac{3}{2+i} = \frac{6+9i}{2+i}\cdot\frac{2-i}{2-i} = \frac{12-6i+18i-9i^2}{2^2 - i^2} = \frac{21+12i}{4+1} = \frac{21+12i}{5} = \frac{21}{5} + \frac{12}{5}i.$

10a $(3+4i)^2 = (3+4i)\cdot(3+4i) = 9 + 24i + 16i^2 = -7 + 24i.$

10b $\frac{3+i}{3+4i} = \frac{3+i}{3+4i}\cdot\frac{3-4i}{3-4i} = \frac{9-12i+3i-4i^2}{3^2-(4i)^2} = \frac{9-9i+4}{9+16} = \frac{13-9i}{25} = \frac{13}{25} - \frac{9}{25}i.$

10c $(2+i)^2 - (2-i)^2 = (2+i)\cdot(2+i) - (2-i)\cdot(2-i) = 4 + 4i + i^2 - (4 - 4i + i^2) = 4 + 4i + i^2 - 4 + 4i - i^2 = 8i.$

10d $i^2\cdot(i^3 - i^4 - i^5) = -1(i^2\cdot i - i^2\cdot i^2 - i^2\cdot i^2\cdot i) = -(-i - 1 - i) = i + 1 + i = 1 + 2i.$

10e $\frac{5}{2+3i} = \frac{5}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{10-15i}{2^2-(3i)^2} = \frac{10-15i}{4+9} = \frac{10-15i}{13} = \frac{10}{13} - \frac{15}{13}i.$

10f $i + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} = \cancel{i - 1 - i + 1} + \cancel{i - 1 - i + 1} + i - 1 = -1 + i.$

11 $\frac{a+bi}{c+di} = \frac{a+bi}{c+di}\cdot\frac{c-di}{c-di} = \frac{ac - adi + bci - bdi^2}{c^2 - (di)^2} = \frac{ac - adi + bci + bd}{c^2 - d^2i^2} = \frac{(ac+bd) - (ad-bc)i}{c^2 + d^2}.$

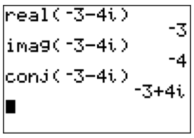
12a $(a+bi)\cdot(a-bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$ is reëel (a en b zijn namelijk beide reëel).

12b $(3+4i)\cdot(3-4i) = 3^2 + 4^2 = 9 + 16 = 25.$
$(100-200i)\cdot(100+200i) = 100^2 + 200^2 = 10\,000 + 40\,000 = 50\,000.$
$(0{,}4i + 0{,}3)\cdot(0{,}3 - 0{,}4i) = (0{,}3 + 0{,}4i)\cdot(0{,}3 - 0{,}4i) = 0{,}3^2 + 0{,}4^2 = 0{,}09 + 0{,}16 = 0{,}25.$

13a $\frac{z+\overline{z}}{2} = \frac{a+bi+a-bi}{2} = \frac{2a}{2} = a = \text{Re}(z).$

13b $\frac{z-\overline{z}}{2i} = \frac{a+bi-(a-bi)}{2i} = \frac{a+bi-a+bi}{2i} = \frac{2bi}{2i} = b = \text{Im}(z).$

13c $\overline{\overline{z}} = \overline{\overline{a+bi}} = \overline{\left(\overline{a+bi}\right)} = \overline{a-bi} = a + bi = z.$

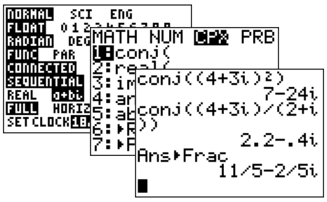


14a $\overline{z_1 + z_2} = \overline{(a+bi)+(c+di)} = \overline{(a+c)+(b+d)i} = a + c - (b+d)i = (a-bi) + c - di = \overline{z_1} + \overline{z_2}.$

14b $\overline{z_1\cdot z_2} = \overline{(a+bi)\cdot(c+di)} = \overline{ac + adi + bci + bdi^2} = \overline{(ac-bd)+(ad+bc)i} = (ac-bd) - (ad+bc)i$ **(1)**
$\overline{z_1}\cdot\overline{z_2} = \overline{a+bi}\cdot\overline{c+di} = ac - adi - bci + bdi^2 = ac - bd - i(ad - bc) = (ac-bd) - (ad+bc)i$ **(2)**
Uit **(1)** en **(2)** volgt dat $\overline{z_1\cdot z_2} = \overline{z_1}\cdot\overline{z_2}.$

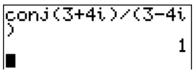
14c $\overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{a+bi}{c+di}\right)} = \overline{\left(\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}\right)} = \overline{\left(\frac{ac-adi+bci-bdi^2}{c^2-(di)^2}\right)} = \overline{\left(\frac{(ac+bd)-(ad-bc)i}{c^2+d^2}\right)} = \frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$ **(1)**
$\frac{\overline{z_1}}{\overline{z_2}} = \frac{\overline{a+bi}}{\overline{c+di}} = \frac{a-bi}{c-di} = \frac{a-bi}{c-di}\cdot\frac{c+di}{c+di} = \frac{ac+adi-bci-bdi^2}{c^2-(di)^2} = \frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$ **(2)**. Uit **(1)** en **(2)** volgt dat $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}.$

15a $\overline{(4+3i)^2} = \left(\overline{4+3i}\right)^2 = (4-3i)\cdot(4-3i) = 16-12i-12i+9i^2 = 16-24i-9 = 7-24i.$

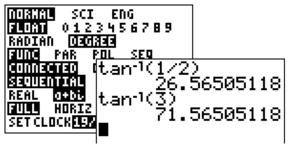


15b $\overline{\left(\frac{4+3i}{2+i}\right)} = \overline{\left(\frac{4+3i}{2+i}\cdot\frac{2-i}{2-i}\right)} = \overline{\left(\frac{8-4i+6i-3i^2}{2^2-i^2}\right)} = \overline{\left(\frac{8+2i+3}{4+1}\right)} = \overline{\left(\frac{11+2i}{5}\right)} = \overline{\left(\frac{11}{5}+\frac{2}{5}i\right)} = \frac{11}{5}-\frac{2}{5}i.$

15c $\frac{\overline{3+4i}}{3-4i} = \frac{3-4i}{3-4i} = 1.$



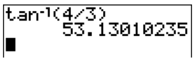
16a De lengte van pijl van de oorsprong naar het punt (2, 1) is $\sqrt{2^2+1^2} = \sqrt{4+1} = \sqrt{5}.$
De lengte van pijl van de oorsprong naar het punt (1, 3) is $\sqrt{1^2+3^2} = \sqrt{1+9} = \sqrt{10}.$

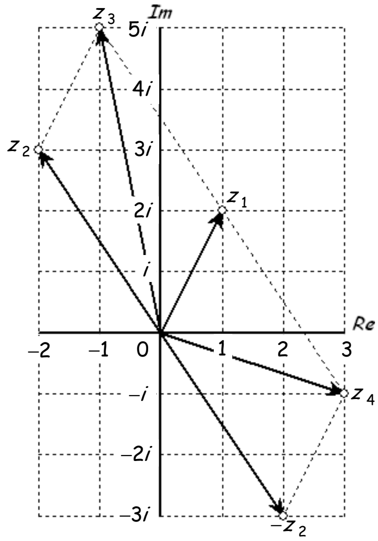


16b $\tan(\alpha) = \frac{1}{2} \Rightarrow \alpha \approx 27°$ en $\tan(\beta) = \frac{3}{1} = 3 \Rightarrow \beta \approx 72°.$ (MODE: DEGREE)

16c De lengte van de som van de pijlen (zie 16b) is 5 en som van de lengten van de pijlen (uit 16a) is $\sqrt{5}+\sqrt{10}.$
De lengte van de som van de pijlen is niet de som van de lengten van de pijlen, want $5 \neq \sqrt{5}+\sqrt{10}.$
(de weg van de oorsprong direct naar het punt (3,4) is korter dan de weg van de oorsprong via (2,1) naar (3,4))

16d $\tan(\gamma) = \frac{4}{3} \Rightarrow \gamma \approx 53°.$ Dus $\gamma \neq \frac{\alpha+\beta}{2}.$





17a Zie z_1 en z_2 in de figuur hiernaast.
$z_1 = 1+2i$ met lengte $|z_1| = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}.$
$z_2 = -2+3i$ met lengte $|z_2| = \sqrt{(-2)^2+3^2} = \sqrt{4+9} = \sqrt{13}.$

17b $z_3 = z_1 + z_2 = (1+2i)+(-2+3i) = -1+5i.$ (zie ook in de figuur hiernaast)

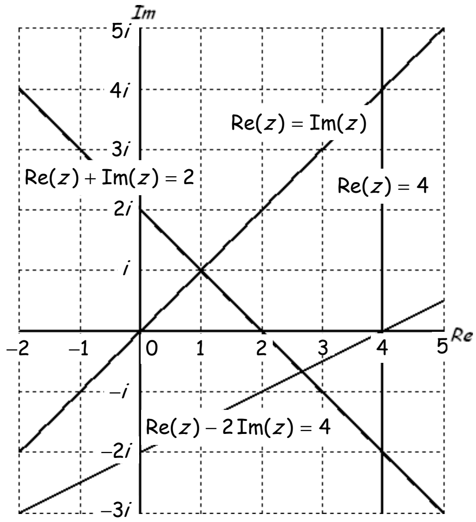
17c Zie de figuur hiernaast. De parallellogram-constructie klopt.

17d $z_4 = z_1 - z_2 = (1+2i)-(-2+3i) = 1+2i+2-3i = 3-i.$

17e Zie de figuur hiernaast. De parallellogram-constructie klopt.

17f $z_5 = z_1 \cdot z_2 = (1+2i)\cdot(-2+3i) = -2+3i-4i+6i^2 = -2-i-6 = -8-i.$

17g $|z_1 \cdot z_2| = |-8-i| = \sqrt{(-8)^2+(-1)^2} = \sqrt{64+1} = \sqrt{65}.$
$|z_1| = \sqrt{5}$ en $|z_2| = \sqrt{13}$ (zie 17a) $\Rightarrow |z_1|\cdot|z_2| = \sqrt{5}\cdot\sqrt{13} = \sqrt{5\cdot 13} = \sqrt{65}.$

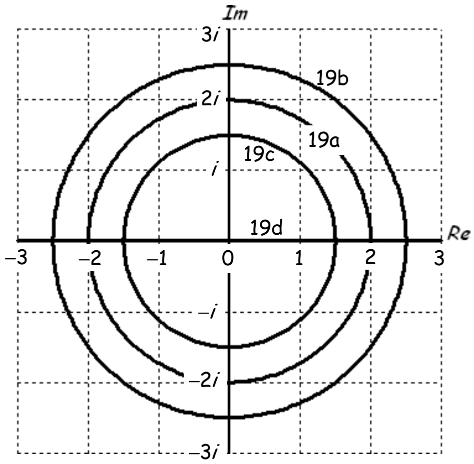


18a $\text{Re}(z) = 4.$
(de lijn $x = 4$ in het xOy-assenstelsel $\Rightarrow$ een verticale lijn)

18b $\text{Re}(z) = \text{Im}(z).$
(de lijn $x = y$ dus $y = x$ in het xOy-assenstelsel)

18c $\text{Re}(z) + \text{Im}(z) = 2.$
(de lijn $x + y = 2 \Rightarrow y = -x + 2$ in het xOy-assenstelsel)

18d $\text{Re}(z) - 2\,\text{Im}(z) = 4.$
(de lijn $x - 2y = 4 \Rightarrow -2y = -x + 4 \Rightarrow y = \frac{1}{2}x - 2$ in het xOy-assenstelsel)



19a $(\text{Re}(z))^2 + (\text{Im}(z))^2 = (|z|)^2 = 4 \Rightarrow |z| = \sqrt{4} = 2$
(dus de lengte van z is 2, dit is een cirkel met middelpunt O en straal 2)

19b $|z| = 2{,}5.$ (dus de lengte van z is $2\frac{1}{2}$)
(dus de lengte van z is $2\frac{1}{2}$, dit is een cirkel met middelpunt O en straal $2\frac{1}{2}$)

19c $z \cdot \overline{z} = (|z|)^2 = 2\frac{1}{4} = \frac{9}{4} \Rightarrow |z| = \sqrt{\frac{9}{4}} = \frac{3}{2}$ (dus de lengte van z is $1\frac{1}{2}$).
(dit is een cirkel met middelpunt O en straal $1\frac{1}{2}$)

19d $z = \overline{z} \Rightarrow a+bi = a-bi \Rightarrow 2bi = 0 \Rightarrow 2b = 0 \Rightarrow b = 0 \Rightarrow \text{Im}(z) = 0.$ Dit is de reële as.
(de lijn $y = 0$ in het xOy-assenstelsel $\Rightarrow$ de x-as in het xOy-assenstelsel)

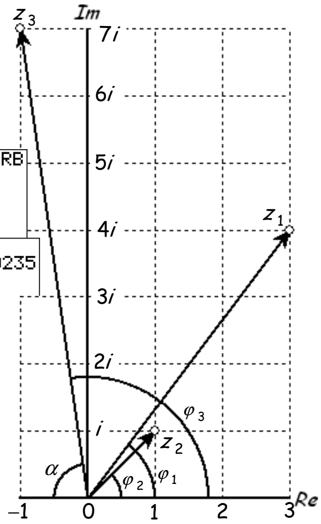
20 Stel $z = a + bi$ dan $\overline{z} = a - bi$ en $z \cdot \overline{z} = (a+bi)\cdot(a-bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$. **(1)**
De lengte van z is $|z| = \sqrt{a^2+b^2} \Rightarrow |z|^2 = a^2 + b^2$ **(2)**. Uit **(1)** en **(2)** volgt $z \cdot \overline{z} = |z|^2$.

21 $|z_1 \cdot z_2| = |(a+bi)\cdot(c+di)| = |ac + adi + bci + bdi^2| = |(ac-bd) + (ad+bc)i| = \sqrt{(ac-bd)^2 + (ad+bc)^2}$
$= \sqrt{(ac)^2 - 2acbd + (bd)^2 + (ad)^2 + 2adbc + (bc)^2} = \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}$ **(1)**
$|z_1|\cdot|z_2| = |a+bi|\cdot|c+di| = \sqrt{a^2+b^2}\cdot\sqrt{c^2+d^2} = \sqrt{(a^2+b^2)\cdot(c^2+d^2)} = \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}$ **(2)**
Uit **(1)** en **(2)** volgt $|z_1 \cdot z_2| = |z_1|\cdot|z_2|$ (hier staat de lengte van $(z_1 \cdot z_2)$ is de lengte van z_1 keer de lengte van z_2).

22a $z_3 = z_1 \cdot z_2 = (3+4i)\cdot(1+i) = 3 + 3i + 4i + 4i^2 = 3 + 7i - 4 = -1 + 7i$.

```
(3+4i)(1+i)
              -1+7i
```

22b Zie de figuur hiernaast.



22c Voor z_1 geldt $\tan(\varphi_1) = \frac{4}{3} \Rightarrow \varphi_1 = \tan^{-1}(\frac{4}{3}) \approx 53°$.
Voor z_2 geldt $\tan(\varphi_2) = \frac{1}{1} = 1 \Rightarrow \varphi_2 = \tan^{-1}(1) = 45°$.
Voor z_3 geldt $\tan(\alpha) = \frac{7}{1} \Rightarrow \varphi_3 = 180° - \tan^{-1}(7) \approx 98°$.
Voor de draaihoeken geldt $\varphi_1 + \varphi_2 = \varphi_3$.

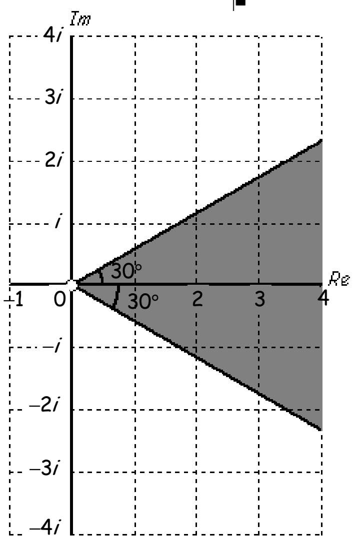
```
tan-1(4/3)
        53.13010235
tan-1(1)
                 45
```

```
tan-1(7)
        81.86989765
180-Ans
        98.13010235
```

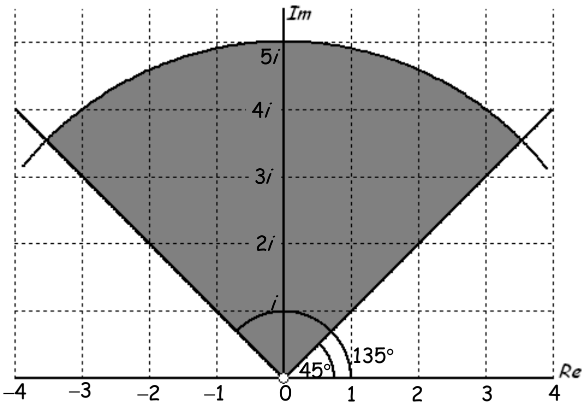
```
MATH NUM CPX PRB
1:conj(
2:real(
3:imag(
4:angle(
5:abs(
```

```
angle(-1+7i)
        98.13010235
```

23a $-30° \le \text{Arg}(z) \le 30°$.
(zie de figuur hiernaast)



23b $45° \le \text{Arg}(z) \le 135° \;\wedge\; |z| \le 5$.
(zie de figuur hieronder)



24a $z = 2 + 2i$ heeft lengte $|z| = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ en
de hoofdwaarde van het argument is $\text{Arg}(z) = 45°$.

```
abs(2+2i)
        2.828427125
√(8)
        2.828427125
angle(2+2i)
                 45
```

24b $z = (1-i)^6$ heeft lengte $|z| = |(1-i)^6| = |1-i|^6 = (\sqrt{1^2 + (-1)^2})^6 = (\sqrt{1+1})^6 = \left((\sqrt{2})^2\right)^3 = 2^3 = 8$ en
de hoofdwaarde van het argument is $\text{Arg}(z) = 90°$.
N.B.: $z = (1-i)^6 = 8i$ (op de imaginaire as) met lengte $|z| = |8i| = 8$ en hoofdwaarde van het argument $\text{Arg}(8i) = 90°$.

```
angle((1-i)^6)
                 90
(1-i)^6
                 8i
```

24c $z = \cos(40°) + i\sin(40°)$ heeft lengte $|z| = 1$ (af te lezen in de eenheidscirkel) en
de hoofdwaarde van het argument is $\text{Arg}(z) = 40°$ (af te lezen in de eenheidscirkel).

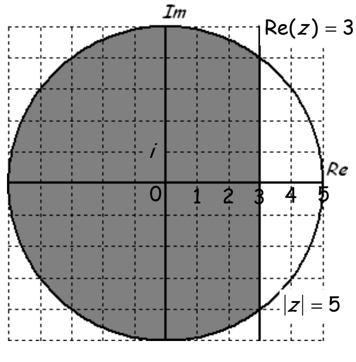
```
abs(cos(40)+isin
(40))
                  1
angle(cos(40)+is
in(40))
                 40
```

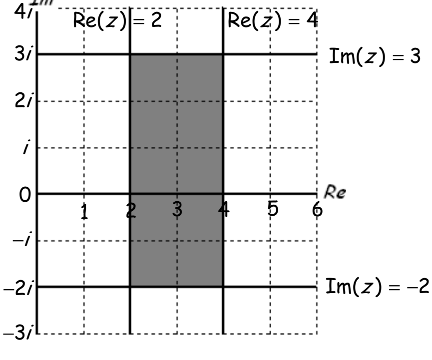
24d $z = 5\cos(140°) + 5i\sin(140°)$ heeft lengte $|z| = 5$ (vergroot de eenheidscirkel) en
de hoofdwaarde van het argument is $\text{Arg}(z) = 140°$ (vergroot de eenheidscirkel).

```
abs(5cos(140)+5i
sin(140))
                  5
angle(5cos(140)+
5isin(140))
                140
```

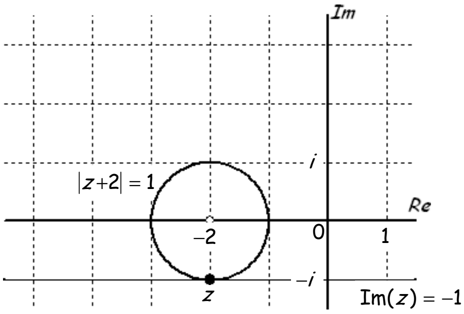
25a $\text{Re}(z) \le 3 \;\wedge\; |z| \le 5$.

25b $2 \le \text{Re}(z) \le 4 \;\wedge\; -2 \le \text{Im}(z) \le 3$.

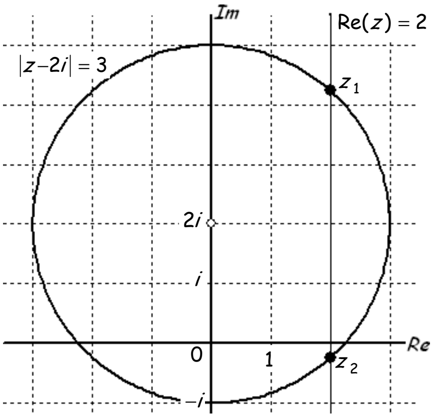




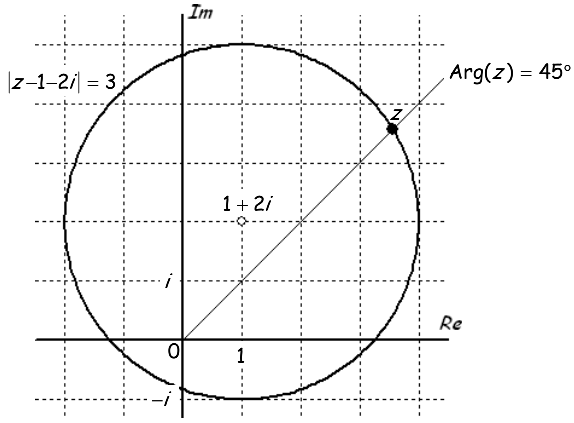
26a $|z+2| = |z-(-2)| = 1$ (de afstand van z tot -2 is 1)
is de cirkel met middelpunt -2 en straal 1.



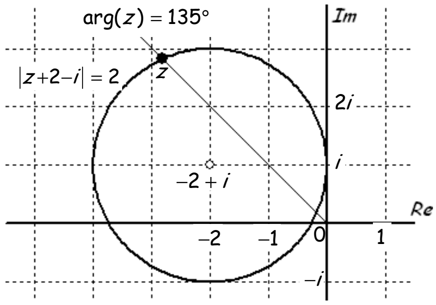
26b $|z-2i| = 3$ (de afstand van z tot $2i$ is 3)
is de cirkel met middelpunt $2i$ en straal 3.



26c $|z-1-2i| = |z-(1+2i)| = 3$ (de afstand van z tot $1+2i$ is 3)
is de cirkel met middelpunt $1+2i$ en straal 3.



26d $|z+2-i| = |z-(-2+i)| = 2$ (de afstand van z tot $-2+i$ is 2)
is de cirkel met middelpunt $-2+i$ en straal 2.



27a $|z+2| = 1 \;\wedge\; \text{Im}(z) = -1$ (zie figuur 26a).

27b $|z-2i| = 3 \;\wedge\; \text{Re}(z) = 2$ (zie figuur 26b).

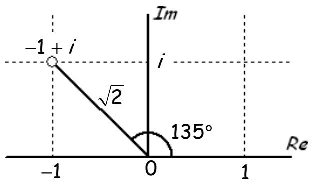
27c $|z-1-2i| = 3 \;\wedge\; \text{Arg}(z) = 45°$ (zie figuur 26c).

27d $|z-2+i| = 2 \;\wedge\; \text{Arg}(z) = 135°$ (zie figuur 26d).

28a $\text{Arg}(z_1) = \text{Arg}(-1+i) = 135°$ en $|z_1| = |-1+i| = \sqrt{(-1)^2+1^2} = \sqrt{2}$.

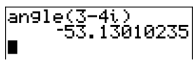
28b $\text{Arg}(z_2) = \text{Arg}(\sqrt{2}\cos(135°) + \sqrt{2}i\sin(135°)) = 135°$ en
$|z_2| = |\sqrt{2}\cos(135°) + \sqrt{2}i\sin(135°)| = \sqrt{2}$ (denk aan de eenheidscirkel).

28c $z_1 = z_2$ omdat $\text{Arg}(z_1) = \text{Arg}(z_2)$ en $|z_1| = |z_2|$.

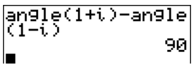


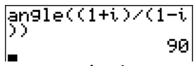
29a $|10+10i| = \sqrt{10^2+10^2} = \sqrt{100\cdot 2} = 10\sqrt{2}$ en $\text{Arg}(10+10i) = 45° \Rightarrow 10+10i = 10\sqrt{2}(\cos(45°) + i\sin(45°))$.

29b $|3-4i| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$ en $\text{Arg}(3-4i) \approx -53{,}1° \Rightarrow 3-4i \approx 5(\cos(-53{,}1°) + i\sin(-53{,}1°))$.



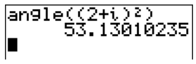
29c $|8| = 8$ en $\text{Arg}(8) = 0° \Rightarrow 8 = 8(\cos(0°) + i\sin(0°))$.



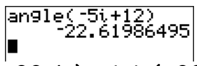


29d $\left|\frac{1+i}{1-i}\right| = \frac{|1+i|}{|1-i|} = \frac{\sqrt{2}}{\sqrt{2}} = 1$ en $\arg(\frac{1+i}{1-i}) = \text{Arg}(1+i) - \text{Arg}(1-i) = 45° - (-45°) = 90° \Rightarrow \frac{1+i}{1-i} = \cos(90°) + i\sin(90°)$.

29e $|(2+i)^2| = |2+i|^2 = \sqrt{2^2+1^2}^{\,2} = \sqrt{5}^{\,2} = 5$ en $\text{Arg}((2+i)^2) \approx 53{,}1° \Rightarrow (2+i)^2 \approx 5(\cos(53{,}1°) + i\sin(53{,}1°))$.

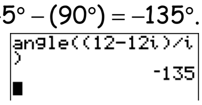


29f $|-5i| = 5$ en $\text{Arg}(-5i) = -90° \Rightarrow 3-4i = 5(\cos(-90°) + i\sin(-90°))$.



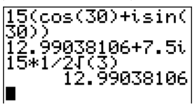
29g $|-5i+12| = \sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = 13$ en $\text{Arg}(12-5i) \approx -22{,}6° \Rightarrow 12-5i = 13(\cos(-22{,}6°) + i\sin(-22{,}6°))$.

29h $\left|\frac{12-12i}{i}\right| = \frac{|12-12i|}{|i|} = \frac{\sqrt{144+144}}{1} = \sqrt{144\cdot 2} = 12\sqrt{2}$ en $\arg(\frac{12-12i}{i}) = \text{Arg}(12-12i) - \text{Arg}(i) = -45° - (90°) = -135°$.
Dus $\frac{1+i}{1-i} = 12\sqrt{2}(\cos(-135°) + i\sin(-135°))$.



30a $15(\cos(30°) + i\sin(30°)) = 15\cdot(\frac{1}{2}\sqrt{3} + i\cdot\frac{1}{2}) = 7\frac{1}{2}\sqrt{3} + 7\frac{1}{2}i$ ($\approx 13{,}0 + 7{,}5i$).

30b $100(\cos(90°) + i\sin(90°)) = 100\cdot(0 + i\cdot 1) = 100i$.
($|z| = 100$ en $\text{Arg}(z) = 90° \Rightarrow$ op de positieve imaginare as)

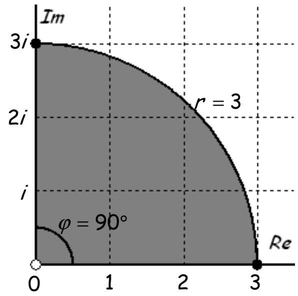


φ	0°	30°	45°	60°	90°
$\sin(\varphi)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\varphi)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

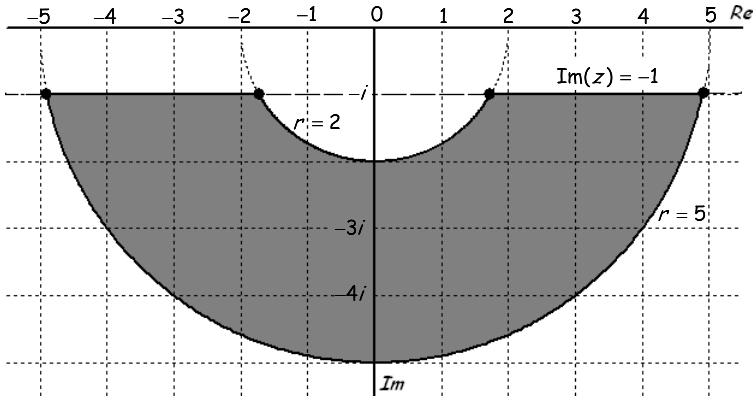
30c $\sqrt{2}(\cos(135°) + i\sin(135°)) = \sqrt{2}\cdot(-\frac{1}{2}\sqrt{2} + i\cdot\frac{1}{2}\sqrt{2}) = -1 + i$.

30d $\sqrt{5}(\cos(-90°) + i\sin(-90°)) = \sqrt{5}\cdot -i = -i\sqrt{5}$. ($|z| = \sqrt{5}$ en $\text{Arg}(z) = -90° \Rightarrow$ op de negatieve imaginare as)

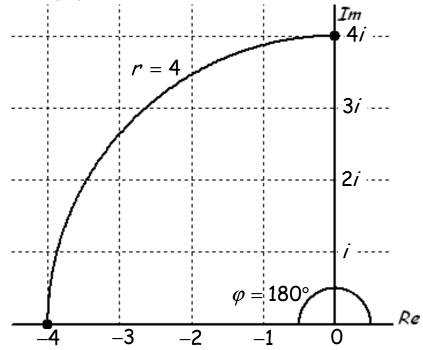
31a $r = |z| \le 3 \;\wedge\; 0° \le \varphi = \text{Arg}(z) \le 90°$.



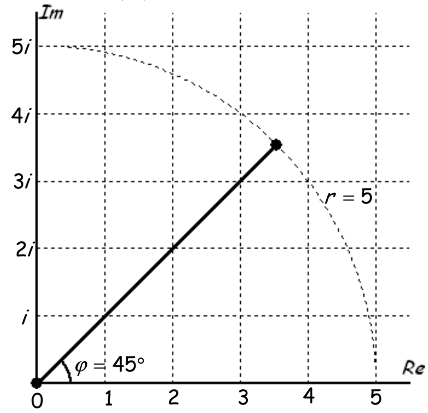
31b $2 \le r = |z| \le 5 \;\wedge\; \text{Im}(z) \le -1$.



31c $r = |z| = 4 \;\wedge\; 90° \le \varphi = \text{Arg}(z) \le 180°$.

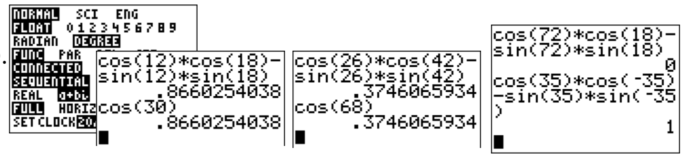


31d $r = |z| \le 5 \;\wedge\; \varphi = \text{Arg}(z) = 45°$.



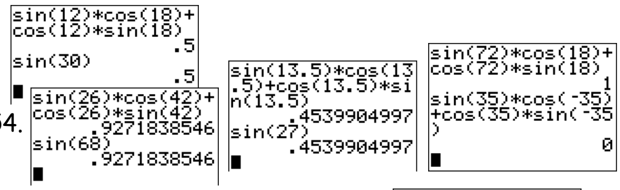
32a $\cos(12°)\cdot\cos(18°) - \sin(12°)\cdot\sin(18°) \approx 0{,}866$ en $\cos(30°) = \frac{1}{2}\sqrt{3} \approx 0{,}866$.
Dus $\cos(12°)\cdot\cos(18°) - \sin(12°)\cdot\sin(18°) = \cos(30°)$.

32b $\cos(26°)\cdot\cos(42°) - \sin(26°)\cdot\sin(42°) = \cos(68°) \approx 0{,}375$.
$\cos(72°)\cdot\cos(18°) - \sin(72°)\cdot\sin(18°) = \cos(90°) = 0$.
$\cos(35°)\cdot\cos(-35°) - \sin(35°)\cdot\sin(-35°) = \cos(0°) = 1$.

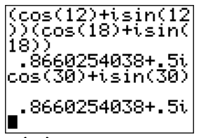


33a $\sin(12°)\cdot\cos(18°) + \cos(12°)\cdot\sin(18°) = 0{,}5$ en $\sin(30°) = 0{,}5$.
Dus $\sin(12°)\cdot\cos(18°) + \cos(12°)\cdot\sin(18°) = \sin(30°)$.

33b $\sin(26°)\cdot\cos(42°) + \cos(26°)\cdot\sin(42°) = \sin(68°) \approx 0{,}927$.
$\sin(13{,}5°)\cdot\cos(13{,}5°) + \cos(13{,}5°)\cdot\sin(13{,}5°) = \sin(27°) \approx 0{,}454$.
$\sin(72°)\cdot\cos(18°) + \cos(72°)\cdot\sin(18°) = \sin(90°) = 1$.
$\sin(35°)\cdot\cos(-35°) + \cos(35°)\cdot\sin(-35°) = \sin(0°) = 0$.



34a $(\cos(12°) + i\sin(12°))\cdot(\cos(18°) + i\sin(18°)) \approx 0{,}866 + 0{,}5i$ en $\cos(30°) + i\sin(30°) \approx 0{,}866 + 0{,}5i$.
Dus $(\cos(12°) + i\sin(12°))\cdot(\cos(18°) + i\sin(18°)) = \cos(30°) + i\sin(30°)$.



34b
$$\begin{aligned}(\cos(\alpha) + i\sin(\alpha))\cdot(\cos(\beta) + i\sin(\beta)) &= \cos(\alpha)\cos(\beta) + i\cos(\alpha)\sin(\beta) + i\sin(\alpha)\cos(\beta) + i^2\sin(\alpha)\sin(\beta)\\ &= \cos(\alpha)\cos(\beta) + i\cos(\alpha)\sin(\beta) + i\sin(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)\\ &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) + i\left(\cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)\right)\\ &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) + i\left(\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)\right)\\ &= \cos(\alpha+\beta) + i\sin(\alpha+\beta).\end{aligned}$$

35a $|z| = |(3-3i)\cdot(1+i)| = |3-3i|\cdot|1+i| = \sqrt{9+9}\cdot\sqrt{1+1} = \sqrt{9\cdot 2}\cdot\sqrt{2} = 3\sqrt{2}\cdot\sqrt{2} = 6$.
$\arg(z) = \arg((3-3i)\cdot(1+i)) = \text{Arg}(3-3i) + \text{Arg}(1+i) = -45° + 45° = 0° = \text{Arg}(z)$.

35b $|z| = \left|(2-2i)\cdot(\sqrt{2}(\cos(45°) + i\sin(45°))\right| = |2-2i|\cdot\left|\sqrt{2}(\cos(45°) + i\sin(45°)\right| = \sqrt{4+4}\cdot\sqrt{2} = \sqrt{4\cdot 2}\cdot\sqrt{2} = 2\cdot\sqrt{2}\cdot\sqrt{2} = 4$.
$\arg\left((2-2i)\cdot(\sqrt{2}(\cos(45°) + i\sin(45°))\right) = \text{Arg}(2-2i) + \text{Arg}\left(\sqrt{2}(\cos(45°) + i\sin(45°)\right) = -45° + 45° = 0° = \text{Arg}(z)$.

35c $|z| = \left|\dfrac{8(\cos(12°) + i\sin(12°))}{2(\cos(17°) + i\sin(17°))}\right| = \dfrac{|8(\cos(12°) + i\sin(12°))|}{|2(\cos(17°) + i\sin(17°))|} = \dfrac{8}{2} = 4$.
$\arg\left(\dfrac{8(\cos(12°) + i\sin(12°))}{2(\cos(17°) + i\sin(17°))}\right) = \text{Arg}\left(8(\cos(12°) + i\sin(12°))\right) - \text{Arg}\left(2(\cos(17°) + i\sin(17°))\right) = 12° - 17° = -5° = \text{Arg}(z)$.

36 $|z| = |1+i| = \sqrt{1+1} = \sqrt{2}$ en $\text{Arg}(z) = \text{Arg}(1+i) = 45°$.

36a $|2z| = |2 \cdot z| = |2| \cdot |z| = 2 \cdot \sqrt{2}$ en $\arg(2z) = \text{Arg}(2) + \text{Arg}(z) = 0° + 45° = 45° = \text{Arg}(2z)$.

36b $|iz| = |i \cdot z| = |i| \cdot |z| = 1 \cdot \sqrt{2}$ en $\arg(iz) = \text{Arg}(i) + \text{Arg}(z) = 90° + 45° = 135° = \text{Arg}(iz)$.

36c $\left|\frac{1}{z}\right| = \frac{|1|}{|z|} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$ en $\arg(\frac{1}{z}) = \text{Arg}(1) - \text{Arg}(z) = 0° - 45° = -45° = \text{Arg}(\frac{1}{z})$.

36d $\left|z^2\right| = |z \cdot z| = |z| \cdot |z| = |z|^2 = \sqrt{2}^2 = 2$ en $\arg(z^2) = \text{Arg}(z) + \text{Arg}(z) = 2\text{Arg}(z) = 2 \cdot 45° = 90° = \text{Arg}(z^2)$.

36e $\left|z^5\right| = |z|^5 = \sqrt{2}^5 = 4\sqrt{2}$ en $\arg(z^5) = 5\text{Arg}(z) = 5 \cdot 45° = 225°$, dus $\text{Arg}(z^5) = 225° - 360° = -135°$.

36f $\left|\frac{2i}{z}\right| = \frac{|2i|}{|z|} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$ en $\arg(\frac{2i}{z}) = \text{Arg}(2i) - \text{Arg}(z) = 90° - 45° = 45° = \text{Arg}(\frac{2i}{z})$.

37 $|z| = \left|(\cos(20°) + i\sin(20°))^{-4}\right| = |\cos(20°) + i\sin(20°)|^{-4} = 1^{-4} = 1$ en

$\arg(z) = \arg\left((\cos(20°) + i\sin(20°))^{-4}\right) = -4\text{Arg}(\cos(20°) + i\sin(20°)) = -4 \cdot 20° = -80° = \text{Arg}(z)$.

I $|\cos(-80°) + i\sin(-80°)| = 1 = |z|$ en $\text{Arg}(\cos(-80°) + i\sin(-80°)) = -80° = \text{Arg}(z)$.

II $\left|\frac{1}{\cos(80°) + i\sin(80°)}\right| = \frac{|1|}{|\cos(80°) + i\sin(80°)|} = \frac{1}{1} = 1 = |z|$ en

$\arg\left(\frac{1}{\cos(80°) + i\sin(80°)}\right) = \text{Arg}(1) - \text{Arg}(\cos(80°) + i\sin(80°)) = 0 - 80° = -80° = \text{Arg}(z)$.

III $|-\cos(80°) - i\sin(-80°)| = |-1 \cdot (\cos(80°) + i\sin(-80°))| = |-1| \cdot |\cos(80°) + i\sin(-80°)| = 1 \cdot 1 = 1 = |z|$ en

$\arg(-1 \cdot (\cos(80°) + i\sin(-80°))) = \text{Arg}(-1) + \text{Arg}(\cos(80°) + i\sin(-80°)) = 180° - 80° = 100° \neq \text{Arg}(z)$.

IV $\left|\frac{-4}{\cos(20°) + i\sin(20°)}\right| = \frac{|-4|}{|\cos(20°) + i\sin(20°)|} = \frac{4}{1} = 4 \neq |z|$. Dus I en II zijn gelijk aan $z = (\cos(20°) + i\sin(20°))^{-4}$.

Toets voorkennis (vóór het voorbeeld boven opgave 38)

1a $\cos(30°) = \frac{1}{2}\sqrt{3}$. 1c $\sin(150°) = \sin(30°) = \frac{1}{2}$.

1b $\sin(45°) = \frac{1}{2}\sqrt{2}$. 1d $\cos(-150°) = -\cos(30°) = -\frac{1}{2}\sqrt{3}$.

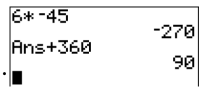
φ	0°	30°	45°	60°	90°
$\sin(\varphi)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\varphi)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

2a $\sin(\alpha) = \frac{1}{2} \Rightarrow \alpha = 30° \lor \alpha = 150°$. 2c $\cos(\alpha) = \frac{1}{2}\sqrt{3} \Rightarrow \alpha = 30° \lor \alpha = -30°$.

2b $\cos(\alpha) = -\frac{1}{2}\sqrt{2} \Rightarrow \alpha = 135° \lor \alpha = -135°$. 2d $\sin(\alpha) = -\frac{1}{2}\sqrt{3} \Rightarrow \alpha = -60° \lor \alpha = -120°$.

38a $(1-i)^6$ (met $|1-i| = \sqrt{2}$ en $\text{Arg}(1-i) = -45°$) $= \left(\sqrt{2}(\cos(-45°) + i\sin(-45°))\right)^6$

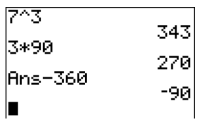
$= \sqrt{2}^6 \cdot (\cos(6 \cdot -45°) + i\sin(6 \cdot -45°)) = 2^3(\cos(-270°) + i\sin(-270°)) = 8(\cos(90°) + i\sin(90°))$.



38b $(\cos(20°) + i\sin(20°))^4 = 1^4 \cdot (\cos(4 \cdot 20°) + i\sin(4 \cdot 20°)) = \cos(80°) + i\sin(80°)$.

38c $(7i)^3$ (met $|7i| = 7$ en $\text{Arg}(7i) = 90°$) $= \left(7(\cos(90°) + i\sin(90°))\right)^3$

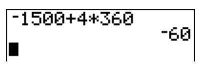
$= 7^3 \cdot (\cos(3 \cdot 90°) + i\sin(3 \cdot 90°)) = 343(\cos(270°) + i\sin(270°)) = 343(\cos(-90°) + i\sin(-90°))$.



39a $(\cos(30°) + i\sin(30°))^2 = \cos(60°) + i\sin(60°) = \frac{1}{2} + \frac{1}{2}i\sqrt{3}$.

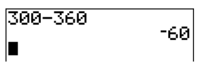
39b $(\cos(45°) + i\sin(45°))^5 = \cos(225°) + i\sin(225°) = -\cos(45°) - i\sin(45°) = -\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}$.

39c $(\cos(300°) + i\sin(300°))^{-5} = \cos(-1500°) + i\sin(-1500°) = \cos(-60°) + i\sin(-60°) = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$.



39d $\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}\right)^5 = (\cos(-45°) + i\sin(-45°))^5 = \cos(-225°) + i\sin(-225°) = -\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}$.

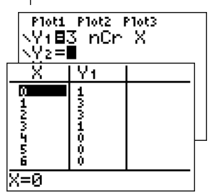
39e $\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right)^{10} = (\cos(30°) + i\sin(30°))^{10} = \cos(300°) + i\sin(300°) = \cos(-60°) + i\sin(-60°) = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$.



40a $(\cos(x) + i\sin(x))^3 = \binom{3}{0} \cdot \cos^3(x) + \binom{3}{1} \cdot \cos^2(x) \cdot i\sin(x) + \binom{3}{2} \cdot \cos(x) \cdot (i\sin(x))^2 + \binom{3}{3} \cdot (i\sin(x))^3$

$= \cos^3(x) + 3 \cdot \cos^2(x) \cdot i\sin(x) + 3 \cdot \cos(x) \cdot i^2\sin^2(x) + i^3\sin^3(x)$

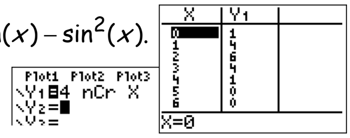
$= \cos^3(x) + 3i\cos^2(x)\sin(x) - 3\cos(x)\sin^2(x) - i\sin^3(x)$.



40b Uit $(\cos(\varphi) + i\sin(\varphi))^3 = \cos(3\varphi) + i\sin(3\varphi)$ volgt $(\cos(x) + i\sin(x))^3 = \cos(3x) + i\sin(3x)$ (door φ te vervangen door x).

40c Uit 40a en 40b volgt dan $\cos(3x)+i\sin(3x)=\cos^3(x)+3i\cos^2(x)\sin(x)-3\cos(x)\sin^2(x)-i\sin^3(x)$.
Dus $\cos(3x)=\cos^3(x)-3\cos(x)\sin^2(x)$ en $i\sin(3x)=3i\cos^2(x)\sin(x)-i\sin^3(x)$.
Dit geeft dan $\cos(3x)=\cos^3(x)-3\cos(x)\sin^2(x)$ en $\sin(3x)=3\cos^2(x)\sin(x)-\sin^3(x)$.

40d $\cos(2x)+i\sin(2x)=(\cos(x)+i\sin(x))^2=\binom{2}{0}\cdot\cos^2(x)+\binom{2}{1}\cdot\cos(x)\cdot i\sin(x)+\binom{2}{2}\cdot(i\sin(x))^2$
$=\cos^2(x)+2\cdot\cos(x)\cdot i\sin(x)+i^2\sin^2(x)=\cos^2(x)+2i\cos(x)\sin(x)-\sin^2(x)$.
Dus $\cos(2x)=\cos^2(x)-\sin^2(x)$ en $\sin(2x)=2\cos(x)\sin(x)$.



40e $\cos(4x)+i\sin(4x)=(\cos(x)+i\sin(x))^4$
$=\binom{4}{0}\cdot\cos^4(x)+\binom{4}{1}\cdot\cos^3(x)\cdot i\sin(x)+\binom{4}{2}\cdot\cos^2(x)\cdot(i\sin(x))^2+\binom{4}{3}\cdot\cos(x)\cdot(i\sin(x))^3+\binom{4}{4}\cdot(i\sin(x))^4$
$=\cos^4(x)+4\cos^3(x)\cdot i\sin(x)+6\cos^2(x)\cdot i^2\sin^2(x)+4\cos(x)\cdot i^3\sin^3(x)+i^4\sin^4(x)$
$=\cos^4(x)+4i\cos^3(x)\sin(x)-6\cos^2(x)\sin^2(x)-4i\cos(x)\sin^3(x)+\sin^4(x)$.
Dus $\cos(4x)=\cos^4(x)-6\cos^2(x)\sin^2(x)+\sin^4(x)$ en $\sin(4x)=4\cos^3(x)\sin(x)-4\cos(x)\sin^3(x)$.

41a $|4i|=4$ en $\text{Arg}(4i)=90°\Rightarrow 4i=4(\cos(90°)+i\sin(90°))$.
$|4i|=4$ en $\arg(4i)=-270°\Rightarrow 4i=4(\cos(-270°)+i\sin(-270°))$.

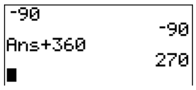
41b $z^2=4i=4(\cos(90°)+i\sin(90°))$ geeft
$z=\sqrt{4}\left(\cos(\frac{90}{2}°)+i\sin(\frac{90}{2}°)\right)=2(\cos(45°)+i\sin(45°))=2\left(\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2}\right)=\sqrt{2}+i\sqrt{2}=z_1$.
$z^2=4i=4(\cos(-270°)+i\sin(-270°))$ geeft
$z=\sqrt{4}\left(\cos(\frac{-270}{2}°)+i\sin(\frac{-270}{2}°)\right)=2(\cos(-135°)+i\sin(-135°))=2\left(-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2}\right)=-\sqrt{2}-i\sqrt{2}=z_2$.

41c Gerson heeft geen gelijk, want $z^2=4i=4(\cos(450°)+i\sin(450°))$ geeft
$z=\sqrt{4}\left(\cos(\frac{450}{2}°)+i\sin(\frac{450}{2}°)\right)=2(\cos(225°)+i\sin(225°))=2\left(-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2}\right)=-\sqrt{2}-i\sqrt{2}=z_2$ (uit 41b).

42a $|z|=1$ en $\text{Arg}(z)=120°$ geeft $|z^3|=1^3=1$ en $\arg(z^3)=3\text{Arg}(z)=360°\Rightarrow|z^3|=1$ en $\text{Arg}(z^3)=0°\Rightarrow z^3=1$.

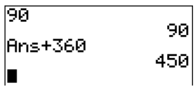
42b $|z|=1$ met $\text{Arg}(z)=-120°$ of $\text{Arg}(z)=0°$ (naast $\text{Arg}(z)=120°$ uit 42a) geven ook $z^3=1$.

43a $z^2=-4i$ (op de negatieve imaginaire as en op afstand 4 van 0) heeft twee oplosssingen voor z in $\mathbb{C}$.
$z^2=4(\cos(-90°)+i\sin(-90°))$ ∨ $z^2=4(\cos(270°)+i\sin(270°))$



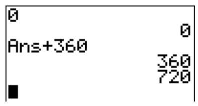
$z=2(\cos(-45°)+i\sin(-45°))$ ∨ $z=2(\cos(135°)+i\sin(135°))$
$z=2\cdot(\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2})=\sqrt{2}-i\sqrt{2}$ ∨ $z=2\cdot(-\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2})=-\sqrt{2}+i\sqrt{2}$.

43b $z^2=9i$ (op de positieve imaginaire as en op afstand 9 van 0) heeft twee oplosssingen voor z in $\mathbb{C}$.
$z^2=9(\cos(90°)+i\sin(90°))$ ∨ $z^2=9(\cos(450°)+i\sin(450°))$



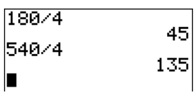
$z=3(\cos(45°)+i\sin(45°))$ ∨ $z=3(\cos(225°)+i\sin(225°))$
$z=3\cdot(\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2})=1\frac{1}{2}\sqrt{2}+1\frac{1}{2}i\sqrt{2}$ ∨ $z=3\cdot(-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2})=-1\frac{1}{2}\sqrt{2}-1\frac{1}{2}i\sqrt{2}$.

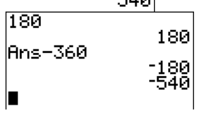
43c $z^3=27$ (op de positieve reële as en op afstand 27 van 0) heeft drie oplosssingen voor z in $\mathbb{C}$.
$z^3=27(\cos(0°)+i\sin(0°))$ ∨ $z^3=27(\cos(360°)+i\sin(360°))$ ∨ $z^3=27(\cos(720°)+i\sin(720°))$



$z=\sqrt[3]{27}(\cos(0°)+i\sin(0°))$ ∨ $z=\sqrt[3]{27}(\cos(120°)+i\sin(120°))$ ∨ $z=\sqrt[3]{27}(\cos(240°)+i\sin(240°))$
$z=3\cdot(1-i\cdot 0)=3$ ∨ $z=3\cdot(-\frac{1}{2}+\frac{1}{2}i\sqrt{3})=-1\frac{1}{2}+1\frac{1}{2}i\sqrt{3}$ ∨ $z=3\cdot(-\frac{1}{2}-\frac{1}{2}i\sqrt{3})=-1\frac{1}{2}-1\frac{1}{2}i\sqrt{3}$.

43d $z^4=-81$ (op de negatieve reële as en op afstand 81 van 0) heeft vier oplosssingen voor z in $\mathbb{C}$.
$z^4=81(\cos(180°)+i\sin(180°))$ ∨ $z^4=81(\cos(540°)+i\sin(540°))$ ∨
$z^4=81(\cos(-180°)+i\sin(-180°))$ ∨ $z^4=81(\cos(-540°)+i\sin(-540°))$





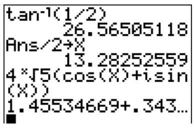
$z=3(\cos(45°)+i\sin(45°))$ ∨ $z=3(\cos(135°)+i\sin(135°))$ ∨
$z=3(\cos(-45°)+i\sin(-45°))$ ∨ $z=3(\cos(-135°)+i\sin(-135°))$
$z=3\cdot(\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2})$ ∨ $z=3\cdot(-\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2})$ ∨ $z=3\cdot(\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2})$ ∨ $z=3\cdot(-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2})$
$z=1\frac{1}{2}\sqrt{2}+1\frac{1}{2}i\sqrt{2}$ ∨ $z=-1\frac{1}{2}\sqrt{2}+1\frac{1}{2}i\sqrt{2}$ ∨ $z=1\frac{1}{2}\sqrt{2}-1\frac{1}{2}i\sqrt{2}$ ∨ $z=-1\frac{1}{2}\sqrt{2}-1\frac{1}{2}i\sqrt{2}$.

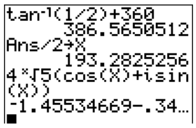
44a $z=\sqrt{2+i} \Rightarrow z^2 = 2+i$ (met $\text{Arg}(z^2)=\tan^{-1}(\frac{1}{2}) \approx 26{,}6°$ en z^2 op afstand $\sqrt{5}$ van 0) heeft twee oplosssingen voor z in $\mathbb{C}$.

$z^2 \approx \sqrt{5}\,(\cos(26{,}6°)+i\sin(26{,}6°)) \quad \vee \quad z^2 \approx \sqrt{5}\,(\cos(386{,}6°)+i\sin(386{,}6°))$

$z \approx \sqrt{\sqrt{5}}\,(\cos(13{,}3°)+i\sin(13{,}3°)) \quad \vee \quad z \approx \sqrt[4]{5}\,(\cos(193{,}3°)+i\sin(193{,}3°))$

$z \approx 1{,}46+0{,}34i \quad \vee \quad z \approx -1{,}46-0{,}34i.$



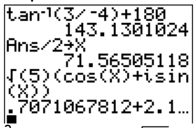


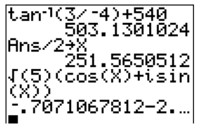
44b $z=\sqrt{-4+3i} \Rightarrow z^2 = -4+3i$ in II (met $\text{Arg}(z^2)=\tan^{-1}(\frac{3}{-4})+180° \approx 143{,}1°$ en z^2 op afstand 5 van 0).

$z^2 \approx 5(\cos(143{,}1°)+i\sin(143{,}1°)) \quad \vee \quad z^2 \approx 5(\cos(503{,}1°)+i\sin(503{,}1°))$

$z \approx \sqrt{5}\,(\cos(71{,}6°)+i\sin(71{,}6°)) \quad \vee \quad z \approx \sqrt{5}\,(\cos(251{,}6°)+i\sin(251{,}6°))$

$z \approx 0{,}71+2{,}12i \quad \vee \quad z \approx -0{,}71-2{,}12i.$



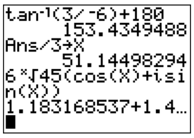


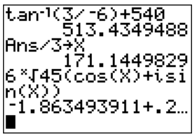
44c $z=\sqrt[3]{-6+3i} \Rightarrow z^3 = -6+3i$ in II (met $\text{Arg}(z^3)=\tan^{-1}(\frac{3}{-6})+180° \approx 153{,}4°$ en z^3 op afstand $\sqrt{45}$ van 0).

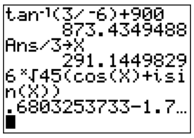
$z^3 \approx \sqrt{45}\,(\cos(153{,}4°)+i\sin(153{,}4°)) \vee z^3 \approx \sqrt{45}\,(\cos(513{,}4°)+i\sin(513{,}4°)) \vee z^3 \approx \sqrt{45}\,(\cos(873{,}4°)+i\sin(873{,}4°))$

$z \approx \sqrt[6]{45}\,(\cos(51{,}1°)+i\sin(51{,}1°)) \quad \vee \quad z \approx \sqrt[6]{45}\,(\cos(171{,}1°)+i\sin(171{,}1°)) \quad \vee \quad z \approx \sqrt[6]{45}\,(\cos(291{,}1°)+i\sin(291{,}1°))$

$z \approx 1{,}18+1{,}47i \quad \vee \quad z \approx -1{,}86+0{,}29i \quad \vee \quad z \approx 0{,}68-1{,}76i.$



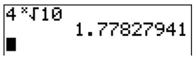




44d $z=\sqrt[4]{10} \Rightarrow z^4 = 10$ (met $\text{Arg}(z^4)=0°$ en z^4 op afstand 10 van 0).

$z^4 = 10(\cos(0°)+i\sin(0°)) \vee z^4 = \sqrt{10}\,(\cos(360°)+i\sin(360°)) \vee$

$z^4 = 10(\cos(720°)+i\sin(720°)) \vee z^4 = 10(\cos(1080°)+i\sin(1080°))$

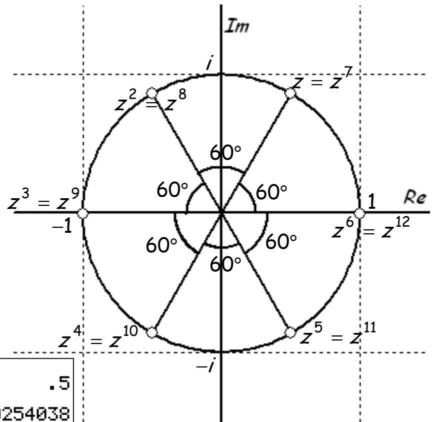


$z=\sqrt[4]{10}\,(\cos(0°)+i\sin 0°)) \approx 1{,}78(1+i\cdot 0) \approx 1{,}78 \vee z=\sqrt[4]{10}\,(\cos(90°)+i\sin(90°)) \approx 1{,}78(0+i\cdot 1) \approx 1{,}78i \quad \vee$

$z=\sqrt[4]{10}\,(\cos(180°)+i\sin(180°)) \approx 1{,}78(-1+i\cdot 0) \approx -1{,}78 \vee z=\sqrt[4]{10}\,(\cos(270°)+i\sin(270°)) \approx 1{,}78(0+i\cdot -1) \approx -1{,}78i.$

45a $z=\frac{1}{2}+\frac{1}{2}i\sqrt{3} = \cos(60°)+i\sin(60°)$ (met $\text{Arg}(z)=60°$ en z op de eenheidscirkel).

Zie het complexe vlak hiernaast. ($\arg(z^2)=2\text{Arg}(z)$, $\arg(z^3)=3\text{Arg}(z)$, ...)



45b $z=\frac{1}{2}+\frac{1}{2}i\sqrt{3} \Rightarrow |z| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\sqrt{3}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1 \Rightarrow \left|z^n\right| = |z|^n = 1^n = 1.$

Of $z=\frac{1}{2}+\frac{1}{2}i\sqrt{3} = 1\cdot(\cos(60°)+i\sin(60°))$ ligt op de eenheidscirkel.

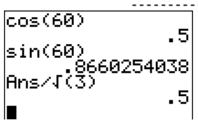
Dus ook $z^n = \cos(n\cdot 60°)+i\sin(n\cdot 60°)$ ligt op de eenheidscirkel.

45c $z^2 = -\frac{1}{2}+\frac{1}{2}i\sqrt{3} = \cos(120°)+i\sin(120°)$ (aflezen in het complexe vlak).

$z^2 = \cos(120°)+i\sin(120°) \quad \vee \quad z^2 = \cos(120°+360°)+i\sin(120°+360°)$

$z = \cos(60°)+i\sin(60°) \quad \vee \quad z = \cos(60°+180°)+i\sin(60°+180°)$

$z=\frac{1}{2}+i\cdot\frac{1}{2}\sqrt{3} = \frac{1}{2}+\frac{1}{2}i\sqrt{3} \quad \vee \quad z=-\frac{1}{2}+i\cdot -\frac{1}{2}\sqrt{3} = -\frac{1}{2}-\frac{1}{2}i\sqrt{3}.$



45d $z^4 = -1 = \cos(180°)+i\sin(180°)$ (aflezen in het complexe vlak).

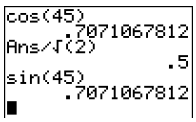
$z^4 = \cos(180°)+i\sin(180°) \quad \vee \quad z^4 = \cos(180°+360°)+i\sin(180°+360°) \quad \vee$

$z^4 = \cos(180°+720°)+i\sin(180°+720°) \quad \vee \quad z^4 = \cos(180°+1080°)+i\sin(180°+1080°)$

$z = \cos(45°)+i\sin(45°) \quad \vee \quad z = \cos(45°+90°)+i\sin(45°+90°) \quad \vee$

$z = \cos(45°+180°)+i\sin(45°+180°) \quad \vee \quad z = \cos(45°+270°)+i\sin(45°+270°)$

$z=\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2} \quad \vee \quad z=-\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2} \quad \vee \quad z=-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2} \quad \vee \quad z=\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2}.$



46a $(z-1)^2 = \frac{1}{2}+\frac{1}{2}i\sqrt{3} = \cos(60°)+i\sin(60°)$ (aflezen in het complexe vlak).

$(z-1)^2 = \cos(60°)+i\sin(60°) \quad \vee \quad (z-1)^2 = \cos(60°+360°)+i\sin(60°+360°)$

$z-1 = \cos(30°)+i\sin(30°) \quad \vee \quad z-1 = \cos(30°+180°)+i\sin(30°+180°)$

$z-1 = \frac{1}{2}\sqrt{3}+\frac{1}{2}i \quad \vee \quad z-1 = -\frac{1}{2}\sqrt{3}-\frac{1}{2}i$

$z = 1+\frac{1}{2}\sqrt{3}+\frac{1}{2}i \quad \vee \quad z = 1-\frac{1}{2}\sqrt{3}-\frac{1}{2}i.$

46b $(z-1-i)^2 = -i = \cos(270°)+i\sin(270°)$ (aflezen in het complexe vlak).

$(z-1-i)^2 = \cos(270°)+i\sin(270°) \quad \vee \quad (z-1-i)^2 = \cos(270°+360°)+i\sin(270°+360°)$

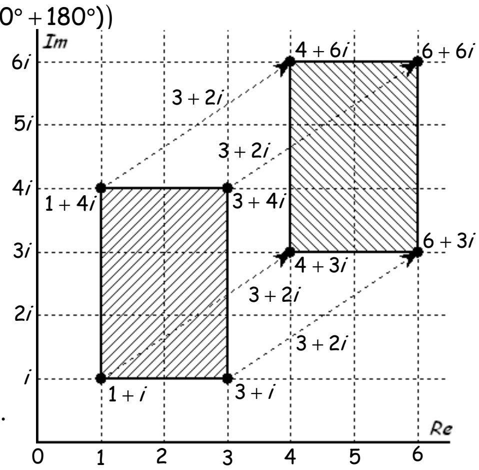
$z-1-i = \cos(135°)+i\sin(135°) \quad \vee \quad z-1-i = \cos(135°+180°)+i\sin(135°+180°)$

$z-1-i = -\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{2} \quad \vee \quad z-1-i = \frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2}$

$z = 1-\frac{1}{2}\sqrt{2}+i(1+\frac{1}{2}\sqrt{2}) \quad \vee \quad z = 1+\frac{1}{2}\sqrt{2}+i(1-\frac{1}{2}\sqrt{2}).$

46c $z^2-4z+4=(z-2)^2=\frac{1}{2}-\frac{1}{2}i\sqrt{3}=\cos(-60°)+i\sin(-60°)$ (aflezen in het complexe vlak).
$(z-2)^2=\cos(-60°)+i\sin(-60°) \quad \vee \quad (z-2)^2=\cos(-60°+360)+i\sin(-60°+360)$
$z-2=\cos(-30°)+i\sin(-30°) \quad \vee \quad z-2=\cos(-30°+180°)+i\sin(-30°+180°)$
$z-2=\frac{1}{2}\sqrt{3}-\frac{1}{2}i \quad \vee \quad z-2=-\frac{1}{2}\sqrt{3}+\frac{1}{2}i$
$z=2+\frac{1}{2}\sqrt{3}-\frac{1}{2}i \quad \vee \quad z=2-\frac{1}{2}\sqrt{3}+\frac{1}{2}i.$

46d $\boxed{z^2-6z}+10=\boxed{(z-3)^2-9}+10=(z-3)^2+1=i\sqrt{3}\Rightarrow(z-3)^2=-1+i\sqrt{3}=2(\cos(120°)+i\sin(120°)).$
$(z-3)^2=2(\cos(120°)+i\sin(120°)) \quad \vee \quad (z-3)^2=2(\cos(120°+360)+i\sin(120°+360))$
$z-3=\sqrt{2}(\cos(60°)+i\sin(60°)) \quad \vee \quad z-3=\sqrt{2}(\cos(60°+180°)+i\sin(60°+180°))$
$z-3=\sqrt{2}(\frac{1}{2}+\frac{1}{2}i\sqrt{3}) \quad \vee \quad z-3=\sqrt{2}(-\frac{1}{2}-\frac{1}{2}i\sqrt{3})$
$z-3=\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{6} \quad \vee \quad z-3=-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{6}$
$z=3+\frac{1}{2}\sqrt{2}+\frac{1}{2}i\sqrt{6} \quad \vee \quad z=3-\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{6}.$

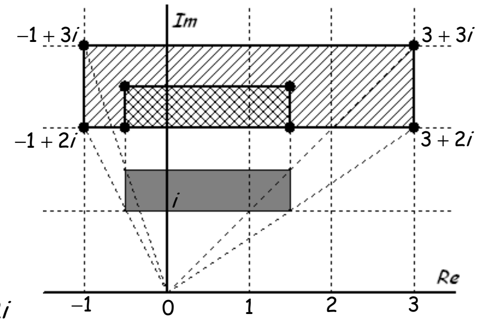


47a Zie de linker rechthoek in het complexe vlak hiernaast.

47b $z=1+i\Rightarrow z+3+2i=1+i+3+2i=4+3i.$
$z=3+i\Rightarrow z+3+2i=3+i+3+2i=6+3i.$
$z=3+4i\Rightarrow z+3+2i=3+4i+3+2i=6+6i.$
$z=1+4i\Rightarrow z+3+2i=1+4i+3+2i=4+6i.$

47c Zie de rechter rechthoek in het complexe vlak hiernaast.
De oorspronkelijke rechthoek is 3 naar rechts en 2 omhoog verschoven.

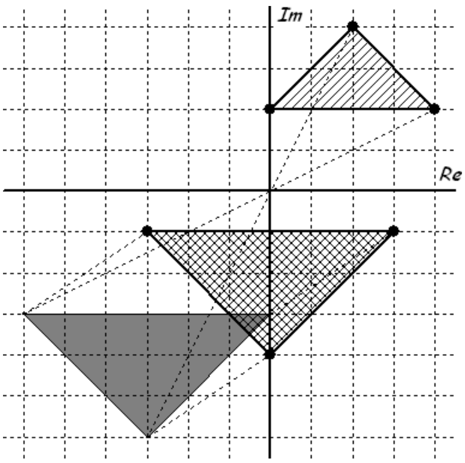
48a Bij $f(z)=\frac{1}{2}z+i$ hoort een vermenigvuldiging met $\frac{1}{2}$ t.o.v. $z=0$
(zie de grijs gemarkeerde rechthoek) gevolgd door de translatie (0, 1)
(zie de dubbel geaceerde rechthoek in de figuur hiernaast).



48b $f(z)=0\Rightarrow\frac{1}{2}z+i=0\Rightarrow\frac{1}{2}z=-i\Rightarrow z=-2i$ ($-2i$ is het nulpunt van $f(z)$).

48c $f(z)=z\Rightarrow\frac{1}{2}z+i=z\Rightarrow-\frac{1}{2}z=-i\Rightarrow z=2i$ ($2i$ is het dekpunt van $f(z)$).

49



49a Bij $f(z)=-1\frac{1}{2}z+3+2i$
hoort een vermenigvuldiging met $-1\frac{1}{2}$ t.o.v. $z=0$
(zie de grijs gemarkeerde driehoek)
gevolgd door de translatie (3, 2)
(zie de dubbel geaceerde driehoek in de figuur hiernaast).

49b $f(z)=0$
$-1\frac{1}{2}z+3+2i=0$
$-1\frac{1}{2}z=-3-2i$
$-3z=-6-4i$
$z=2+\frac{4}{3}i.$

49c $f(z)=z$
$-1\frac{1}{2}z+3+2i=z$
$-2\frac{1}{2}z=-3-2i$
$-5z=-6-4i$
$z=\frac{6}{5}+\frac{4}{5}i.$

50a $f(z)=0$
$3z+2-4i=0$
$3z=-2+4i$
$z=-\frac{2}{3}+\frac{4}{3}i$ (nulpunt).

$f(z)=z$
$3z+2-4i=z$
$2z=-2+4i$
$z=-1+2i$ (dekpunt).

50b $g(z)=0$
$\frac{1}{3}z+5=0$
$\frac{1}{3}z=-5$
$z=-15$ (nulpunt).

$g(z)=z$
$\frac{1}{3}z+5=z$
$-\frac{2}{3}z=-5$
$-2z=-15\Rightarrow z=\frac{-15}{-2}$ (dekpunt).

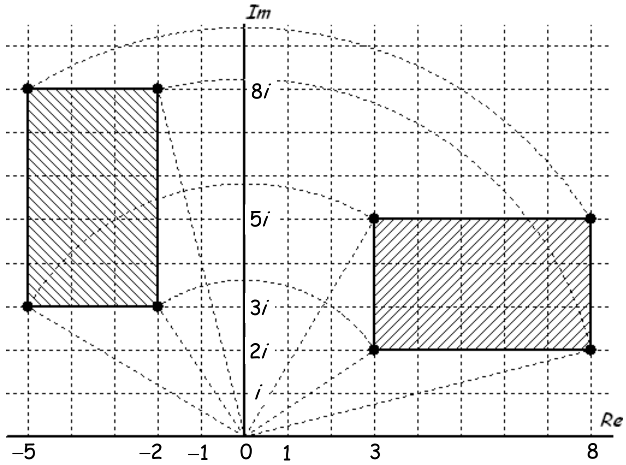
51a $f(z)=z\Rightarrow az+5-2i=z\Rightarrow az-z=-5+2i\Rightarrow(a-1)z=-5+2i\Rightarrow z=\frac{-5+2i}{a-1}$ dus geen dekpunt voor $a=1$.

51b $f(z)=0\Rightarrow az+5-2i=0\Rightarrow az=-5+2i\Rightarrow z=\frac{-5+2i}{a}$ dus geen nulpunt voor $a=0$ (de noemer wordt dan 0).

52a $f(1+2i)=0\Rightarrow3\cdot(1+2i)+a+bi=0\Rightarrow3+6i+a+bi=0\Rightarrow(3+a)+(6+b)i=0\Rightarrow a=-3\wedge b=-6.$

52b $f(1+2i)=1+2i\Rightarrow3\cdot(1+2i)+a+bi=1+2i\Rightarrow(3+a)+(6+b)i=1+2i\Rightarrow3+a=1\wedge6+b=2\Rightarrow a=-2\wedge b=-4.$

53a Zie de rechter rechthoek in het complexe vlak hiernaast.

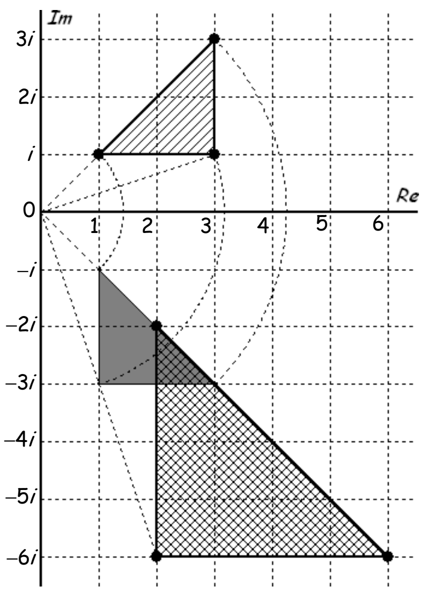


53b $i(3+2i) = 3i + 2i^2 = 3i - 2 = -2 + 3i$ (zie de figuur hiernaast)
$i(8+2i) = 8i + 2i^2 = 8i - 2 = -2 + 8i$ (zie de figuur hiernaast)
$i(8+5i) = 8i + 5i^2 = 8i - 5 = -5 + 8i$ (zie de figuur hiernaast)
$i(3+5i) = 3i + 5i^2 = 3i - 5 = -5 + 3i$ (zie de figuur hiernaast).

53c $|iz| = |i \cdot z| = |i| \cdot |z| = 1 \cdot |z| = |z|$ en
$\arg(iz) = \arg(i \cdot z) = \text{Arg}(i) + \text{Arg}(z) = 90° + \text{Arg}(z)$.
Dus een rotatie om $z = 0$ over $90°$ (tegen de klok in).

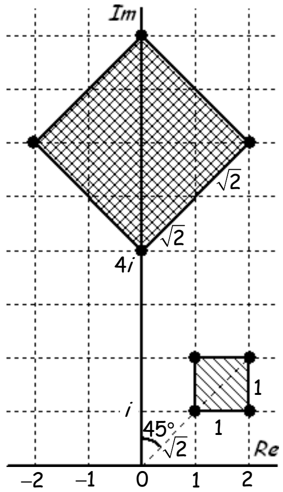
53d $\frac{1}{2} + \frac{1}{2}\sqrt{3} = \cos(60°) + i\sin(60°)$ (ligt dus op de eenheidscirkel).
$\left|(\frac{1}{2} + \frac{1}{2}\sqrt{3}) \cdot z\right| = \left|\frac{1}{2} + \frac{1}{2}\sqrt{3}\right| \cdot |z| = 1 \cdot |z| = |z|$ en
$\arg\left((\frac{1}{2} + \frac{1}{2}\sqrt{3}) \cdot z\right) = \text{Arg}(\frac{1}{2} + \frac{1}{2}\sqrt{3}) + \text{Arg}(z) = 60° + \text{Arg}(z)$.
Dus een rotatie om $z = 0$ over $60°$ (tegen de klok in).

54a Zie de bovenste driehoek in het complexe vlak hiernaast.



54b $-2i(1+i) = -2i - 2i^2 = -2i + 2 = 2 - 2i$,
$-2i(3+i) = -6i - 2i^2 = -6i + 2 = 2 - 6i$ en
$-2i(3+3i) = -6i - 6i^2 = -6i + 6 = 6 - 6i$.
(zie de dubbel gearceerde driehoek in de figuur hiernaast).

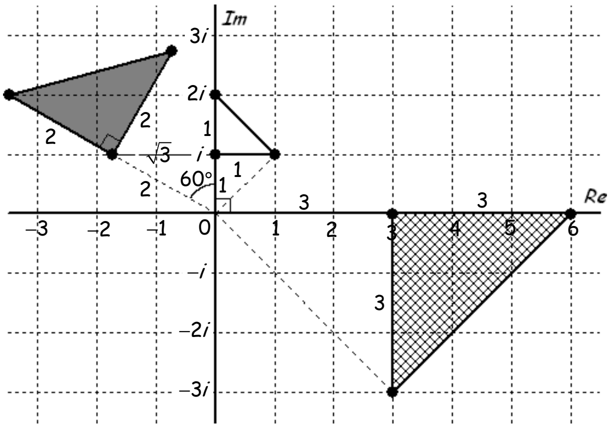
54c $|-2iz| = |-2i \cdot z| = |-2i| \cdot |z| = 2 \cdot |z|$ en
$\arg(-2iz) = \arg(-2i \cdot z) = \text{Arg}(-2i) + \text{Arg}(z) = -90° + \text{Arg}(z)$.
Dus een rotatie om $z = 0$ over $-90°$ (met de klok mee)
en een vermenigvuldiging t.o.v. $z = 0$ met 2.
(eerst vermenigvuldigen en dan roteren mag ook)



55a $|2+2i| = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ en $\text{Arg}(2+2i) = 45°$.
Dus een rotatie om $z = 0$ over $45°$ en
een vermenigvuldiging t.o.v. $z = 0$ met $2\sqrt{2}$.
$1 \le \text{Re}(z) \le 2 \;\wedge\; 1 \le \text{Im}(z) \le 2$ is in het complexe vlak het gearceerde vierkant met zijde 1.
(in een vierkant met zijde 1 is de diagonaal $\sqrt{2}$ lang)
Het beeld is het dubbel gearceerde vierkant.
Berekeningen: $\sqrt{2} \cdot 2\sqrt{2} = 4$ en $1 \cdot 2\sqrt{2} = 2\sqrt{2}$.

55b $|2+2i| = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ en $\text{Arg}(2+2i) = 45°$.
Dus de functie $f(z) = (2+2i)z$ geeft bij
het domein $1 \le |z| \le 2 \;\wedge\; 30° \le \text{Arg}(z) \le 60°$
het bereik $1 \cdot 2\sqrt{2} \le |z| \le 2 \cdot 2\sqrt{2} \;\wedge\; 30° + 45° \le \text{Arg}(z) \le 60° + 45°$, dus $2\sqrt{2} \le |z| \le 4\sqrt{2} \;\wedge\; 75° \le \text{Arg}(z) \le 105°$.

56a $\left|1 + i\sqrt{3}\right| = \sqrt{(1)^2 + \sqrt{3}^2} = \sqrt{1+3} = 2$
en $\text{Arg}(1 + i\sqrt{3}) = \tan^{-1}(\frac{\sqrt{3}}{1}) = \tan^{-1}(\sqrt{3}) = 60°$.
(zie de grijs gemarkeerde driehoek hieronder)

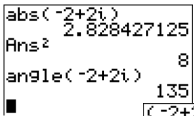


56b $|-3i| = \sqrt{3^2} = 3$ en $\text{Arg}(-3i) = -90°$.
(zie dubbel gearceerde driehoek in de figuur onder 56a)

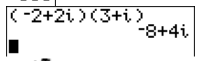
57a $\left|\sqrt{3} - i\right| = \sqrt{3+1} = 2$ en
$\arg(\sqrt{3} - i) = \tan^{-1}(\frac{-1}{\sqrt{3}}) = -30°$.
Domein $10 \le |z| \le 20 \;\wedge\; 90° \le \text{Arg}(z) \le 180°$
geeft bij de functie $f(z) = (\sqrt{3} - i)z$ als
bereik $20 \le |z| \le 40 \;\wedge\; 60° \le \text{Arg}(z) \le 150°$.

57b Domein $|z| \ge 3 \;\wedge\; 0° \le \text{Arg}(z) \le 90°$
geeft bij de functie $f(z) = (\sqrt{3} - i)z$
als bereik $|z| \ge 6 \;\wedge\; -30° \le \text{Arg}(z) \le 60°$.

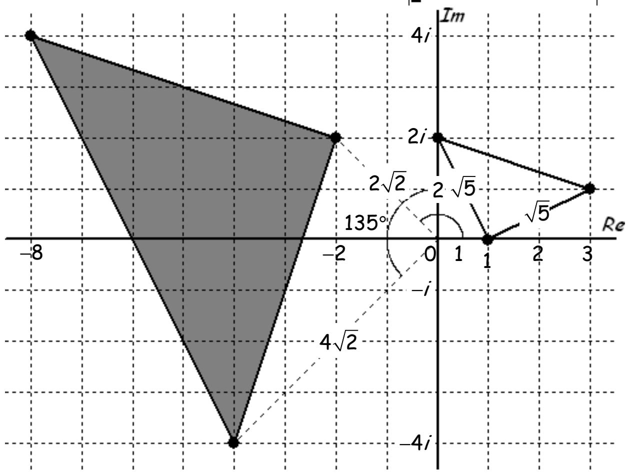
58a $|-2+2i| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ en $\text{Arg}(-2+2i) = 135°$.
Dus een rotatie om $z = 0$ over 135° en een vermenigvuldiging t.o.v. $z = 0$ met $2\sqrt{2}$.
Of bereken de drie beeldpunten van de drie hoekpunten:



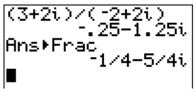
$f(1) = (-2+2i) \cdot 1 = -2+2i$, $\quad f(3) = (-2+2i) \cdot (3+i) = -6-2i+6i+2i^2 = -6+4i-2 = -8+4i$ en



$f(2i) = (-2+2i) \cdot 2i = -4i + 4i^2 = -4i - 4 = -4 - 4i$.
Zie de grijs gemarkeerde driehoek in de figuur hiernaast.



58b $f(z) = 3 + 2i$
$(-2+2i)z = 3 + 2i$
$z = \frac{3+2i}{-2+2i} = \frac{3+2i}{-2+2i} \cdot \frac{-2-2i}{-2-2i}$
$= \frac{-6-6i-4i-4i^2}{4+4} = \frac{-6-10i+4}{8}$
$= \frac{-2-10i}{8} = -\frac{1}{4} - \frac{5}{4}i.$

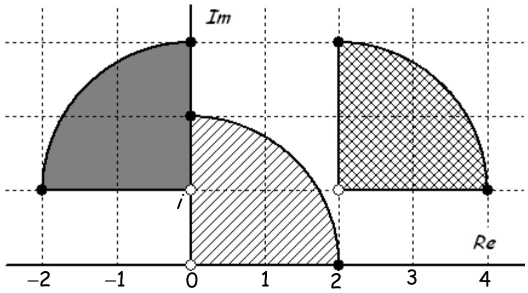


58c $f(z) = f(\frac{1}{z})$
$(-2+2i)z = (-2+2i)\frac{1}{z}$
$z = \frac{1}{z}$
$z^2 = 1$
$z = 1 \quad \vee \quad z = -1.$

59a Zie de kwartcirkel met middelpunt $z = 0$ in de figuur hiernaast.

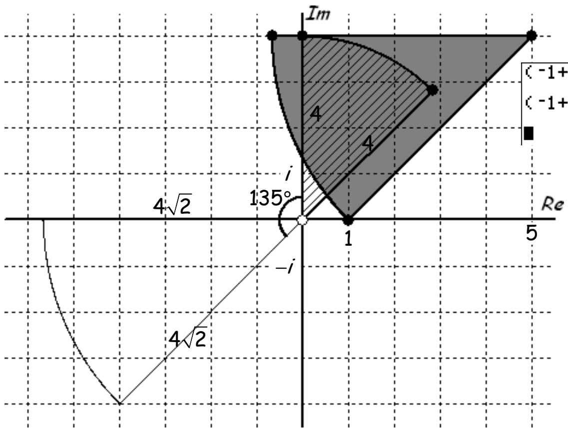
59b De kwartcirkel (uit 59a) 2 naar rechts en 1 omhoog verschuiven.
(zie de dubbel gearceerde kwartcirkel in de figuur hiernaast)

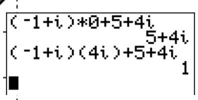
59c De kwartcirkel (uit 59a) eerst vermenigvuldigen met i
met $|i| = 1$ en $\text{Arg}(i) = 90°$.(d.i. een rotatie om $z = 0$ over $\text{Arg}(i) = 90°$)
en daarna 1 omhoog verschuiven. (zie de grijs gemarkeerde kwartcirkel)

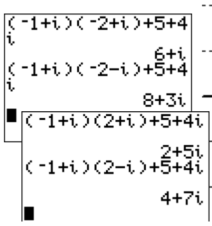


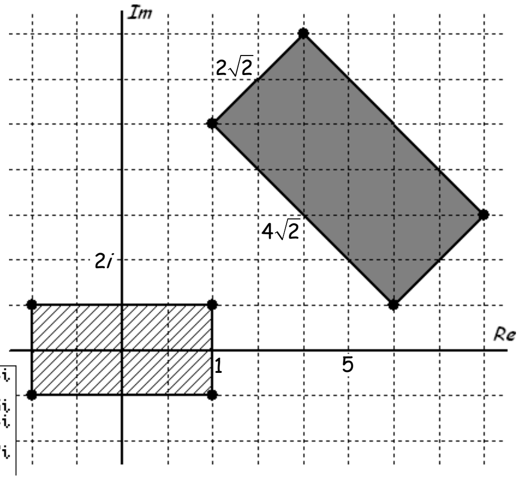
60a $f(z) = 10 + i$
$(-1+i)z + 5 + 4i = 10 + i \Rightarrow (-1+i)z = 5 - 3i$
$z = \frac{5-3i}{-1+i} = \frac{5-3i}{-1+i} \cdot \frac{-1-i}{-1-i} = \frac{-5-5i+3i+3i^2}{1+1} = \frac{-5-2i-3}{2} = \frac{-8-2i}{2} = -4 - i.$

60b $|-1+i| = \sqrt{2}$ en $\text{Arg}(-1+i) = 135°$ dus vermenigvuldiging met $\sqrt{2}$, rotatie over t.o.v. 0 over 135° en translatie (5, 4).
Het beeld van $|z| \le 4 \quad \wedge \quad 45° \le \text{Arg}(z) \le 90°$ is de grijs gemarkeerde cirkelsector in de figuur hieronder.









60c Het beeld van $-2 \le \text{Re}(z) \le 2 \quad \wedge \quad -1 \le \text{Im}(z) \le 1$ is de grijs gemarkeerde rechthoek in de figuur hierboven.

60d De beeldfiguur van vierkant V heeft zijde $\sqrt{10} \Rightarrow V$ heeft zijde $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5} \Rightarrow \text{Opp.}(V) = \sqrt{5} \cdot \sqrt{5} = 5.$

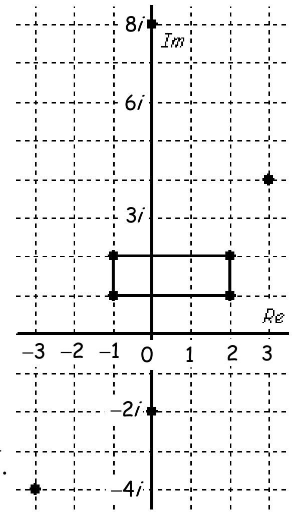
61a $f(z) = 0$
$(1+i\sqrt{3})z - 2 + i = 0$
$(1+i\sqrt{3})z = 2 - i$
$z = \frac{2-i}{1+i\sqrt{3}} = \frac{2-i}{1+i\sqrt{3}} \cdot \frac{1-i\sqrt{3}}{1-i\sqrt{3}} = \frac{2-2i\sqrt{3}-i-\sqrt{3}}{4}$
$= \frac{2-\sqrt{3}}{4} - \frac{i+2i\sqrt{3}}{4} = \frac{1}{2} - \frac{1}{4}\sqrt{3} - (\frac{1}{4} + \frac{1}{2}\sqrt{3})i$ (nulpunt).

$f(z) = z$
$(1+i\sqrt{3})z - 2 + i = z$
$i\sqrt{3} \cdot z = 2 - i$
$z = \frac{2-i}{i\sqrt{3}} = \frac{2-i}{i\sqrt{3}} \cdot \frac{-i\sqrt{3}}{-i\sqrt{3}} = \frac{-2i\sqrt{3}-\sqrt{3}}{3} = -\frac{1}{3}\sqrt{3} - \frac{2}{3}i\sqrt{3}$ (dekpunt).

61b $g(z)=0$
$-2iz+1-3i=0$
$-2iz=-1+3i$
$z=\frac{-1+3i}{-2i}=\frac{-1+3i}{-2i}\cdot\frac{i}{i}=\frac{-i-3}{2}=-\frac{3}{2}-\frac{1}{2}i$ (nulpunt).

$g(z)=z$
$-2iz+1-3i=z$
$(-1-2i)z=-1+3i$
$z=\frac{-1+3i}{-1-2i}=\frac{-1+3i}{-1-2i}\cdot\frac{-1+2i}{-1+2i}=\frac{1-2i-3i-6}{5}=-1-i$ (dekpunt).

62a Zie de rechthoek in de figuur hiernaast.



62b Zie de vier punten in de figuur hiernaast. (hieronder staat de berekening)
$(-1+i)^2=(-1+i)\cdot(-1+i)=1-2i-1=-2i.$
$(2+i)^2=(2+i)\cdot(2+i)=4+4i-1=3+4i.$
$(2+2i)^2=(2+2i)\cdot(2+2i)=4+8i-4=8i.$
$(-1+2i)^2=(-1+2i)\cdot(-1+2i)=1-4i-4=-3-4i.$

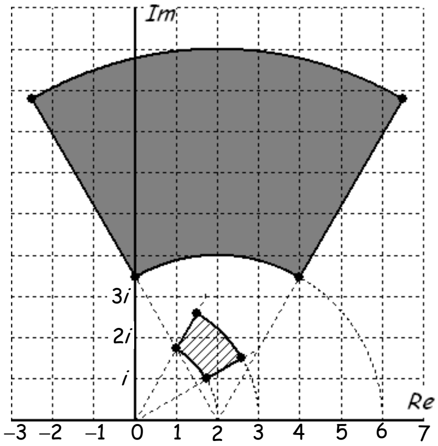
```
(-1+i)²          -2i
(2+i)²          3+4i
(2+2i)²           8i
(-1+2i)²       -3-4i
```

62c Nee, want $-2i$, $3+4i$, $8i$ en $-3-4i$ zijn niet de hoekpunten van een rechthoek.

63a $f(z)=0$
$z^2+2=0$
$z^2=-2$
$z^2=2i^2$
$z=i\sqrt{2}\ \vee\ z=-i\sqrt{2}.$
(de nulpunten van f)

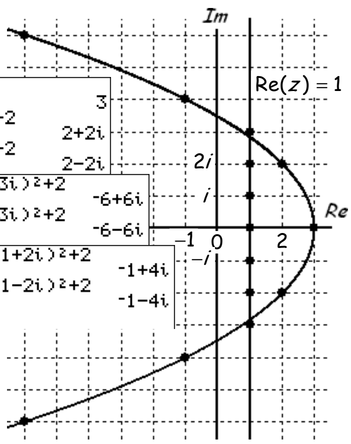
63b $f(z)=z$
$z^2+2=z$
$\boxed{z^2-z}+2=0$
$\boxed{(z-\frac{1}{2})^2-\frac{1}{4}}+2=0$
$(z-\frac{1}{2})^2+1\frac{3}{4}=0$
$(z-\frac{1}{2})^2=-\frac{7}{4}$
$(z-\frac{1}{2})^2=\frac{7}{4}i^2$
$z-\frac{1}{2}=\pm\frac{1}{2}i\sqrt{7}$
$z=\frac{1}{2}+\frac{1}{2}i\sqrt{7}\ \vee\ z=\frac{1}{2}-\frac{1}{2}i\sqrt{7}.$
(de dekpunten van f)

63c Zie de figuur hiernaast. (vlakdeel en beeld in één figuur)
$|z^2|=|z|^2$ en $\arg(z^2)=2\text{Arg}(z)$.
$|z|$ eerst kwadrateren en $\text{Arg}(z)$ verdubbelen,
daarna nog 2 naar rechts verschuiven.



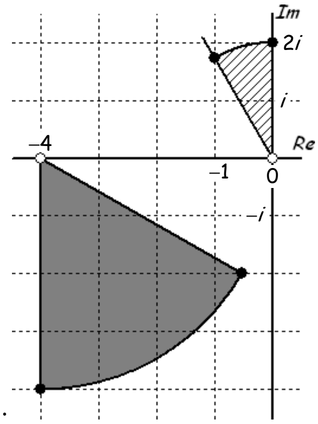
63d $f(1)=1^2+2=3$
$f(1+i)=(1+i)^2+2=2+2i$
$f(1-i)=(1-i)^2+2=2-2i$
$f(1+2i)=(1+2i)^2+2=-1+4i$
$f(1-2i)=(1-2i)^2+2=-1-4i$
$f(1+3i)=(1+3i)^2+2=-6+6i$
$f(1-3i)=(1-3i)^2+2=-6-6i.$
Zie de figuur hiernaast.
($\text{Re}(z)=1$ en de parabool in één figuur)

```
1²+2                3
(1+i)²+2         2+2i
(1-i)²+2         2-2i
(1+3i)²+2       -6+6i
(1-3i)²+2       -6-6i
(1+2i)²+2       -1+4i
(1-2i)²+2       -1-4i
```



64a $f(z)=0$
$iz^2-4=0$
$iz^2=4$ (keer $-i$)
$z^2=-4i=4\cdot -i=4(\cos(-90°)+i\sin(-90°))\ \vee\ z^2=4(\cos(270°)+i\sin(270°))$
$z=2(\cos(-45°)+i\sin(-45°))=2\left(\frac{1}{2}\sqrt{2}+i\cdot-\frac{1}{2}\sqrt{2}\right)=\sqrt{2}-i\sqrt{2}\ \vee$
$z=2(\cos(135°)+i\sin(135°))=2\left(-\frac{1}{2}\sqrt{2}+i\cdot\frac{1}{2}\sqrt{2}\right)=-\sqrt{2}+i\sqrt{2}.$

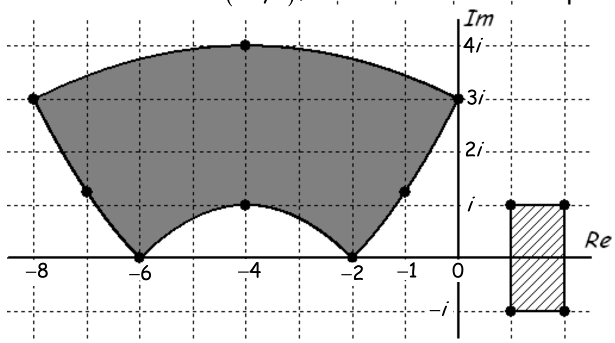
```
-90/2               -45
Ans+360/2           135
```



64b Zie de figuur hiernaast. (vlakdeel en beeld in één figuur)
$|i\cdot z^2|=|i|\cdot|z|^2=|z|^2$ en $\arg(i\cdot z^2)=\text{Arg}(i)+\arg(z^2)=90°+2\text{Arg}(z)$.
$|z|$ kwadrateren, $\text{Arg}(z)$ verdubbelen er nog eens 90° bij optellen en translatie $(-4,0)$.

64c $f(1-i)=i(1-i)^2-4=-2$
$f(2-i)=i(2-i)^2-4=3i$
$f(2+i)=i(2+i)^2-4=-8+3i$
$f(1+i)=i(1+i)^2-4=-6$
$f(2)=i\cdot 2^2-4=-4+4i$
$f(1)=i\cdot 1^2-4=-4+i$
$f(1\frac{1}{2}+i)=i(1\frac{1}{2}+i)^2-4=-7+1\frac{1}{4}i$
$f(1\frac{1}{2}-i)=i(1\frac{1}{2}-i)^2-4=-1+1\frac{1}{4}i$. Zie de figuur hiernaast.

```
i(1-i)²-4           -2
i(2-i)²-4           3i
i(2+i)²-4        -8+3i
i(1+i)²-4           -6
i*2²-4           -4+4i
i*1²-4            -4+i
i(1.5+i)²-4   -7+1.25i
i(1.5-i)²-4   -1+1.25i
```



65a Het beeld $f(z) = -2-2i$ (andere snijpunt van kromme met lijnstuk) komt van $z = 1-i$ (snijpunt van $R(z) = \text{Im}(z)$ en $R(z) = 1$), want het beeld $f(z) = -2+2i$ (zie de cursor in de rechter helft) komt van $z = 1+i$ (zie de cursor in de linker helft).

65b $\left|z^3\right| = |z|^3$ en $\arg(z^3) = 3\text{Arg}(z)$.
Voor alle punten van een lijn door $z = 0$ geldt dus dat ze op één lijn liggen waarvan de hoek t.o.v. de reële as drie keer zo groot is geworden. Het beeld van de lijn is dus weer een rechte lijn.

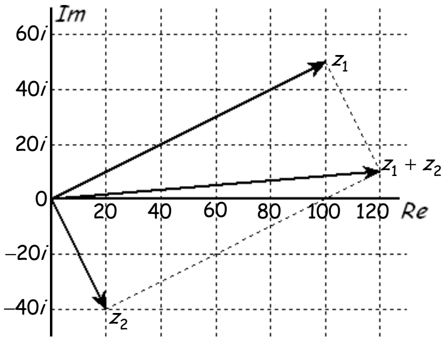
65c Voor de punten z^3 op de reële as geldt: $\text{Arg}(z^3) = 3\text{Arg}(z) = 0°$ of $\text{Arg}(z^3) = 3\text{Arg}(z) = 180°$.
Voor de punten z op $\text{Re}(z) = 1$ geldt: $-90° < \text{Arg}(z) < 90°$.
Voor de punten z op $\text{Re}(z) = 1$ die we zoeken is $\text{Arg}(z) = -60°$ of $\text{Arg}(z) = 60°$. Dus $z = 1 + i\sqrt{3} \lor z = 1 - i\sqrt{3}$.

66a Zie de vectoren in het complexe vlak hiernaast.

66b $z_1 + z_2 = (100+50i) + (20-40i) = 120+10i$.
$|z_1 + z_2| = |120+10i| = \sqrt{120^2 + 10^2} \approx 120{,}4$ en
$\arg(z_1 + z_2) = \text{Arg}(120+10i) \approx 4{,}8°$.

```
120²+10²
              14500
√(14500)
        120.4159458
abs(120+10i)
        120.4159458
```

```
angle(120+10i)
        4.763641691
```



67 $z_1 = 120,\ z_2 = 250\left(\cos(35°) + i\sin(35°)\right)$,
$z_3 = 200\left(\cos(100°) + i\sin(100°)\right)$ en
$z_4 = 180\left(\cos(170°) + i\sin(170°)\right)$.
$|z_1 + z_2 + z_3 + z_4| \approx 388$ Newton en $\text{Arg}(z_1 + z_2 + z_3 + z_4) \approx 73°$.

```
abs(120+250(cos(
35)+isin(35))+20
0(cos(100)+isin(
100))+180(cos(17
0)+isin(170)))
        388.3529083
```

```
angle(120+250(co
s(35)+isin(35))+
200(cos(100)+isi
n(100))+180(cos(
170)+isin(170)))
        73.11573679
```

68 Bij de krachten $\vec{F}_z$, $\vec{F}_1$ en $\vec{F}_2$ (zie figuur 8.15 onder opdracht 69) horen de complexe getallen z_z, z_1 en z_2 met
$z_z = -1200i,\ z_1 = 600\left(\cos(66°) + i\sin(66°)\right)$ en $z_z + z_1 + z_2 = 0$.
$z_z + z_1 + z_2 = 0 \Rightarrow z_2 = -z_z - z_1 = 1200i - 600\left(\cos(66°) + i\sin(66°)\right)$.
De GR geeft tenslotte $|z_2| \approx 696$ (N) en $\text{Arg}(z_2) \approx 111°$.
Dus de spankracht is 696 Newton en $\alpha \approx 180° - 111° = 69°$.

```
abs(1200i-600(co
s(66)+isin(66)))
        696.0564208
```

```
angle(1200i-600(
cos(66)+isin(66)
))
        110.5244229
180-Ans
        69.4755771
```

69 Bij de krachten $\vec{F}_z$, $\vec{F}_1$, $\vec{F}_2$ en $\vec{F}_r$ horen de complexe getallen z_z, z_1, z_2 en z_r met
$z_z = -200i,\ z_1 = 300\left(\cos(25°) + i\sin(25°)\right),\ z_r = 300\left(\cos(70°) + i\sin(70°)\right)$ en $z_r = z_z + z_1 + z_2$.
$z_r = z_z + z_1 + z_2 \Rightarrow z_2 = z_r - z_z - z_1 = 300\left(\cos(70°) + i\sin(70°)\right) + 200i - 300\left(\cos(25°) + i\sin(25°)\right)$.
De GR geeft dan $|z_2| \approx 393$ (N) en $\text{Arg}(z_2) \approx 115°$.
Dus grootte van $\vec{F}_2$ is 393 Newton en $\alpha \approx 180° - 115° = 65°$.

```
abs(300(cos(70)+
isin(70))+200i-3
00(cos(25)+isin(
25)))
        393.4078068
```

```
angle(300(cos(70
)+isin(70))+200i
-300(cos(25)+isi
n(25)))
        115.4870705
180-Ans
        64.51292948
```

70 Bij de krachten $\vec{F}_z$, $\vec{F}_1$, $\vec{F}_2$ en $\vec{F}_t$ horen de complexe getallen
$z_z = -500i,\ z_1 = 300\left(\cos(-160°) + i\sin(-160°)\right),\ z_2 = 150\left(\cos(-40°) + i\sin(-40°)\right)$ met $z_z + z_1 + z_2 + z_t = 0$.
$z_z + z_1 + z_2 + z_t = 0 \Rightarrow z_t = -z_z - z_1 - z_2 = 500i - 300\left(\cos(-160°) + i\sin(-160°)\right) - 150\left(\cos(-40°) + i\sin(-40°)\right)$.
De GR geeft vervolgens $|z_t| \approx 719$ (N) en $\text{Arg}(z_t) \approx 77°$.
Dus grootte van $\vec{F}_2$ is 719 Newton en $\alpha \approx 90° - 77° = 13°$.

```
abs(500i-300(cos
(-160)+isin(-160
))-150(cos(-40)+
isin(-40)))
        718.6961698
```

```
angle(500i-300(c
os(-160)+isin(-1
60))-150(cos(-40
)+isin(-40)))
        76.56354546
90-Ans
        13.43645454
```

71 Bij de snelheden horen de complexe getallen $z_1 = 4\left(\cos(-155°) + i\sin(-155°)\right)$ en $z_2 = 3i$.
De GR geeft vervolgens $|z_1 + z_2| \approx 3{,}9$ (km/uur) en $\text{Arg}(z_1 + z_2) \approx 160°$.
Dus resulterende snelheid is 3,9 km/uur en de koers is 290°.
(de koersvan het Noorden is 0° en dan de graden tellen met de wijzers van de klok mee).

```
abs(4(cos(-155)+
isin(-155))+3i)
        3.854498893
```

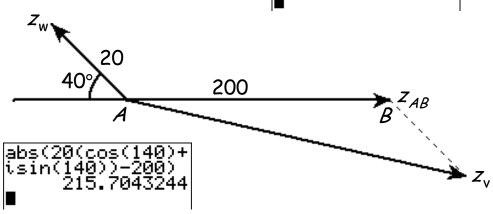
```
angle(4(cos(-155
)+isin(-155))+3i
)
        160.1389737
360-(160-90)
                290
```

72 Bij de snelheden horen de complexe getallen $z_w = 50\left(\cos(45°) + i\sin(45°)\right)$ en $z_v = 250\left(\cos(-80°) + i\sin(-80°)\right)$.
De GR geeft vervolgens $|z_w + z_v| \approx 225{,}08$ (km/uur).
Het vliegtuig doet er $\frac{1000}{\text{Ans}} \approx 4{,}443$ uur over. Dit is 4 uur en 27 minuten.

```
abs(50(cos(45)+i
sin(45))+250(cos
(-80)+isin(-80))
)
        225.079073
```

```
1000/Ans
        4.442883058
Ans-4
        .4428830579
Ans*60
        26.57298347
```

73 Bij de snelheden horen de complexe getallen
$z_w = 20\left(\cos(140°) + i\sin(140°)\right),\ z_{AB} = 200$ met $z_w + z_v = z_{AB}$.
$z_w + z_{AB} = z_v \Rightarrow z_v = z_{AB} - z_w = 20\left(\cos(140°) + i\sin(140°)\right) - 200$
De GR geeft vervolgens $|z_v| = |z_{AB} - z_w| \approx 216$ (km/uur).
De snelheid van het vliegtuig t.o.v. de omringende lucht is 216 km/uur.




```
abs(20(cos(140)+
isin(140))-200)
        215.7043244
```

Diagnostische toets

D1a $2x-1+2i=4x-2+4i$
$-2x=-1+2i$
$x=\frac{1}{2}-i.$

D1b $(2x+1)^2+9=0$
$(2x+1)^2=-9$
$(2x+1)^2=9i^2$
$2x+1=3i \quad \vee \quad 2x+1=-3i$
$2x=-1+3i \quad \vee \quad 2x=-1-3i$
$x=-\frac{1}{2}+\frac{3}{2}i \quad \vee \quad x=-\frac{1}{2}-\frac{3}{2}i.$

D1c $\boxed{x^2-4x}+10=0$
$\boxed{(x-2)^2-4}+10=0$
$(x-2)^2+6=0$
$(x-2)^2=-6$
$(x-2)^2=6i^2$
$x-2=i\sqrt{6} \quad \vee \quad x-2=-i\sqrt{6}$
$x=2+i\sqrt{6} \quad \vee \quad x=2-i\sqrt{6}.$

D1d $\boxed{4x^2+16x}+17=0$
$\boxed{(2x+4)^2-16}+17=0$
$(2x+4)^2+1=0$
$(2x+4)^2=-1$
$(2x+4)^2=i^2$
$2x+4=i \quad \vee \quad 2x+4=-i$
$2x=-4+i \quad \vee \quad 2x=-4-i$
$x=-2+\frac{1}{2}i \quad \vee \quad x=-2-\frac{1}{2}i.$

D2a $(2-i)-(5-8i)=2-i-5+8i=-3+7i.$

D2b $(6-2i)(3+2i)=18+12i-6i-4i^2=18+6i+4=22+6i.$

D2c $\frac{3-i}{4+i}=\frac{3-i}{4+i}\cdot\frac{4-i}{4-i}=\frac{12-3i-4i+i^2}{16-4i+4i-i^2}=\frac{12-7i-1}{16+1}=\frac{11-7i}{17}=\frac{11}{17}-\frac{7}{17}i.$

D2d $\frac{2-5i}{6i}=\frac{2-5i}{6i}\cdot\frac{-i}{-i}=\frac{-2i+5i^2}{-6i^2}=\frac{-5-2i}{6}=-\frac{5}{6}-\frac{1}{3}i.$

D2e $\overline{(2-3i)^2}=\overline{(2-3i)(2-3i)}=\overline{4-6i-6i+9i^2}=\overline{4-12i-9}=\overline{-5-12i}=-5+12i.$

D2f $\frac{\overline{2-i}}{2+3i}=\frac{2+i}{2+3i}=\frac{2+i}{2+3i}\cdot\frac{2-3i}{2-3i}=\frac{4-6i+2i-3i^2}{4-6i+6i-9i^2}=\frac{4-4i+3}{4+9}=\frac{7-4i}{13}=\frac{7}{13}-\frac{4}{13}i.$

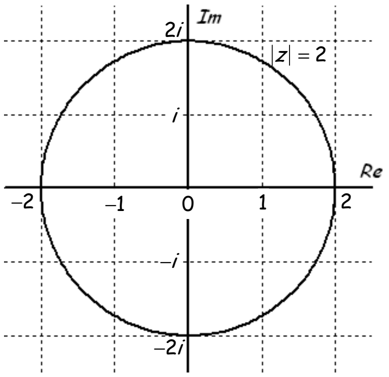
D3a $|3-3i|=\sqrt{3^2+(-3)^2}=\sqrt{9+9}=\sqrt{9\cdot 2}=3\sqrt{2}$ en $\text{Arg}(3-3i)=-45°.$

D3b $\left|(2+2i)^6\right|=|2+2i|^6=\left(\sqrt{2^2+2^2}\right)^6=\left(\sqrt{8}\right)^6=\left(\left(\sqrt{8}\right)^2\right)^3=8^3=512$ en
$\arg\left((2+2i)^6\right)=6\cdot\arg(2+2i)=6\cdot 45°=270°\Rightarrow \text{Arg}(2+2i)=-90°.$

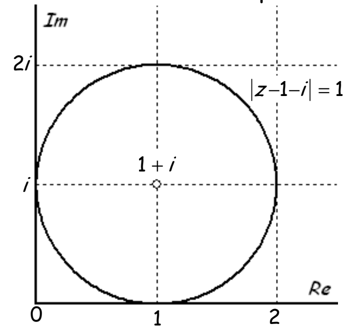
D3c $|\cos(65°)+i\sin(65°)|=1$ en $\text{Arg}(\cos(65°)+i\sin(65°))=65°.$

D3d $|10\cos(105°)+10i\sin(105°)|=|10(\cos(105°)+i\sin(105°))|=10$ en $\text{Arg}(10\cos(105°)+10i\sin(105°))=105°.$

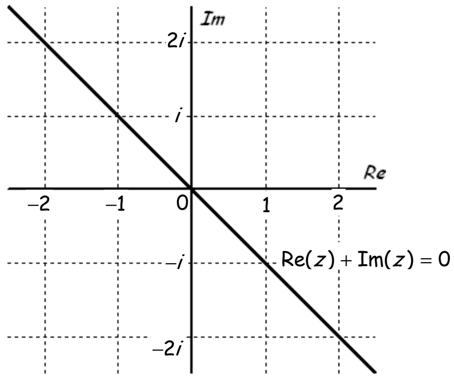
D4a $|z|=|z-0|=2$ (de afstand van z tot 0 is 2)
is de cirkel met middelpunt 0 en straal 2.



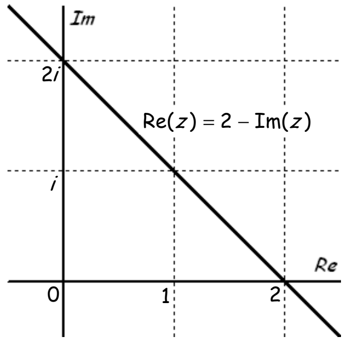
D4b $|z-1-i|=|z-(1+i)|=1$ (de afstand van z tot $1+i$ is 1)
is de cirkel met middelpunt $1+i$ en straal 1.



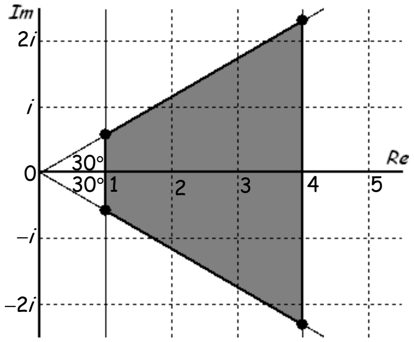
D4c $\text{Re}(z)+\text{Im}(z)=0$
$\text{Im}(z)=-\text{Re}(z).$ ($y=-x$ in het xOy-vlak)



D4d $\text{Re}(z)=2-\text{Im}(z)$
$\text{Im}(z)=2-\text{Re}(z).$ ($y=2-x$ in het xOy-vlak)



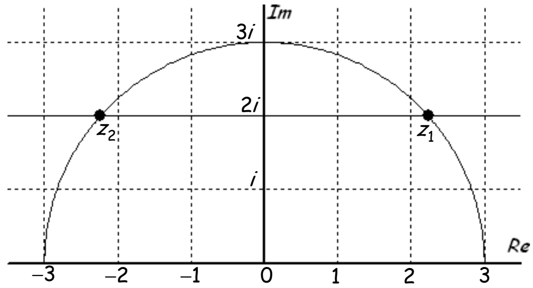
D4e $1 \le \text{Re}(z) \le 4 \ \wedge\ -30° \le \text{Arg}(z) \le 30°$.



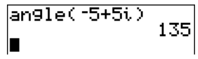
D4f $z \cdot \bar{z} = 9 \ \wedge\ \text{Im}(z) = 2$

$|z|^2 = 9 \ \wedge\ \text{Im}(z) = 2$

$|z| = 3 \ \wedge\ \text{Im}(z) = 2$ (twee punten op de cirkel $|z| = 3$).

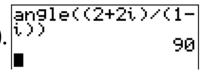


D5a $|-5+5i| = \sqrt{5^2+5^2} = \sqrt{25 \cdot 2} = 5\sqrt{2}$ en $\text{Arg}(-5+5i) = 135° \Rightarrow -5+5i = 5\sqrt{2} \cdot (\cos(135°) + i\sin(135°))$.

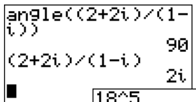


D5b $|-3| = 3$ en $\text{Arg}(-3) = 180°$ (lukt niet met angle(...)) $\Rightarrow -3 = 3 \cdot (\cos(180°) + i\sin(180°))$.

D5c $\left|\frac{2+2i}{1-i}\right| = \frac{|2+2i|}{|1-i|} = \frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4} = 2$ en $\arg(\frac{2+2i}{1-i}) = \text{Arg}(2+2i) - \text{Arg}(1-i) = 45° - (-45°) = 90° = \text{Arg}(\frac{2+2i}{1-i})$.



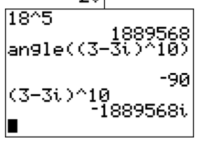
Dus $\frac{2+2i}{1-i} = 2 \cdot (\cos(90°) + i\sin(90°))$ (dit is dus $2i$).



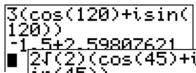
D5d $\left|(3-3i)^{10}\right| = |3-3i|^{10} = \sqrt{3^2+3^2}^{10} = \sqrt{18}^{10} = \left(\sqrt{18}^2\right)^5 = 18^5 = 1889\,568$ en

$\arg\left((3-3i)^{10}\right) = 10 \cdot \arg(3-3i) = 10 \cdot -45° = -450° \Rightarrow \text{Arg}\left((3-3i)^{10}\right) = -90°$.

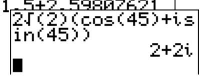
Hieruit volgt: $(3-3i)^{10} = 1889\,568 \cdot (\cos(-90°) + i\sin(-90°))$ (dit is dus $-1889568i$).



D6a $3 \cdot (\cos(120°) + i\sin(120°)) = 3 \cdot \left(\frac{1}{2} + i \cdot \frac{1}{2}\sqrt{3}\right) = 1\frac{1}{2} + 1\frac{1}{2} i\sqrt{3} \approx 1{,}5 + 2{,}6i$.



D6b $2\sqrt{2} \cdot (\cos(45°) + i\sin(45°)) = 2\sqrt{2} \cdot \left(\frac{1}{2}\sqrt{2} + i \cdot \frac{1}{2}\sqrt{2}\right) = 2 + 2i$.



D7a $(\cos(60°) + i\sin(60°))^4 = \cos(240°) + i\sin(240°) = -\frac{1}{2} + i \cdot -\frac{1}{2}\sqrt{3} = -\frac{1}{2} - \frac{1}{2} i\sqrt{3}$.

D7b $\left(\frac{1}{2} - \frac{1}{2} i\sqrt{3}\right)^5 = (\cos(-60°) + i\sin(-60°))^5 = \cos(-300°) + i\sin(-300°) = \cos(60°) + i\sin(60°) = \frac{1}{2} + \frac{1}{2} i\sqrt{3}$.

D8a $\left|z^2\right| = |z|^2 = 3^2 = 9$ en $\arg(z^2) = 2\text{Arg}(z) = 2 \cdot 40° = 80° = \text{Arg}(z^2)$.

D8b $\left|5z^3\right| = 5|z|^3 = 5 \cdot 3^3 = 5 \cdot 27 = 135$ en $\arg(5z^3) = \text{Arg}(5) + \arg(z^3) = 0° + 3\text{Arg}(z) = 0° + 3 \cdot 40° = 120° = \text{Arg}(5z^3)$.

D8c $\left|\frac{1}{z^2}\right| = \frac{1}{|z|^2} = \frac{1}{3^2} = \frac{1}{9}$ en $\arg(\frac{1}{z^2}) = \text{Arg}(1) - \arg(z^2) = 0° - 2\text{Arg}(z) = -2 \cdot 40° = -80° = \text{Arg}(\frac{1}{z^2})$.

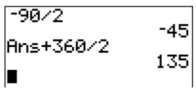
D8d $|2iz| = |2i| \cdot |z| = 2 \cdot 3 = 6$ en $\arg(2iz) = \text{Arg}(2i) + \arg(z) = 90° + 40° = 130° = \text{Arg}(2iz)$.

D9a $z^2 = -16i$ met $\left|z^2\right| = |16i| = 16$ en $\text{Arg}(z^2) = \text{Arg}(-16i) = -90°$.

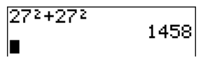
$z^2 = 16 \cdot (\cos(-90°) + i\sin(-90°)) \ \vee\ z^2 = 16 \cdot (\cos(270°) + i\sin(270°))$

$z = 4 \cdot (\cos(-45°) + i\sin(-45°)) \ \vee\ z = 4 \cdot (\cos(135°) + i\sin(135°))$

$z = 4 \cdot \left(\frac{1}{2}\sqrt{2} + i \cdot -\frac{1}{2}\sqrt{2}\right) = 2\sqrt{2} - 2i\sqrt{2} \ \vee\ z = 4 \cdot \left(-\frac{1}{2}\sqrt{2} + i \cdot \frac{1}{2}\sqrt{2}\right) = -2\sqrt{2} + 2i\sqrt{2}$.



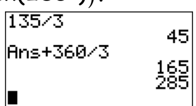
D9b $z^3 = -27 + 27i$ met $\left|z^3\right| = \sqrt{27^2 + 27^2} = \sqrt{27^2 \cdot 2} = 27\sqrt{2}$ en $\text{Arg}(z^3) = \text{Arg}(-27 + 27i) = 135°$.



$z^3 = 27\sqrt{2} \cdot (\cos(135°) + i\sin(135°)) \ \vee\ z^3 = 27\sqrt{2} \cdot (\cos(495°) + i\sin(495°)) \ \vee\ z^3 = 27\sqrt{2} \cdot (\cos(855°) + i\sin(855°))$

$z = \sqrt[3]{27\sqrt{2}} \cdot (\cos(45°) + i\sin(45°)) \ \vee\ z = \sqrt[3]{27\sqrt{2}} \cdot (\cos(165°) + i\sin(165°)) \ \vee\ z = \sqrt[3]{27\sqrt{2}} \cdot (\cos(285°) + i\sin(285°))$

$z = \sqrt[6]{1458} \cdot \left(\frac{1}{2}\sqrt{2} + \frac{1}{2} i\sqrt{2}\right) \ \vee\ z = \sqrt[6]{1458} \cdot (\cos(165°) + i\sin(165°)) \ \vee\ z = \sqrt[6]{1458} \cdot (\cos(285°) + i\sin(285°))$.



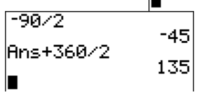
D9c $(z-2i)^2 = -4i$ met $\left|(z-2i)^2\right| = |-4i| = 4$ en $\text{Arg}\left((z-2i)^2\right) = \text{Arg}(-4i) = -90°$.

$(z-2i)^2 = 4 \cdot (\cos(-90°) + i\sin(-90°)) \ \vee\ (z-2i)^2 = 4 \cdot (\cos(270°) + i\sin(270°))$

$z - 2i = 2 \cdot (\cos(-45°) + i\sin(-45°)) \ \vee\ z - 2i = 2 \cdot (\cos(135°) + i\sin(135°))$

$z - 2i = 2 \cdot \left(\frac{1}{2}\sqrt{2} + i \cdot -\frac{1}{2}\sqrt{2}\right) = \sqrt{2} - i\sqrt{2} \ \vee\ z - 2i = 2 \cdot \left(-\frac{1}{2}\sqrt{2} + i \cdot \frac{1}{2}\sqrt{2}\right) = -\sqrt{2} + i\sqrt{2}$

$z = \sqrt{2} - i\sqrt{2} + 2i = \sqrt{2} + (2 - \sqrt{2})i \ \vee\ z = -\sqrt{2} + i\sqrt{2} + 2i = -\sqrt{2} + (2 + \sqrt{2})i$.



D9d $\boxed{z^2+4z}+5=i \Rightarrow \boxed{(z+2)^2-4}+5=i \Rightarrow (z+2)^2+1=i \Rightarrow (z+2)^2=-1+i$

met $\left|(z+2)^2\right|=|-1+i|=\sqrt{2}$ en $\text{Arg}(z^2)=\text{Arg}(-1+i)=135°$.

$z^2=\sqrt{2}\cdot(\cos(135°)+i\sin(135°)) \;\vee\; z^2=\sqrt{2}\cdot(\cos(495°)+i\sin(495°))$

$z=\sqrt{\sqrt{2}}\cdot\left(\cos(67\tfrac{1}{2}°)+i\sin(67\tfrac{1}{2}°)\right) \;\vee\; z=\sqrt{\sqrt{2}}\cdot\left(\cos(247\tfrac{1}{2}°)+i\sin(247\tfrac{1}{2}°)\right)$

$z=\sqrt[4]{2}\cdot\left(\cos(67\tfrac{1}{2}°)+i\sin(67\tfrac{1}{2}°)\right) \;\vee\; z=\sqrt[4]{2}\cdot\left(\cos(247\tfrac{1}{2}°)+i\sin(247\tfrac{1}{2}°)\right)$.

```
135/2          67.5
Ans+360/2     247.5
```

D10a $z=\sqrt{i} \Rightarrow z^2=i$ met $\left|z^2\right|=|i|=1$ en $\text{Arg}(z^2)=\text{Arg}(i)=90°$.

$z^2=\cos(90°)+i\sin(90°) \;\vee\; z^2=\cos(450°)+i\sin(450°)$

$z=\cos(45°)+i\sin(45°) \;\vee\; z=\cos(225°)+i\sin(225°)$

$z=\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}i\sqrt{2} \;\vee\; z=-\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}i\sqrt{2}$.

D10b $z=\sqrt[4]{-16} \Rightarrow z^4=-16$ met $\left|z^4\right|=|-16|=16$ en $\text{Arg}(z^4)=\text{Arg}(-16)=180°$.

$z^4=16\cdot(\cos(180°)+i\sin(180°)) \;\vee\; z^4=16\cdot(\cos(540°)+i\sin(540°)) \;\vee$

$z^4=16\cdot(\cos(900°)+i\sin(900°)) \;\vee\; z^4=16\cdot(\cos(1280°)+i\sin(1280°))$

$z=2(\cos(45°)+i\sin(45°)) \;\vee\; z=2(\cos(135°)+i\sin(135°)) \;\vee$

$z=2(\cos(225°)+i\sin(225°)) \;\vee\; z=2(\cos(315°)+i\sin(315°))$

$z=2\cdot\left(\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}i\sqrt{2}\right) \;\vee\; z=2\cdot\left(-\tfrac{1}{2}\sqrt{2}+\tfrac{1}{2}i\sqrt{2}\right) \;\vee\; z=2\cdot\left(-\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}i\sqrt{2}\right) \;\vee\; z=2\cdot\left(\tfrac{1}{2}\sqrt{2}-\tfrac{1}{2}i\sqrt{2}\right)$

$z=\sqrt{2}+i\sqrt{2} \;\vee\; z=-\sqrt{2}+i\sqrt{2} \;\vee\; z=-\sqrt{2}-i\sqrt{2} \;\vee\; z=\sqrt{2}-i\sqrt{2}$.

```
4×√16              2
```

```
180/4             45
Ans+360/4        135
                 225
                 315
```

D11a $f(z)=0$

$2iz+3-i=0$

$2iz=-3+i$

```
(-3+i)/(2i)
          .5+1.5i
```

$z=\frac{-3+i}{2i}=\frac{-3+i}{2i}\cdot\frac{-i}{-i}=\frac{3i-i^2}{-2i^2}=\frac{1+3i}{2}=\frac{1}{2}+\frac{3}{2}i$.

$f(z)=z$

$2iz+3-i=z$

$2iz-z=-3+i$

$(-1+2i)z=-3+i$

```
(-3+i)/(-1+2i)
             1+i
```

$z=\frac{-3+i}{-1+2i}=\frac{-3+i}{-1+2i}\cdot\frac{-1-2i}{-1-2i}=\frac{3+6i-i+2}{1+4}=\frac{5+5i}{5}=1+i$.

D11b $g(z)=0$

$(1+i)z+2-3i=0$

$(1+i)z=-2+3i$

```
(-2+3i)/(1+i)
          .5+2.5i
```

$z=\frac{-2+3i}{1+i}=\frac{-2+3i}{1+i}\cdot\frac{1-i}{1-i}=\frac{-2+2i+3i+3}{1-i^2}=\frac{1+5i}{2}=\frac{1}{2}+\frac{5}{2}i$.

$g(z)=z$

$(1+i)z+2-3i=z$

$(1+i)z-z=-2+3i$

$iz=-2+3i$

```
(-2+3i)/i
            3+2i
```

$z=\frac{-2+3i}{i}=\frac{-2+3i}{i}\cdot\frac{-i}{-i}=\frac{2i+3}{1}=3+2i$.

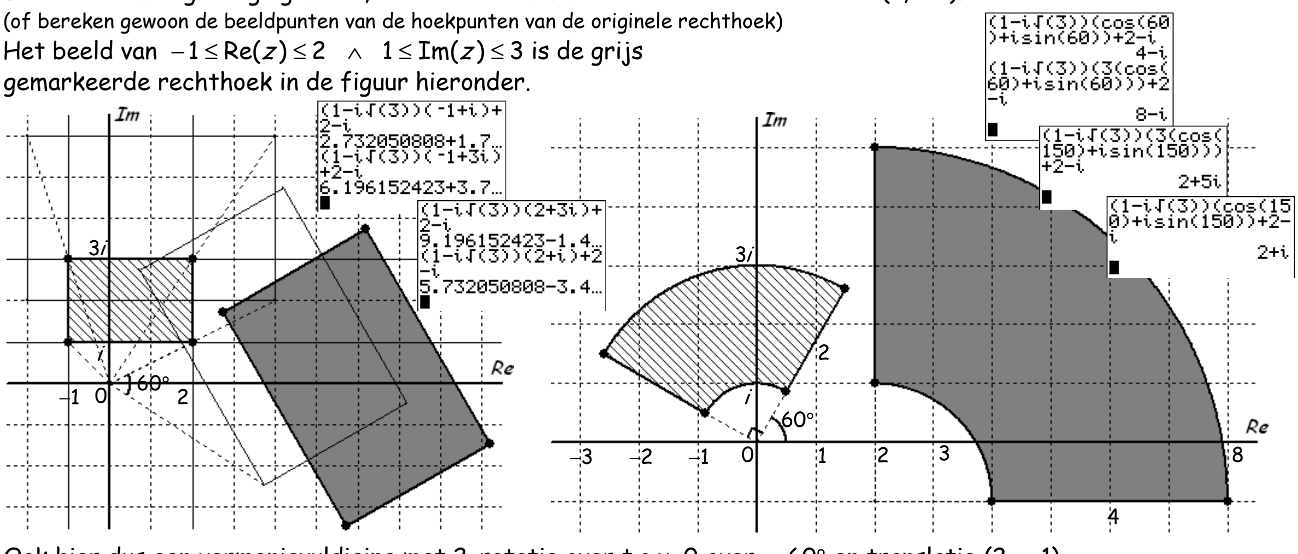
D12a $\left|1-i\sqrt{3}\right|=\sqrt{4}=2$ en $\text{Arg}(1-i\sqrt{3})=-60°$.

```
angle(1-i√(3))
             -60
```

Dus een vermenigvuldiging met 2, rotatie over t.o.v. 0 over $-60°$ en translatie $(2,\,-1)$.

(of bereken gewoon de beeldpunten van de hoekpunten van de originele rechthoek)

Het beeld van $-1\le \text{Re}(z)\le 2 \;\wedge\; 1\le \text{Im}(z)\le 3$ is de grijs gemarkeerde rechthoek in de figuur hieronder.



D12b Ook hier dus een vermenigvuldiging met 2, rotatie over t.o.v. 0 over $-60°$ en translatie $(2,\,-1)$.

(of bereken de beeldpunten van de hoekpunten van de originele gebied)

Het beeld van $1\le|z|\le 3 \;\wedge\; 60°\le \text{Arg}(z)\le 150°$ is het grijs gemarkeerde gebied in de figuur hierboven.

$\cos(60°)+i\sin(60°)$, $\cos(150°)+i\sin(150°)$, $3(\cos(60°)+i\sin(60°))$ en $3(\cos(150°)+i\sin(150°))$ zijn de de vier hoekpunten van $1\le|z|\le 3 \;\wedge\; 60°\le \text{Arg}(z)\le 150°$.

D13 Bij de krachten $\overline{F}_1$, $\overline{F}_2$ en $\overline{F}_3$ horen de complexe getallen
$z_1 = 1200\left(\cos(20°) + i\sin(20°)\right)$, $z_2 = 400\left(\cos(-65°) + i\sin(-65°)\right)$ en
$z_3 = 1500\left(\cos(145°) + i\sin(145°)\right)$ met $z_r = z_1 + z_2 + z_3$.
De GR geeft vervolgens $|z_r| \approx 911$ (N) en $\text{Arg}(z_r) \approx 86°$.
De resultante is 911 Newton groot en maakt een hoek van 86° met de positieve reële as.

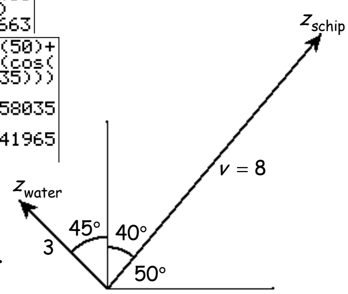
```
abs(1200(cos(20)
+isin(20))+400(c
os(-65)+isin(-65
))+1500(cos(145)
+isin(145)))
      910.8039622
```

```
angle(1200(cos(2
0)+isin(20))+400
(cos(-65)+isin(-
65))+1500(cos(14
5)+isin(145)))
      85.72148235
```

D14a Bij de snelheden horen de complexe getallen
$z_{\text{schip}} = 8\left(\cos(50°) + i\sin(50°)\right)$ en $z_{\text{water}} = 3\left(\cos(135°) + i\sin(135°)\right)$.
$z_{\text{resultante}} = z_{\text{schip}} + z_{\text{water}}$
$= 8\left(\cos(50°) + i\sin(50°)\right) + 3\left(\cos(135°) + i\sin(135°)\right)$.
De GR geeft vervolgens $|z_{\text{resultante}}| \approx 8{,}8$ (knopen) en $\text{Arg}(z_r) \approx 77°$.
Dus werkelijke snelheid van het schip is 8,8 knopen en de koers is $90° - 70° = 20°$.

```
abs(8(cos(50)+is
in(50))+3(cos(13
5)+isin(135)))
      8.785412663
```

```
angle(8(cos(50)+
isin(50))+3(cos(
135)+isin(135)))
      69.88758035
90-Ans
      20.11241965
```



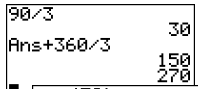
D14b Stel dat de eigen snelheid van het schip v knopen is.
Dan $z_{\text{resultante}} = z_{\text{schip}} + z_{\text{water}} = v\left(\cos(50°) + i\sin(50°)\right) + 3\left(\cos(135°) + i\sin(135°)\right)$.
Er geldt nu: $\text{Re}(z_{\text{resultante}}) = 0$ (naar het noorden dus zuiver imaginair)
$v\cos(50°) + 3\cos(135°) = 0$
$v\cos(50°) = -3\cos(135°)$
$v = \frac{-3\cos(135°)}{\cos(50)} \approx 3{,}3$.
De stuurman moet zijn snelheid met $8 - 3{,}3 = 4{,}7$ knopen verlagen.

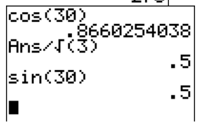
Gemengde opgaven 8. Complexe getallen

G33a $\boxed{z^2+2z}+2=\boxed{(z+1)^2-1}+2=(z+1)^2+1=0$
$(z+1)^2=-1$
$(z+1)^2=i^2$
$z+1=i \;\vee\; z+1=-i$
$z=-1+i \;\vee\; z=-1-i.$

G33c $2iz+1=-z+2i$
$z+2iz=-1+2i$
$(1+2i)z=-1+2i$
$z=\frac{-1+2i}{1+2i}=\frac{-1+2i}{1+2i}\cdot\frac{1-2i}{1-2i}=\frac{-1+2i+2i+4}{1+4}=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i.$

G33b $z^3=i$ met $\left|z^3\right|=|i|=1$ en $\text{Arg}(z)=\text{Arg}(i)=90°.$
$z^3=\cos(90°)+i\sin(90°) \;\vee\; z^3=\cos(450°)+i\sin(450°) \;\vee\; z^3=\cos(810°)+i\sin(810°)$
$z=\cos(30°)+i\sin(30°) \;\vee\; z=\cos(150°)+i\sin(150°) \;\vee\; z=\cos(270°)+i\sin(270°)$
$z=\frac{1}{2}\sqrt{3}+\frac{1}{2}i \;\vee\; z=-\frac{1}{2}\sqrt{3}+\frac{1}{2}i \;\vee\; z=-i.$

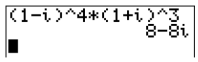




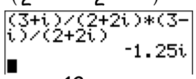
G33d $(z-2i)^2=-4$ met $\left|(z-2i)^2\right|=|-4|=4$ en $\text{Arg}((z-2i)^2)=\text{Arg}(-4)=180°.$
$(z-2i)^2=4\left(\cos(180°)+i\sin(180°)\right) \;\vee\; (z-2i)^2=4\left(\cos(540°)+i\sin(540°)\right)$
$z-2i=2\left(\cos(90°)+i\sin(90°)\right) \;\vee\; z-2i=2\left(\cos(270°)+i\sin(270°)\right)$
$z-2i=2i \;\vee\; z-2i=-2i$
$z=4i \;\vee\; z=0.$

of $(z-2i)^2=-4$
$(z-2i)^2=4i^2$
$z-2i=2i \;\vee\; z-2i=-2i$
$z=4i \;\vee\; z=0.$

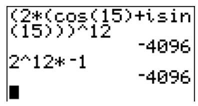
G34a $(1-i)^4\cdot(1+i)^3=\left(\sqrt{2}\left(\cos(-45°)+i\sin(-45°)\right)\right)^4\cdot\left(\sqrt{2}\left(\cos(45°)+i\sin(45°)\right)\right)^3$
$=\sqrt{2}^4\cdot\left(\cos(-180°)+i\sin(-180°)\right)\cdot\sqrt{2}^3\cdot\left(\cos(135°)+i\sin(135°)\right)$
$=\sqrt{2}^7\cdot\left(\cos(-45°)+i\sin(-45°)\right)=8\sqrt{2}\cdot\left(\frac{1}{2}\sqrt{2}-\frac{1}{2}i\sqrt{2}\right)=8-8i.$



G34b $\frac{3+i}{2+2i}\cdot\frac{3-i}{2+2i}=\frac{9-3i+3i+1}{4+4i+4i-4}=\frac{10}{8i}=\frac{10}{8i}\cdot\frac{-i}{-i}=\frac{-10i}{8}=-\frac{5}{4}i.$



G34c $\left(2\left(\cos(15°)+i\sin(15°)\right)\right)^{12}=2^{12}\cdot\left(\cos(180°)+i\sin(180°)\right)=2^{12}\cdot(-1+i\cdot 0)=-2^{12}=-4\,096.$



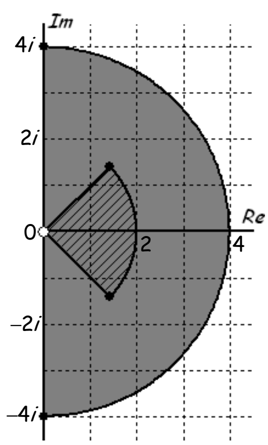
G34d $\left(\cos(60°)-i\sin(60°)\right)^2\cdot\left(\cos(60°)+i\sin(60°)\right)=\left(\cos(60°)-i\sin(60°)\right)\cdot\left(\cos(60°)-i\sin(60°)\right)\cdot\left(\cos(60°)+i\sin(60°)\right)$
$=\left(\cos(60°)-i\sin(60°)\right)\cdot 1$ (gebruik $z\cdot\overline{z}=|z|^2$) $=\frac{1}{2}-\frac{1}{2}i\sqrt{3}.$

G35a $f(z)=0$
$(2+i)z+3-i=0$
$(2+i)z=-3+i$
$z=\frac{-3+i}{2+i}=\frac{-3+i}{2+i}\cdot\frac{2-i}{2-i}=\frac{-6+3i+2i+1}{5}=\frac{-5+5i}{5}=-1+i.$
$f(z)=z$
$(2+i)z+3-i=z$
$(2+i)z-z=-3+i$
$(1+i)z=-3+i$
$z=\frac{-3+i}{1+i}=\frac{-3+i}{1+i}\cdot\frac{1-i}{1-i}=\frac{-3+3i+i+1}{2}=\frac{-2+4i}{2}=-1+2i.$

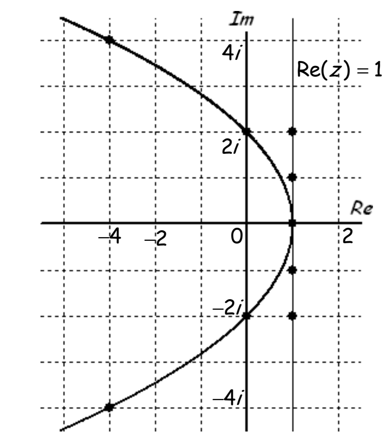
G35b $g(z)=0$
$z^2+4=0$
$z^2=-4$
$z^2=4i^2$
$z=2i \;\vee\; z=-2i.$

$g(z)=z$
$z^2+4=z$
$z^2-z+4=0$
$(z-\frac{1}{2})^2-\frac{1}{4}+4=0$
$(z-\frac{1}{2})^2+3\frac{3}{4}=0$
$(z-\frac{1}{2})^2=-3\frac{3}{4}$
$(z-\frac{1}{2})^2=\frac{15}{4}i^2$
$z-\frac{1}{2}=\frac{1}{2}i\sqrt{15} \;\vee\; z-\frac{1}{2}=-\frac{1}{2}i\sqrt{15}$
$z=\frac{1}{2}+\frac{1}{2}i\sqrt{15} \;\vee\; z=\frac{1}{2}-\frac{1}{2}i\sqrt{15}.$

G36a $\left|z^2\right|=|z|^2$ en $\arg(z^2)=2\text{Arg}(z).$
Het beeld van $|z|\le 2 \;\wedge\; -45°\le\text{Arg}(z)\le 45°$
is het grijs gemarkeerde gebied hiernaast.



G36b $f(1+bi)=(1+bi)^2=(1+bi)\cdot(1+bi)$
$=1+bi+bi+b^2i^2=1+2bi-b^2.$
$b=0\Rightarrow f(1+0i)=f(1)=1$
$b=1\Rightarrow f(1+i)=1+2i-1=2i$
$b=2\Rightarrow f(1+2i)=1+4i-4=-3+4i$
$b=-1\Rightarrow f(1-i)=1-2i-1=-2i$
$b=-2\Rightarrow f(1-2i)=1-4i-4=-3-4i.$
Het beeld van $\text{Re}(z)=1$ is de parabool rechts.

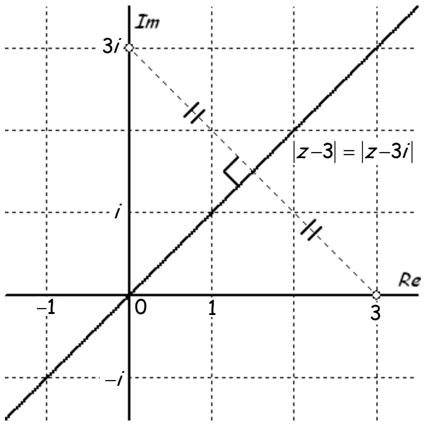


G36c $\text{Im}(z)=4$, dus stel $z=a+4i.$
$f(z)=f(a+4i)=(a+4i)^2=(a+4i)(a+4i)=a^2+8ai+16i^2=a^2-16+8ai.$
Op de reële as is $\text{Re}(z)=0\Rightarrow a^2-16=0\Rightarrow a=4 \;\vee\; a=-4.$
Dus $z=4+4i$ en $z=-4+4i$ worden op de imaginaire as afgebeeld.

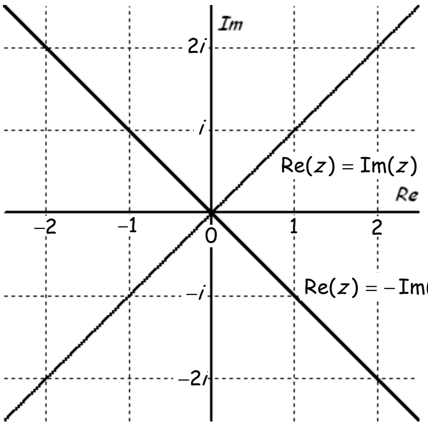
G37a Vanuit punt A' (het complex getal $z = 3 + 2i$) de afbeeldingen in omgekeerde volgorde toepassen, levert A weer op.
Dus A' eerst roteren om $z = 0$ over $-(-90°)$, dus over 90° en daarna de translatie $(-3, -1)$.
$3 + 2i \xrightarrow{\text{roteren om } z=0 \text{ over } 90°} i \cdot (3 + 2i) = 3i - 2 = -2 + 3i \xrightarrow{\text{translatie } (-3,-1)} (-2 + 3i) - 3 - i = -5 + 2i.$
Dus bij A hoort het getal $z = -5 + 2i$.

G37b $z \xrightarrow{\text{roteren om } z=0 \text{ over } 45° \text{ en vermenigvuldigen met } \sqrt{2}} (1 + i)z \xrightarrow{\text{translatie } (2,-3)} (1 + i)z + 2 - 3i.$
$f(z) = z \Rightarrow (1 + i)z + 2 - 3i = z \Rightarrow (1 + i)z - z = -2 + 3i \Rightarrow iz = -2 + 3i \Rightarrow z = \frac{-2 + 3i}{i} = \frac{-2 + 3i}{i} \cdot \frac{-i}{-i} = \frac{2i + 3}{1} = 3 + 2i.$

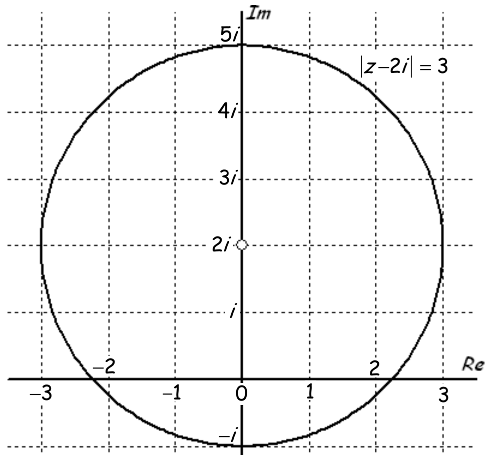
G38a $\left|\frac{z - 3}{z - 3i}\right| = 1 \Rightarrow \frac{|z - 3|}{|z - 3i|} = 1 \Rightarrow |z - 3| = |z - 3i|.$
De afstand van z tot 3 moet gelijk zijn aan de afstand van z tot $3i$.
De getallen z die hieraan voldoen liggen op de middelloodlijn van 3 en $3i$.



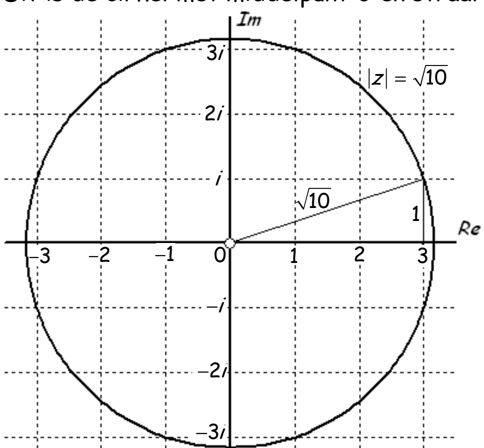
G38b $(\text{Re}(z))^2 = (\text{Im}(z))^2 \Rightarrow \text{Re}(z) = \text{Im}(z) \quad \vee \quad \text{Re}(z) = -\text{Im}(z).$



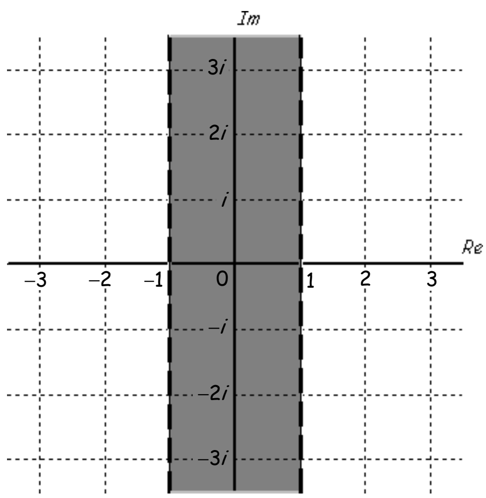
G38c $(z - 2i) \cdot (\overline{z - 2i}) = 9 \Rightarrow |z - 2i|^2 = 9 \Rightarrow |z - 2i| = 3.$
De afstand van z tot $2i$ moet 3 zijn.
Dit is de cirkel met middelpunt $2i$ en straal 3.



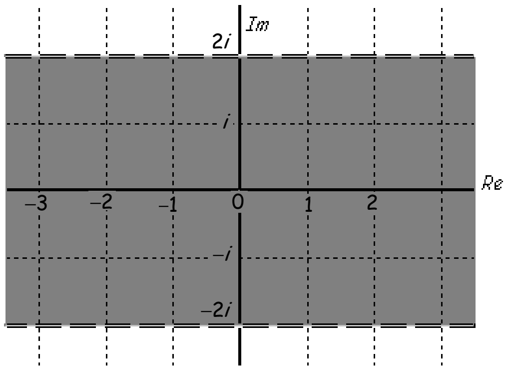
G38d $(\text{Re}(z))^2 + (\text{Im}(z))^2 = 10 \Rightarrow |z|^2 = 10 \Rightarrow |z| = \sqrt{10}.$
Dit is de cirkel met middelpunt 0 en straal $\sqrt{10}$.



G38e $|z + \overline{z}| < 2$ we weten: $z + \overline{z} = a + bi + a - bi = 2a = 2\text{Re}(z)$
$|2\text{Re}(z)| = 2|\text{Re}(z)| < 2 \Rightarrow |\text{Re}(z)| < 1 \Rightarrow -1 < \text{Re}(z) < 1.$
$z + \overline{z} = a + bi + a - bi = 2a = 2\text{Re}(z)$



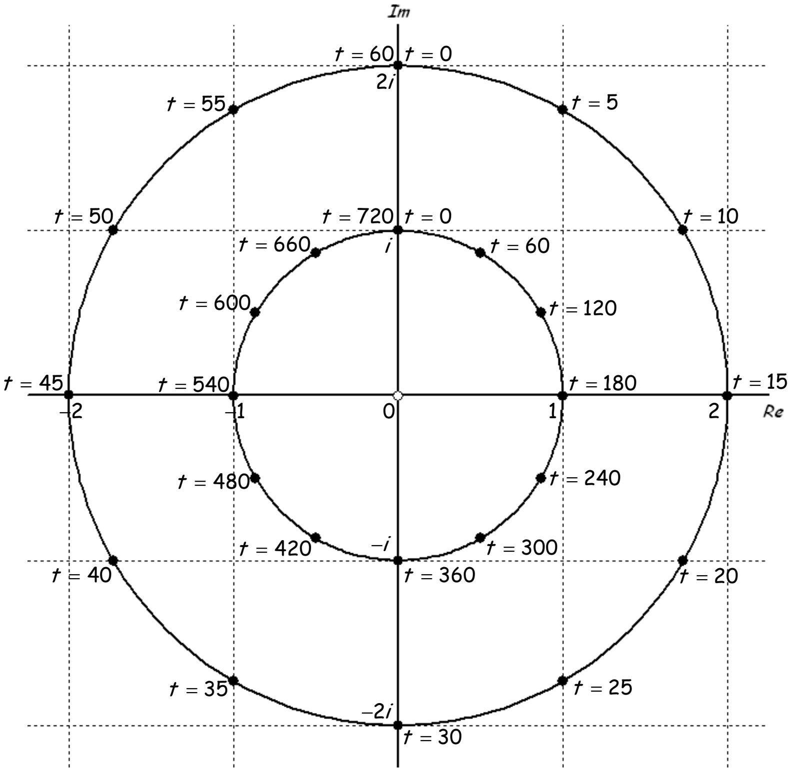
G38f $|z - \overline{z}| < 4$ we weten: $z - \overline{z} = a + bi - (a - bi) = 2bi = 2i\,\text{Im}(z)$
$|2i\,\text{Im}(z)| = |2i| \cdot |\text{Im}(z)| = 2|\text{Im}(z)| < 4 \Rightarrow |\text{Im}(z)| < 2 \Rightarrow -2 < \text{Im}(z) < 2.$



G39a $|z+4i|$ is de afstand van z tot $-4i$ en $|z-4i|$ is de afstand van z tot $4i \Rightarrow |z+4i| = 3|z-4i|$.

G39b $z = a + bi$ geeft
$|a+bi+4i| = 3|a+bi-4i|$
$|a+(b+4)i| = 3|a+(b-4)i|$
$\sqrt{a^2+(b+4)^2} = 3\sqrt{a^2+(b-4)^2}$
$a^2+(b+4)^2 = 9\left(a^2+(b-4)^2\right)$
$a^2+b^2+8b+16 = 9\left(a^2+b^2-8b+16\right)$
$a^2+b^2+8b+16 = 9a^2+9b^2-72b+144$
$-8a^2-8b^2+80b-128 = 0$
$a^2+\underline{b^2-10b}+16 = 0$
$a^2+\underline{(b-5)^2-25}+16 = 0$
$a^2+(b-5)^2-9 = 0$
$a^2+(b-5)^2 = 9.$

G39c $a^2+(b-5)^2 = 9$
$|a+(b-5)i| = 3$
$|a+bi-5i| = 3$
$|z-5i| = 3.$
Dit geeft de cirkel met middelpunt $5i$ en straal 3.



G40a Zie de punten in het complexe vlak hierboven op de cirkel $|z| = 2$.

G40b Zie de punten in het complexe vlak hierboven op de cirkel $|z| = 1$.

G40c Op $t = 0$ wijzen beide wijzers naar de 12. De wijzer staan op elkaar als
$\cos(90-6t)^\circ + i\sin(90-6t)^\circ = \cos(90-\frac{1}{2}t)^\circ + i\sin(90-\frac{1}{2}t)^\circ$
$90-6t = 90-\frac{1}{2}t \;\vee\; 90-6t+360 = 90-\frac{1}{2}t \;\vee\; 90-6t+2\cdot 360 = 90-\frac{1}{2}t \;\vee\; ...$
$-5\frac{1}{2}t = 0 \;\vee\; -5\frac{1}{2}t = -360 \;\vee\; -5\frac{1}{2}t = -2\cdot 360 \;\vee\; ...$
$t = 0 \;\vee\; t = \frac{360}{5{,}5} = \frac{720}{11} = 65\frac{5}{11} \;\vee\; ...$

```
360/5.5
          65.45454545
Ans-65▸Frac
                 5/11
```

Dus na $65\frac{5}{11}$ minuut staan de wijzers van de klok voor het eerst weer op elkaar. Dit is om $5\frac{5}{11}$ minuut over één.

G40d $90-6t+9\cdot 360 = 90-\frac{1}{2}t$
$-5\frac{1}{2}t = -9\cdot 360$
$t = \frac{9\cdot 360}{5{,}5} = \frac{9\cdot 360}{11} = 589\frac{1}{11}$

```
9*360/5.5
          589.0909091
Ans-589▸Frac
          .0909090909
```

Dit is $49\frac{1}{11}$ minuut na 9 uur, ofwel $10\frac{10}{11}$ voor 10.

G40e $90-6t-180 = 90-\frac{1}{2}t$
$-5\frac{1}{2}t = 180$
$t = \frac{180}{5{,}5} = \frac{360}{11} = 32\frac{8}{11}$
Dit is $32\frac{8}{11}$ minuut voor 12 uur, ofwel $27\frac{3}{11}$ over 11.

G41 Bij de resulterende snelheid hoort het complexe getal $z_r = 300(\cos(37^\circ)+i\sin(37^\circ))$. (150 km in 30 min $\Rightarrow$ 300km/u)
Bij de wind hoort het complexe getal $z_w = 60(\cos(135^\circ)+i\sin(135^\circ))$.
Voor de snelheid van het vliegtuig geldt $z_v + z_r = z_w \Rightarrow z_v = z_w - z_r$.
De GR geeft $|z_v| = |z_w - z_r| \approx 314$ (km/u) en $\text{Arg}(z_w - z_r) \approx 26^\circ$.
De piloot moet 314 km/uur vliegen op een koers van $90^\circ - 26^\circ = 64^\circ$

```
abs(300(cos(37)+
isin(37))-60(cos
(135)+isin(135))
)
          314.022661
```

```
angle(300(cos(37
)+isin(37))-60(c
os(135)+isin(135
)))
          26.09334238
90-Ans
          63.90665762
```