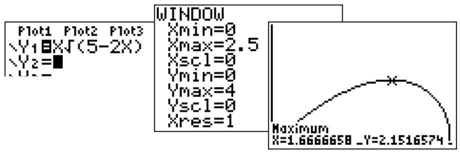
1a $PQ = y_Q = f(1\frac{1}{2}) = \sqrt{5 - 2 \cdot 1\frac{1}{2}} = \sqrt{5-3} = \sqrt{2}.$

$A = O_{OPQR} = OP \cdot PQ = 1\frac{1}{2} \cdot \sqrt{2} = 1\frac{1}{2}\sqrt{2}.$

1b $PQ = y_Q = f(p) = \sqrt{5 - 2 \cdot p} = \sqrt{5 - 2p}.$

$A = O_{OPQR} = OP \cdot PQ = p \cdot \sqrt{5-2p} = p\sqrt{5-2p}.$



1c $A = p\sqrt{5-2p}$ (optie maximum) $\Rightarrow A_{max} \approx 2{,}15$ (voor $p \approx 1{,}67$).

2a $OQ = x_Q = x_P = p$ en $PQ = y_P = f(p) = \sqrt{3-p}.$

$A = O_{OPQ} = \frac{1}{2} \cdot OQ \cdot PQ = \frac{1}{2} \cdot p \cdot \sqrt{3-p} = \frac{1}{2}p\sqrt{3-p}.$

2b $A = \frac{1}{2}p\sqrt{3-p} \Rightarrow \frac{dA}{dp} = \frac{1}{2} \cdot \sqrt{3-p} + \frac{1}{2}p \cdot \frac{1}{2 \cdot \sqrt{3-p}} \cdot -1 = \frac{\sqrt{3-p}}{2} \cdot \frac{2\sqrt{3-p}}{2\sqrt{3-p}} - \frac{p}{4\sqrt{3-p}} = \frac{2(3-p)-p}{4\sqrt{3-p}} = \frac{6-2p-p}{4\sqrt{3-p}} = \frac{6-3p}{4\sqrt{3-p}}.$

2c $\frac{dA}{dp} = 0 \Rightarrow \frac{6-3p}{4\sqrt{3-p}} = 0$ (teller $= 0$) $\Rightarrow 6 - 3p = 0 \Rightarrow 6 = 3p \Rightarrow p = 2.$

$A_{max} = A(2) = \frac{1}{2} \cdot 2 \cdot \sqrt{3-2} = 1 \cdot \sqrt{1} = 1.$

2d $L = OP = \sqrt{OQ^2 + PQ^2} = \sqrt{p^2 + \sqrt{3-p}^2} = \sqrt{p^2 + 3 - p} = \sqrt{p^2 - p + 3}.$

2e $L = \sqrt{p^2 - p + 3} \Rightarrow \frac{dL}{dp} = \frac{1}{2 \cdot \sqrt{p^2 - p + 3}} \cdot (2p - 1) = \frac{2p-1}{2\sqrt{p^2-p+3}}.$

$\frac{dL}{dp} = 0 \Rightarrow \frac{2p-1}{2\sqrt{p^2-p+3}} = 0$ (teller $= 0$) $\Rightarrow 2p - 1 = 0 \Rightarrow 2p = 1 \Rightarrow p = \frac{1}{2}.$

$L_{min} = L(\frac{1}{2}) = \sqrt{\frac{1}{4} - \frac{1}{2} + 3} = \sqrt{-\frac{1}{4} + 3} = \sqrt{2\frac{3}{4}} = \sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{\sqrt{4}} = \frac{\sqrt{11}}{2} = \frac{1}{2}\sqrt{11}.$

3a $A = O_{OSP} = \frac{1}{2} \cdot OS \cdot y_P = \frac{1}{2} \cdot 4 \cdot p\sqrt{8-2p} = 2p\sqrt{8-2p}.$

$A = 2p\sqrt{8-2p} \Rightarrow \frac{dA}{dp} = 2 \cdot \sqrt{8-2p} + 2p \cdot \frac{1}{2 \cdot \sqrt{8-2p}} \cdot -2 = 2 \cdot \frac{\sqrt{8-2p}}{1} \cdot \frac{\sqrt{8-2p}}{\sqrt{8-2p}} - \frac{2p}{\sqrt{8-2p}} = \frac{2(8-2p)-2p}{\sqrt{8-2p}} = \frac{16-6p}{\sqrt{8-2p}}.$

$\frac{dA}{dp} = 0 \Rightarrow \frac{16-6p}{\sqrt{8-2p}} = 0$ (teller $= 0$) $\Rightarrow 16 - 6p = 0 \Rightarrow 16 = 6p \Rightarrow p = \frac{16}{6} = \frac{8}{3}.$

$A_{max} = A(\frac{8}{3}) = 2 \cdot \frac{8}{3} \cdot \sqrt{8 - 2 \cdot \frac{8}{3}} = \frac{16}{3} \cdot \sqrt{8 - \frac{16}{3}} = \frac{16}{3} \cdot \sqrt{\frac{8}{3}} = \frac{16}{3} \cdot \frac{\sqrt{8}}{\sqrt{3}} = \frac{16}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16}{3} \cdot \frac{2\sqrt{6}}{3} = \frac{32}{9}\sqrt{6} = 3\frac{5}{9}\sqrt{6}.$

3b $A = O_{QSP} = \frac{1}{2} \cdot QS \cdot y_P = \frac{1}{2} \cdot (4 - p) \cdot p\sqrt{8-2p} = (2p - \frac{1}{2}p^2)\sqrt{8-2p}.$

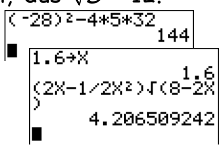
3c $A = (2p - \frac{1}{2}p^2)\sqrt{8-2p} \Rightarrow \frac{dA}{dp} = (2-p) \cdot \sqrt{8-2p} + (2p - \frac{1}{2}p^2) \cdot \frac{1}{2 \cdot \sqrt{8-2p}} \cdot -2 = \frac{(2-p) \cdot \sqrt{8-2p}}{1} \cdot \frac{\sqrt{8-2p}}{\sqrt{8-2p}} - \frac{2p - \frac{1}{2}p^2}{\sqrt{8-2p}}$

$= \frac{(2-p) \cdot (8-2p) - (2p - \frac{1}{2}p^2)}{\sqrt{8-2p}} = \frac{16 - 4p - 8p + 2p^2 - 2p + \frac{1}{2}p^2}{\sqrt{8-2p}} = \frac{2\frac{1}{2}p^2 - 14p + 16}{\sqrt{8-2p}} = \frac{5p^2 - 28p + 32}{2\sqrt{8-2p}}.$

3d $\frac{dA}{dp} = 0 \Rightarrow \frac{5p^2 - 28p + 32}{2\sqrt{8-2p}} = 0$ (teller $= 0$) $\Rightarrow 5p^2 - 28p + 32 = 0$ met $D = (-28)^2 - 4 \cdot 5 \cdot 32 = 144,$ dus $\sqrt{D} = 12.$

Dit geeft $p = \frac{28+12}{2 \cdot 5} = \frac{40}{10} = 4$ (voldoet niet, omdat dan de noemer nul is) $\vee\ p = \frac{28-12}{2 \cdot 5} = \frac{16}{10} = 1{,}6.$

$A_{max} = A(1{,}6) = (2 \cdot 1{,}6 - \frac{1}{2} \cdot 1{,}6^2)\sqrt{8 - 2 \cdot 1{,}6} \approx 4{,}21.$



4a $A = O_{OPQ} = \frac{1}{2} \cdot QP \cdot y_P = \frac{1}{2} \cdot 2x_P \cdot y_P = x_P \cdot y_P = p \cdot (3 - \frac{1}{2}p^2) = 3p - \frac{1}{2}p^3.$

4b $A = 3p - \frac{1}{2}p^3 \Rightarrow \frac{dA}{dp} = 3 - 1\frac{1}{2}p^2$ en $\frac{dA}{dp} = 0 \Rightarrow 3 - 1\frac{1}{2}p^2 = 0 \Rightarrow 3 = 1\frac{1}{2}p^2 \Rightarrow 3p^2 = 6 \Rightarrow p^2 = 2 \Rightarrow p = \sqrt{2}.$

$A_{max} = A(\sqrt{2}) = \sqrt{2}(3 - \frac{1}{2} \cdot 2) = \sqrt{2} \cdot 2 = 2\sqrt{2}.$

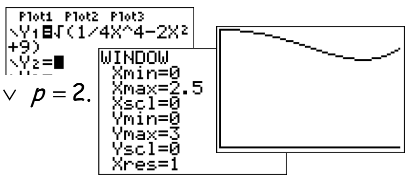
4c $L = OP = \sqrt{(x_P)^2 + (y_P)^2} = \sqrt{p^2 + (3 - \frac{1}{2}p^2)^2} = \sqrt{p^2 + (3 - \frac{1}{2}p^2)(3 - \frac{1}{2}p^2)} = \sqrt{p^2 + 9 - 3p^2 + \frac{1}{4}p^4} = \sqrt{\frac{1}{4}p^4 - 2p^2 + 9}.$

$L = \sqrt{\frac{1}{4}p^4 - 2p^2 + 9} \Rightarrow \frac{dL}{dp} = \frac{1}{2 \cdot \sqrt{\frac{1}{4}p^4 - 2p^2 + 9}} \cdot (p^3 - 4p) = \frac{p^3 - 4p}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}}.$

$\frac{dL}{dp} = 0 \Rightarrow \frac{p^3 - 4p}{2\sqrt{\frac{1}{4}p^4 - 2p^2 + 9}} = 0$ (teller $= 0$) $\Rightarrow$

$p^3 - 4p = 0 \Rightarrow p(p^2 - 4) = 0 \Rightarrow p = 0 \ \vee\ p^2 = 4 \Rightarrow p = 0 \ \vee\ p = -2 \ \vee\ p = 2.$

L_{min} (zie een plot) $= L(2) = \sqrt{\frac{1}{4} \cdot 2^4 - 2 \cdot 2^2 + 9} = \sqrt{4 - 8 + 9} = \sqrt{5}.$



5a Noem $x_P = p$ dan $y_P = f(p) = p\sqrt{p} \Rightarrow P(p, p\sqrt{p})$ en $A(4, 0)$.
$L = AP = \sqrt{(4-p)^2 + (p\sqrt{p} - 0)^2} = \sqrt{(4-p)(4-p) + (p\sqrt{p})^2} = \sqrt{16 - 8p + p^2 + p^2 \cdot p} = \sqrt{p^3 + p^2 - 8p + 16}$.

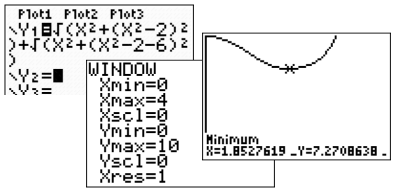
5b $L = \sqrt{p^3 + p^2 - 8p + 16} \Rightarrow \frac{dL}{dp} = \frac{1}{2 \cdot \sqrt{p^3 + p^2 - 8p + 16}} \cdot (3p^2 + 2p - 8) = \frac{3p^2 + 2p - 8}{2\sqrt{p^3 + p^2 - 8p + 16}}$.
$\frac{dL}{dp} = 0 \Rightarrow \frac{3p^2 + 2p - 8}{2\sqrt{p^3 + p^2 - 8p + 16}} = 0$ (teller = 0) $\Rightarrow 3p^2 + 2p - 8 = 0$ met $D = 2^2 - 4 \cdot 3 \cdot -8 = 4 + 96 = 100$.
Dit geeft $p = \frac{-2 - \sqrt{100}}{2 \cdot 3} = \frac{-2 - 10}{6} = -2$ (voldoet niet omdat $p > 0$) $\vee$ $p = \frac{-2 + \sqrt{100}}{2 \cdot 3} = \frac{-2 + 10}{6} = \frac{8}{6} = \frac{4}{3}$.
Het gevraagde punt is $(\frac{4}{3}, \frac{4}{3}\sqrt{\frac{4}{3}}) = (\frac{4}{3}, \frac{4}{3}\frac{\sqrt{4}}{\sqrt{3}}) = (\frac{4}{3}, \frac{4}{3}\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}) = (\frac{4}{3}, \frac{8}{9}\sqrt{3})$.

6a Noem $x_P = p$ dan $y_P = f(p) = p \cdot \sqrt[3]{8-p}$. (maak een schets)
$A = O_{OSP} = \frac{1}{2} \cdot OS \cdot y_P = \frac{1}{2} \cdot 8 \cdot p \cdot \sqrt[3]{8-p} = 4p \cdot \sqrt[3]{8-p}$.
$A = 4p \cdot \sqrt[3]{8-p} = 4p \cdot (8-p)^{\frac{1}{3}} \Rightarrow \frac{dA}{dp} = 4 \cdot \sqrt[3]{8-p} + 4p \cdot \frac{1}{3} \cdot (8-p)^{-\frac{2}{3}} \cdot -1 = 4 \cdot \sqrt[3]{8-p} - \frac{4p}{3(8-p)^{\frac{2}{3}}}$.
$\frac{dA}{dp} = 0 \Rightarrow 4 \cdot \sqrt[3]{8-p} - \frac{4p}{3(8-p)^{\frac{2}{3}}} = 0 \Rightarrow \frac{4(8-p)^{\frac{1}{3}}}{1} = \frac{4p}{3(8-p)^{\frac{2}{3}}} \Rightarrow 12(8-p) = 4p \Rightarrow 96 - 12p = 4p \Rightarrow 96 = 16p \Rightarrow p = 6$.
$A_{max} = A(6) = 4 \cdot 6 \cdot \sqrt[3]{8-6} = 24 \cdot \sqrt[3]{2}$.

6b $x_P = p$ dan $y_P = f(p) = p \cdot \sqrt[3]{8-p}$ (maak een schets) geeft $A = O_{OPQ} = \frac{1}{2} \cdot OQ \cdot PQ = \frac{1}{2} \cdot p \cdot p \cdot \sqrt[3]{8-p} = \frac{1}{2}p^2 \cdot \sqrt[3]{8-p}$.

6c $A = \frac{1}{2}p^2 \cdot \sqrt[3]{8-p} = \frac{1}{2}p^2 \cdot (8-p)^{\frac{1}{3}} \Rightarrow \frac{dA}{dp} = p \cdot \sqrt[3]{8-p} + \frac{1}{2}p^2 \cdot \frac{1}{3} \cdot (8-p)^{-\frac{2}{3}} \cdot -1 = p \cdot \sqrt[3]{8-p} - \frac{p^2}{6(8-p)^{\frac{2}{3}}}$.
$\left[\frac{dA}{dp}\right]_{p=7} = 7 \cdot \sqrt[3]{8-7} - \frac{7^2}{6(8-7)^{\frac{2}{3}}} = 7 \cdot 1 - \frac{49}{6 \cdot 1} = 7 - \frac{49}{6} \neq 0 \Rightarrow A$ heeft geen extreem (dus ook geen maximum) voor $p = 7$.

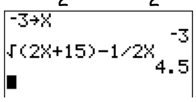
7 Noem $x_B = b$ dan $y_B = f(b) = b^2 - 2$ (maak een schets)
$L = OB + AB = \sqrt{b^2 + (b^2 - 2)^2} + \sqrt{b^2 + (b^2 - 2 - 6)^2}$
$= \sqrt{b^2 + (b^2 - 2)(b^2 - 2)} + \sqrt{b^2 + (b^2 - 8)(b^2 - 8)}$
$= \sqrt{b^2 + b^4 - 2b^2 - 2b^2 + 4} + \sqrt{b^2 + b^4 - 8b^2 - 8b^2 + 64}$
$= \sqrt{b^4 - 3b^2 + 4} + \sqrt{b^4 - 15b^2 + 64}$.
L_{min} (optie minimum) $\approx 7{,}27$ (voor $b \approx 1{,}85$ of voor $b \approx -1{,}85$).

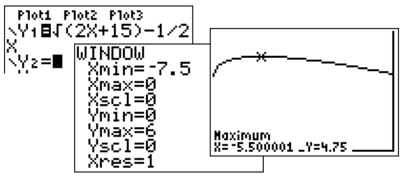


8a $p = -3 \Rightarrow L = y_A - y_B = \sqrt{2 \cdot -3 + 15} - \frac{1}{2} \cdot -3 = \sqrt{9} + 1\frac{1}{2} = 3 + 1\frac{1}{2} = 4\frac{1}{2}$.

8b $L = y_A - y_B = f(p) - g(p) = \sqrt{2p + 15} - \frac{1}{2}p$.

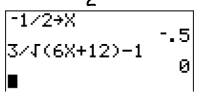
8c AB is maximaal (optie maximum) voor $p = -5{,}5$.



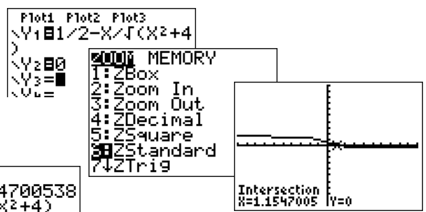


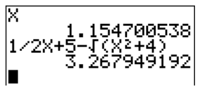
9a $L = y_A - y_B = f(p) - g(p) = \sqrt{6p + 12} - (p + 2) = \sqrt{6p + 12} - p - 2$.

9b $L = \sqrt{6p + 12} - p - 2 \Rightarrow \frac{dL}{dp} = \frac{1}{2 \cdot \sqrt{6p + 12}} \cdot 6 - 1 = \frac{3}{\sqrt{6p + 12}} - 1$.
$\frac{dL}{dp} = 0 \Rightarrow \frac{3}{\sqrt{6p + 12}} - 1 = 0 \Rightarrow \frac{3}{\sqrt{6p + 12}} = \frac{1}{1} \Rightarrow \sqrt{6p + 12} = 3$ (kwadrateren) $\Rightarrow 6p + 12 = 9 \Rightarrow 6p = -3 \Rightarrow p = -\frac{1}{2}$ (voldoet).
$L_{min} = L(-\frac{1}{2}) = \sqrt{6 \cdot -\frac{1}{2} + 12} + \frac{1}{2} - 2 = \sqrt{9} - 1\frac{1}{2} = 3 - 1\frac{1}{2} = 1\frac{1}{2}$.



10 $L = CD = y_D - y_C = g(p) - f(p) = \frac{1}{2}p + 5 - \sqrt{p^2 + 4}$.
$L = \frac{1}{2}p + 5 - \sqrt{p^2 + 4} \Rightarrow \frac{dL}{dp} = \frac{1}{2} - \frac{1}{2 \cdot \sqrt{p^2 + 4}} \cdot 2p = \frac{1}{2} - \frac{p}{\sqrt{p^2 + 4}}$.
$\frac{dL}{dp} = 0 \Rightarrow \frac{1}{2} - \frac{p}{\sqrt{p^2 + 4}} = 0$ (optie intersect) $\Rightarrow p \approx 1{,}155$.
$L_{max} = L(Ans) = \frac{1}{2}Ans + 5 - \sqrt{Ans^2 + 4} \approx 3{,}27$.





11 $L = AB = y_A - y_B = f(p) - g(p) = \frac{1}{2}\sin(2p) - \left(\cos(p) - 1\frac{1}{2}\right) = \frac{1}{2}\sin(2p) - \cos(p) + 1\frac{1}{2}.$

$L = \frac{1}{2}\sin(2p) - \cos(p) + 1\frac{1}{2} \Rightarrow \frac{dL}{dp} = \frac{1}{2}\cos(2p) \cdot 2 + \sin(p) = \cos(2p) + \sin(p).$

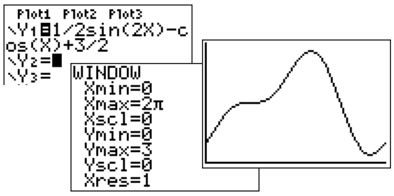
$\frac{dL}{dp} = 0 \Rightarrow \cos(2p) + \sin(p) = 0 \Rightarrow \cos(2p) = -\sin(p) \Rightarrow \sin(2p + \frac{1}{2}\pi) = \sin(p + \pi).$

$2p + \frac{1}{2}\pi = p + \pi + k \cdot 2\pi \ \vee \ 2p + \frac{1}{2}\pi = \pi - (p + \pi) + k \cdot 2\pi$

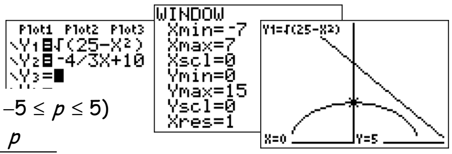
p op $[0, 2\pi]$ geeft $p = \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ 3p = -\frac{1}{2}\pi + k \cdot 2\pi$

$p = \frac{1}{2}\pi + k \cdot 2\pi \ \vee \ p = -\frac{1}{6}\pi + k \cdot \frac{2}{3}\pi$

$p = \frac{1}{2}\pi \ \vee \ p = \frac{7}{6}\pi \ \vee \ p = \frac{11}{6}\pi.$



L_{max} (zie een plot van L) $= L(1\frac{1}{6}\pi) = \frac{1}{2}\sin(2\frac{1}{3}\pi) - \cos(1\frac{1}{6}\pi) + 1\frac{1}{2} = \frac{1}{2} \cdot \frac{1}{2}\sqrt{3} - -\frac{1}{2}\sqrt{3} + 1\frac{1}{2} = \frac{1}{4}\sqrt{3} + \frac{1}{2}\sqrt{3} + 1\frac{1}{2} = \frac{3}{4}\sqrt{3} + 1\frac{1}{2}.$



12a $L = AB = y_B - y_A = g(p) - f(p) = -\frac{4}{3}p + 10 - \sqrt{25 - p^2}.$

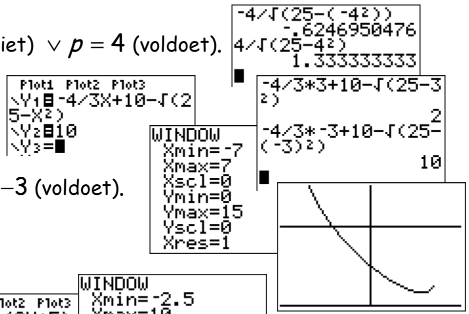
(vanwege de wortel moet gelden: $25 - p^2 \geq 0 \Rightarrow -p^2 \geq -25 \Rightarrow p^2 \leq 25 \Rightarrow -5 \leq p \leq 5$)

$L = -\frac{4}{3}p + 10 - \sqrt{25 - p^2} \Rightarrow \frac{dL}{dp} = -\frac{4}{3} - \frac{1}{2 \cdot \sqrt{25 - p^2}} \cdot -2p = -\frac{4}{3} + \frac{p}{\sqrt{25 - p^2}}.$

$\frac{dL}{dp} = 0 \Rightarrow -\frac{4}{3} + \frac{p}{\sqrt{25 - p^2}} = 0 \Rightarrow \frac{p}{\sqrt{25 - p^2}} = \frac{4}{3} \Rightarrow 3p = 4\sqrt{25 - p^2}$ (kwadrateren) $\Rightarrow 9p^2 = 16(25 - p^2) \Rightarrow$

$9p^2 = 16 \cdot 25 - 16p^2 \Rightarrow 25p^2 = 16 \cdot 25 \Rightarrow p^2 = 16 \Rightarrow p = -4$ (voldoet niet) $\vee \ p = 4$ (voldoet).

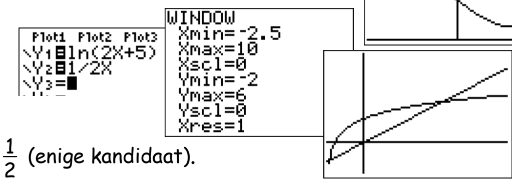
$L_{min} = L(4) = -\frac{4}{3} \cdot 4 + 10 - \sqrt{25 - 4^2} = -\frac{16}{3} + 10 - \sqrt{9} = -5\frac{1}{3} + 7 = 1\frac{2}{3}.$



12b $AB = 10 \Rightarrow -\frac{4}{3}p + 10 - \sqrt{25 - p^2} = 10 \Rightarrow -\frac{4}{3}p = \sqrt{25 - p^2}$ (kwadrateren) $\Rightarrow$

$\frac{16}{9}p^2 = 25 - p^2 \Rightarrow \frac{25}{9}p^2 = 25 \Rightarrow p^2 = 9 \Rightarrow p = 3$ (voldoet niet) $\vee \ p = -3$ (voldoet).

$AB > 10$ (zie een plot en houd rekening met het domein) $\Rightarrow -5 \leq p < -3.$



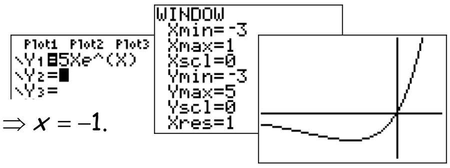
13 $L = CD = f(p) - g(p) = \ln(2p + 5) - \frac{1}{2}p.$

(vanweg de ln(... moet zeker gelden: $2p + 5 > 0 \Rightarrow 2p > -5 \Rightarrow p > -2\frac{1}{2}$)

$L = \ln(2p + 5) - \frac{1}{2}p \Rightarrow \frac{dL}{dp} = \frac{1}{2p + 5} \cdot 2 - \frac{1}{2} = \frac{2}{2p + 5} - \frac{1}{2}.$

$\frac{dL}{dp} = 0 \Rightarrow \frac{2}{2p + 5} - \frac{1}{2} = 0 \Rightarrow \frac{2}{2p + 5} = \frac{1}{2} \Rightarrow 2p + 5 = 4 \Rightarrow 2p = -1 \Rightarrow p = -\frac{1}{2}$ (enige kandidaat).

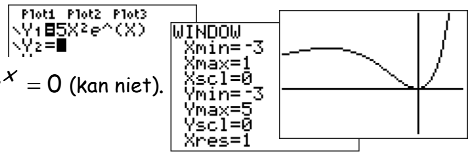
$L_{max} = L(-\frac{1}{2}) = \ln(2 \cdot -\frac{1}{2} + 5) - \frac{1}{2} \cdot -\frac{1}{2} = \ln(4) + \frac{1}{4}.$



14a $f(x) = 5x \cdot e^x \Rightarrow f'(x) = 5 \cdot e^x + 5x \cdot e^x = (5x + 5) \cdot e^x.$

$f'(x) = 0 \Rightarrow (5x + 5) \cdot e^x = 0 \Rightarrow 5x + 5 = 0 \ \vee \ e^x = 0$ (kan niet) $\Rightarrow 5x = -5 \Rightarrow x = -1.$

Het minimum van f is $f(-1) = 5 \cdot -1 \cdot e^{-1} = -\frac{5}{e}$. Dus $B_f = \left[-\frac{5}{e}, \rightarrow\right\rangle$.



14b $g(x) = 5x^2 \cdot e^x \Rightarrow g'(x) = 10x \cdot e^x + 5x^2 \cdot e^x = (5x^2 + 10x) \cdot e^x.$

$g'(x) = 0 \Rightarrow (5x^2 + 10x) \cdot e^x = 0 \Rightarrow 5x(x + 2) \cdot e^x = 0 \Rightarrow x = 0 \ \vee \ x = -2 \ \vee \ e^x = 0$ (kan niet).

Het maximum (zie een plot) van g is $g(-2) = 5 \cdot (-2)^2 \cdot e^{-2} = 5 \cdot 4 \cdot e^{-2} = \frac{20}{e^2}.$

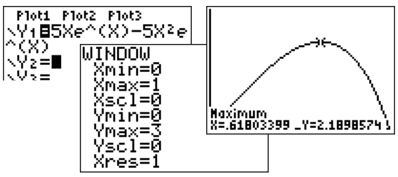
Het minimum (zie een plot) van g is $g(0) = 5 \cdot 0^2 \cdot e^0 = 5 \cdot 0 \cdot 1 = 0.$

14c $L = AB = g(p) - f(p) = 5p^2e^p - 5pe^p = (5p^2 - 5p)e^p$. (met $p < 0$)

$L = (5p^2 - 5p)e^p \Rightarrow \frac{dL}{dp} = (10p - 5) \cdot e^p + (5p^2 - 5p) \cdot e^p = (10p - 5 + 5p^2 - 5p) \cdot e^p = (5p^2 + 5p - 5) \cdot e^p.$

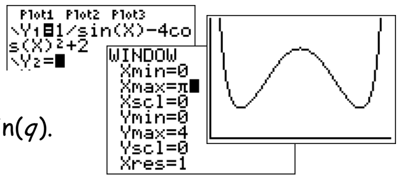
$\frac{dL}{dp} = 0 \Rightarrow 5(p^2 + p - 1) \cdot e^p = 0$ ($e^p = 0$ kan niet) $\Rightarrow p^2 + p - 1 = 0$ met $D = 1^2 - 4 \cdot 1 \cdot -1 = 5 \Rightarrow p = \frac{-1 - \sqrt{5}}{2}$ (want $p < 0$).

Dus L is maximaal voor $p = \frac{-1 - \sqrt{5}}{2}.$



14d $L = CD = f(q) - g(q) = 5qe^q - 5q^2e^q$. (met $0 < q < 1$)

L_{max} (optie maximum) $\approx 2{,}190$ (voor $q \approx 0{,}618$).



15a $L = AB = f(q) - g_4(q) = \frac{1}{\sin(q)} - \left(4\cos^2(q) - 2\right) = \frac{1}{\sin(q)} - 4\cos^2(q) + 2.$

$L = \frac{1}{\sin(q)} - 4\cos^2(q) + 2 = (\sin(q))^{-1} - 4(\cos(q))^2 + 2 \Rightarrow$

$\frac{dL}{dq} = -(\sin(q))^{-2} \cdot \cos(q) - 4 \cdot 2(\cos(q))^1 \cdot -\sin(q) = -\frac{\cos(q)}{\sin^2(q)} + 8\cos(q)\sin(q).$

$\frac{dL}{dq} = 0$ (met $0 < q < \pi$) $\Rightarrow -\frac{\cos(q)}{\sin^2(q)} + 8\cos(q)\sin(q) = 0 \Rightarrow 8\cos(q)\sin(q) = \frac{\cos(q)}{\sin^2(q)} \Rightarrow 8\cos(q)\sin^3(q) = \cos(q) \Rightarrow$

$\cos(q) = 0 \ \vee \ 8\sin^3(q) = 1 \Rightarrow q = \frac{1}{2}\pi$ (geeft maximum) $\vee \ \sin^3(q) = \frac{1}{8} \Rightarrow \sin(q) = \frac{1}{2}$ (geeft minimum) $\Rightarrow q = \frac{1}{6}\pi \ \vee \ q = \frac{5}{6}\pi$.

Dus AB is minimaal voor $q = \frac{1}{6}\pi$ en $q = \frac{5}{6}\pi$.

15b $f(x) = \frac{1}{\sin(x)} = (\sin(x))^{-1} \Rightarrow f'(x) = -(\sin(x))^{-2} \cdot \cos(x) = -\frac{\cos(x)}{\sin^2(x)}$.

$g_p(x) = p\cos^2(x) - 2 = p(\cos(x))^2 - 2 \Rightarrow g_p'(x) \Rightarrow p \cdot 2(\cos(x))^1 \cdot -\sin(x) = -2p\sin(x)\cos(x)$.

Voor raken geldt (met $0 < x < \pi$):

$f(x) = g_p(x)$ (dezelfde waarden) $\wedge$ $f'(x) = g_p'(x)$ (dezelfde helling)

$\frac{1}{\sin(x)} = p\cos^2(x) - 2$ $\wedge$ $-\frac{\cos(x)}{\sin^2(x)} = -2p\sin(x)\cos(x)$

$p\sin(x)\cos^2(x) - 2\sin(x) = 1$ ❶ $\wedge$ $2p\sin^3(x)\cos(x) = \cos(x)$ ❷

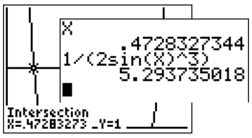
❷ geeft: $\cos(x) = 0 \ \vee \ 2p\sin^3(x) = 1 \Rightarrow x = \frac{1}{2}\pi$ (voldoet niet aan ❶) $\vee \ 2p\sin^3(x) = 1 \Rightarrow p\sin(x) = \frac{1}{2\sin^2(x)}$ ❸

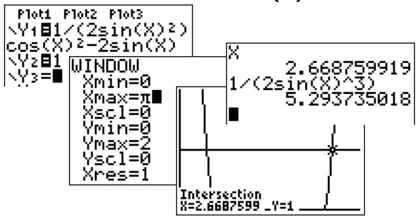
❸ invullen in ❶ $\Rightarrow \frac{1}{2\sin^2(x)}\cos^2(x) - 2\sin(x) = 1 \Rightarrow x \approx 0{,}4728$ en $x \approx 2{,}6688$.

$x \approx 0{,}4728$ invullen in ❸ $\Rightarrow p = \frac{1}{2\sin^3(\text{Ans})} \approx 5{,}29$.

$x \approx 2{,}6688$ invullen in ❸ $\Rightarrow p = \frac{1}{2\sin^3(\text{Ans})} \approx 5{,}29$.

Dus de grafieken raken elkaar voor $p \approx 5{,}29$.

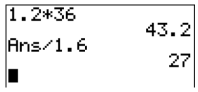




16 $\left.\begin{array}{l} I = l \cdot b \cdot h = 2x \cdot x \cdot h = 2x^2h \\ I = 40 \end{array}\right\} \Rightarrow 2x^2h = 40 \Rightarrow h = \frac{40}{2x^2} = \frac{20}{x^2}$.

17a $K = K_{\text{bodem}} + K_{\text{zijwanden}} = 2x \cdot x \cdot 0{,}4 + 2 \cdot 2x \cdot h \cdot 0{,}2 + 2 \cdot x \cdot h \cdot 0{,}2 = 0{,}8x^2 + 0{,}8xh + 0{,}4xh = 0{,}8x^2 + 1{,}2xh$.

$\left.\begin{array}{l} \left.\begin{array}{l} I = l \cdot b \cdot h = 2x \cdot x \cdot h = 2x^2h \\ I = 72 \end{array}\right\} \Rightarrow 2x^2h = 72 \Rightarrow h = \frac{72}{2x^2} = \frac{36}{x^2} \\ K = 0{,}8x^2 + 1{,}2xh \end{array}\right\} \Rightarrow K = 0{,}8x^2 + 1{,}2x \cdot \frac{36}{x^2} = 0{,}8x^2 + \frac{43{,}2}{x}$.

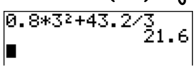


17b $K = 0{,}8x^2 + \frac{43{,}2}{x} = 0{,}8x^2 + 43{,}2x^{-1} \Rightarrow \frac{dK}{dx} = 1{,}6x - 43{,}2x^{-2} = 1{,}6x - \frac{43{,}2}{x^2}$.

$\frac{dK}{dx} = 0 \Rightarrow 1{,}6x - \frac{43{,}2}{x^2} = 0 \Rightarrow 1{,}6x = \frac{43{,}2}{x^2} \Rightarrow 1{,}6x^3 = 43{,}2 \Rightarrow x^3 = \frac{43{,}2}{1{,}6} = 27 \Rightarrow x = \sqrt[3]{27} = 3$ (de enige kandidaat).

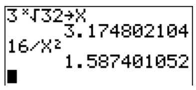
K is minimaal bij de afmetingen $l = 2x = 2 \cdot 3 = 6$ (dm) bij $b = x = 3$ (dm) bij $h = \frac{36}{x^2} = \frac{36}{9} = 4$ (dm).

$K_{\text{min}} = K(3) = 0{,}8 \cdot 3^2 + \frac{43{,}2}{3} = 21{,}60$ (€).



18a $M = O_{\text{bodem}} + O_{\text{zijwanden}} = x \cdot x + 4 \cdot x \cdot h = x^2 + 4xh$.

$\left.\begin{array}{l} \left.\begin{array}{l} I = l \cdot b \cdot h = x \cdot x \cdot h = x^2h \\ I = 16 \end{array}\right\} \Rightarrow x^2h = 16 \Rightarrow h = \frac{16}{x^2} \\ M = x^2 + 4xh \end{array}\right\} \Rightarrow M = x^2 + 4x \cdot \frac{16}{x^2} = x^2 + \frac{64}{x}$.



18b $M = x^2 + \frac{64}{x} = x^2 + 64x^{-1} \Rightarrow \frac{dM}{dx} = 2x - 64x^{-2} = 2x - \frac{64}{x^2}$.

$\frac{dM}{dx} = 0 \Rightarrow 2x - \frac{64}{x^2} = 0 \Rightarrow 2x = \frac{64}{x^2} \Rightarrow 2x^3 = 64 \Rightarrow x^3 = \frac{64}{2} = 32 \Rightarrow x = \sqrt[3]{32}$ (de enige kandidaat voor een minimum).

M is minimaal bij de afmetingen $l = x = \sqrt[3]{32} \approx 3{,}17$ (dm) bij $b = x = \sqrt[3]{32} \approx 3{,}17$ (dm) bij $h = \frac{16}{x^2} = \frac{16}{\sqrt[3]{32}^2} \approx 1{,}59$ (dm).

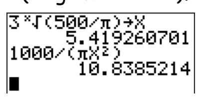
19a $O = O_{\text{bodem+deksel}} + O_{\text{omhulsel}} = 2 \cdot \pi r^2 + 2\pi r \cdot h = 2\pi r^2 + 2\pi rh$.

$\left.\begin{array}{l} \left.\begin{array}{l} I = G \cdot h = \pi r^2 \cdot h = \pi r^2 h \\ I = 1000 \text{ (cm}^3) \end{array}\right\} \Rightarrow \pi r^2 h = 1000 \Rightarrow h = \frac{1000}{\pi r^2} \\ O = 2\pi r^2 + 2\pi rh \end{array}\right\} \Rightarrow O = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$.

19b $O = 2\pi r^2 + \frac{2000}{r} = 2\pi r^2 + 2\,000r^{-1} \Rightarrow \frac{dO}{dr} = 4\pi r - 2\,000r^{-2} = 4\pi r - \frac{2000}{r^2}$.

$\frac{dO}{dr} = 0 \Rightarrow 4\pi r - \frac{2000}{r^2} = 0 \Rightarrow 4\pi r = \frac{2000}{r^2} \Rightarrow 4\pi r^3 = 2\,000 \Rightarrow r^3 = \frac{2000}{4\pi} = \frac{500}{\pi} \Rightarrow r = \sqrt[3]{\frac{500}{\pi}}$ (enige kandidaat).

O is minimaal bij een straal $r = \sqrt[3]{\frac{500}{\pi}} \approx 5{,}4$ (cm) en hoogte $h = \frac{1000}{\pi r^2} \approx 10{,}8$ (cm).

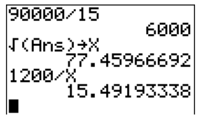


20a $K = K_{\text{langs bos}} + K_{\text{in weiland}} = y \cdot 60 + x \cdot 15 + y \cdot 15 = 15x + 75y.$

$\left.\begin{array}{l} \left.\begin{array}{l} O = l \cdot b = x \cdot y = xy \\ O = 1200 \end{array}\right\} \Rightarrow xy = 1200 \Rightarrow y = \frac{1200}{x} \\ K = 15x + 75y \end{array}\right\} \Rightarrow K = 15x + 75 \cdot \frac{1200}{x} = 15x + \frac{90000}{x}.$

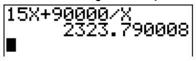
20b $K = 15x + \frac{90000}{x} = 15x + 90\,000x^{-1} \Rightarrow \frac{dK}{dx} = 15 - 90\,000x^{-2} = 15 - \frac{90000}{x^2}.$

$\frac{dK}{dx} = 0 \Rightarrow 15 - \frac{90000}{x^2} = 0 \Rightarrow 15 = \frac{90000}{x^2} \Rightarrow 15x^2 = 90\,000 \Rightarrow x^2 = \frac{90000}{15} = 6\,000 \ (x > 0) \Rightarrow x = \sqrt{6\,000}.$



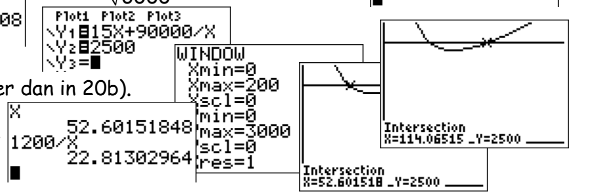
K is minimaal bij de afmetingen $l = x = \sqrt{6\,000} \approx 77{,}5$ (m) bij $b = y = \frac{1200}{x} = \frac{1200}{\sqrt{6000}} \approx 15{,}5$ (m).

$K_{\min} = K(\sqrt{6\,000}) = 15 \cdot \sqrt{6\,000} + \frac{90000}{\sqrt{6000}} \approx 2\,324$ (€).



20c $K = 15x + \frac{90000}{x} = 2\,500$ (intersect) $\Rightarrow x \approx 52{,}6$ en $x \approx 114{,}1$ (nog langer dan in 20b).

Hij kiest de afmetingen $l = x \approx 52{,}6$ (m) bij $b = y = \frac{1200}{x} \approx 22{,}8$ (m).



21a $I_{\text{potje}} = G \cdot h = \pi r^2 h = 500 \Rightarrow h = \frac{500}{\pi r^2}.$

$O_{\text{deksel}} = O_{\text{bovenkant}} + O_{\text{rand}} = \pi r^2 + 2\pi r \cdot 1 = \pi r^2 + 2\pi r.$

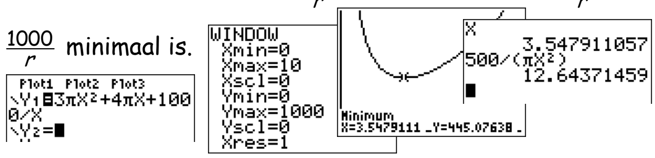
$O_{\text{potje}} = O_{\text{bodem}} + O_{\text{zijkant}} = \pi r^2 + 2\pi r \cdot h = \pi r^2 + 2\pi rh = \pi r^2 + 2\pi r \cdot \frac{500}{\pi r^2} = \pi r^2 + \frac{1000}{r}.$

$K = K_{\text{deksel}} + K_{\text{potje}} = \left(\pi r^2 + 2\pi r\right) \cdot 2a + \left(\pi r^2 + \frac{1000}{r}\right) \cdot a = 2\pi ar^2 + 4\pi ar + \pi ar^2 + \frac{1000a}{r} = 3\pi ar^2 + 4\pi ar + \frac{1000a}{r}.$

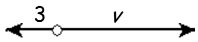
21b $K = a\left(3\pi r^2 + 4\pi r + \frac{1000}{r}\right)$ is minimaal als $3\pi r^2 + 4\pi r + \frac{1000}{r}$ minimaal is.

De optie minimum geeft K is minimaal voor $r \approx 3{,}5$.

De straal $r \approx 3{,}5$ (cm) en de hoogte $h = \frac{500}{\pi r^2} \approx 12{,}6$ (cm).



22a De snelheid van de boot is $v - 3$ (km/uur) $\Rightarrow v - 3 = \frac{5}{t} \Rightarrow t \cdot (v - 3) = 5 \Rightarrow t = \frac{5}{v - 3}.$

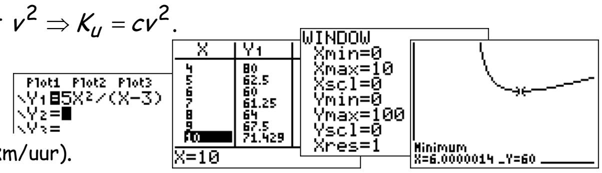


22b De brandstofkosten K_u per uur zijn evenredig met $v^2 \Rightarrow K_u = cv^2.$

$K = K_u \cdot t = cv^2 \cdot \frac{5}{v - 3} = \frac{5cv^2}{v - 3}.$

22c $K = \frac{5cv^2}{v - 3} = c \cdot \frac{5v^2}{v - 3}$ is minimaal als $\frac{5v^2}{v - 3}$ minimaal is.

De optie minimum geeft K is minimaal voor $v \approx 6$ (km/uur).



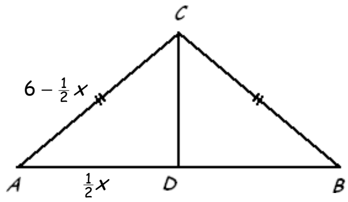
23a $AB + BC + AC = 12$ met $AB = x$ en $BC = AC \Rightarrow x + AC + AC = 12 \Rightarrow x + 2AC = 12 \Rightarrow 2AC = 12 - x \Rightarrow AC = 6 - \frac{1}{2}x.$

23b In ΔACD is $AD^2 + CD^2 = AC^2$

$\left(\frac{1}{2}x\right)^2 + CD^2 = \left(6 - \frac{1}{2}x\right)^2$

$\frac{1}{4}x^2 + CD^2 = 36 - 6x + \frac{1}{4}x^2$

$CD^2 = 36 - 6x \Rightarrow CD = \sqrt{36 - 6x}.$



23c $O_{\Delta ABC} = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot x \cdot \sqrt{36 - 6x} = \frac{1}{2}x\sqrt{36 - 6x}.$

23d $O = \frac{1}{2}x\sqrt{36 - 6x} \Rightarrow \frac{dO}{dx} = \frac{1}{2} \cdot \sqrt{36 - 6x} + \frac{1}{2}x \cdot \frac{1}{2\sqrt{36 - 6x}} \cdot -6$

$= \frac{\sqrt{36 - 6x}}{2} - \frac{3x}{2\sqrt{36 - 6x}} = \frac{\sqrt{36 - 6x}}{2} \cdot \frac{\sqrt{36 - 6x}}{\sqrt{36 - 6x}} - \frac{3x}{2\sqrt{36 - 6x}} = \frac{36 - 6x}{2\sqrt{36 - 6x}} - \frac{3x}{2\sqrt{36 - 6x}} = \frac{36 - 9x}{2\sqrt{36 - 6x}}.$

23e $\frac{dO}{dx} = 0 \Rightarrow \frac{36 - 9x}{2\sqrt{36 - 6x}} = 0$ (teller = 0) $\Rightarrow 36 - 9x = 0 \Rightarrow 36 = 9x \Rightarrow x = 4$ (de enige kandidaat).

$O_{\max} = O(4) = \frac{1}{2} \cdot 4 \cdot \sqrt{36 - 6 \cdot 4} = 2 \cdot \sqrt{36 - 24} = 2 \cdot \sqrt{12} = 2 \cdot \sqrt{4 \cdot 3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}.$

💻

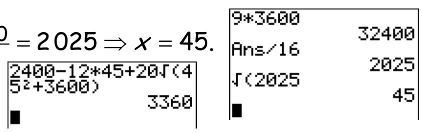
24a $K = K_{AP} + K_{PB} + K_{PC} = (200 - x) \cdot 12 + 2 \cdot \sqrt{x^2 + 3600} \cdot 10 = 2\,400 - 12x + 20 \cdot \sqrt{x^2 + 3600}.$

24b $K = 2\,400 - 12x + 20 \cdot \sqrt{x^2 + 3600} \Rightarrow \frac{dK}{dx} = -12 + 20 \cdot \frac{1}{2\sqrt{x^2 + 3600}} \cdot 2x = -12 + \frac{20x}{\sqrt{x^2 + 3600}}.$

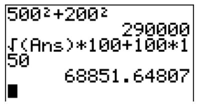
$\frac{dK}{dx} = 0 \Rightarrow -12 + \frac{20x}{\sqrt{x^2 + 3600}} = 0 \Leftrightarrow \frac{20x}{\sqrt{x^2 + 3600}} = 12 \Rightarrow 20x = 12\sqrt{x^2 + 3600} \Rightarrow 5x = 3\sqrt{x^2 + 3600}$ (kwadrateren) $\Rightarrow$

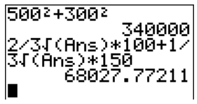
$25x^2 = 9\left(x^2 + 3600\right) \Rightarrow 25x^2 = 9x^2 + 32\,400 \Rightarrow 16x^2 = 32\,400 \Rightarrow x^2 = \frac{32400}{16} = 2\,025 \Rightarrow x = 45.$

$K_{\min} = K(45) = 2\,400 - 12 \cdot 45 + 20 \cdot \sqrt{45^2 + 3\,600} = 3\,360$ (€).



25a $AB' = \sqrt{500^2 + 200^2} = \sqrt{290\,000} \Rightarrow K = \sqrt{290\,000} \cdot 100 + 100 \cdot 150 \approx 68\,852$ (€).





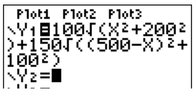
25b $AB = \sqrt{500^2 + 300^2} = \sqrt{340\,000}$.

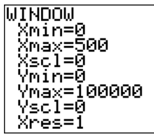
$\left.\begin{array}{l} AC : BC = 2 : 1 \\ AC + BC = AB \end{array}\right\} \Rightarrow AC = \frac{2}{3}AB$ en $BC = \frac{1}{3}AB$. Dus $K = \frac{2}{3}\sqrt{340\,000} \cdot 100 + \frac{1}{3}\sqrt{340\,000} \cdot 150 \approx 68\,028$ (€).

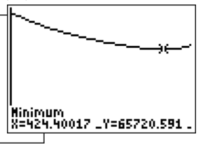
25c $AP = \sqrt{x^2 + 200^2}$ en $BP = \sqrt{(500 - x)^2 + 100^2}$.

$K = 100 \cdot \sqrt{x^2 + 200^2} + 150 \cdot \sqrt{(500 - x)^2 + 100^2}$ (optie minimum geeft)

$K_{\text{min}} \approx 65\,721$ (€ voor $x \approx 424$).





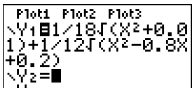


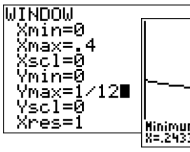
26a $AP = \sqrt{x^2 + 0{,}1^2} = \sqrt{x^2 + 0{,}01}$ en $BP = \sqrt{(0{,}4 - x)^2 + 0{,}2^2} = \sqrt{0{,}16 - 0{,}8x + x^2 + 0{,}04} = \sqrt{x^2 - 0{,}8x + 0{,}2}$.

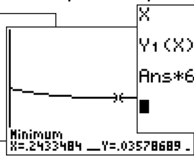
$t = \frac{AP}{18} + \frac{BP}{12} = \frac{1}{18}AP + \frac{1}{12}BP = \frac{1}{18}\sqrt{x^2 + 0{,}01} + \frac{1}{12}\sqrt{x^2 - 0{,}8x + 0{,}2}$.

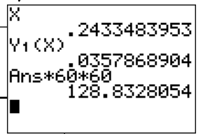
26b $t = \frac{1}{18}\sqrt{x^2 + 0{,}01} + \frac{1}{12}\sqrt{x^2 - 0{,}8x + 0{,}2}$ (optie minimum geeft)

$t_{\text{min}} \approx 0{,}0358$ (uur). Dit is afgerond 129 seconden.









27a De afgelegde afstand in het water is $\sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$.

De afgelegde afstand over land is $10 - x$.

$t = \frac{1}{4}\sqrt{x^2 + 4} + \frac{1}{12}(10 - x) = \frac{1}{4}\sqrt{x^2 + 4} + \frac{5}{6} - \frac{1}{12}x$.

27b $t = \frac{1}{4}\sqrt{x^2 + 4} + \frac{5}{6} - \frac{1}{12}x \Rightarrow \frac{dt}{dx} = \frac{1}{4} \cdot \frac{1}{2\sqrt{x^2+4}} \cdot 2x - \frac{1}{12} = \frac{x}{4\sqrt{x^2+4}} - \frac{1}{12}$.

$\frac{dt}{dx} = 0 \Rightarrow \frac{x}{4\sqrt{x^2+4}} - \frac{1}{12} = 0 \Rightarrow \frac{x}{4\sqrt{x^2+4}} = \frac{1}{12} \Rightarrow 12x = 4\sqrt{x^2 + 4} \Rightarrow 3x = \sqrt{x^2 + 4}$ (kwadrateren) $\Rightarrow$

$9x^2 = x^2 + 4 \Rightarrow 8x^2 = 4 \Rightarrow x^2 = \frac{1}{2}$ $(x > 0) \Rightarrow x = \sqrt{\frac{1}{2}} = \sqrt{\frac{1}{2} \cdot \frac{2}{2}} = \frac{1}{2}\sqrt{2}$ (enige kandidaat voor het minimum).

28a $AP = x \Rightarrow PD = 20 - x$.

Pythagoras is ΔAPD: $AD = \sqrt{PD^2 - AP^2} = \sqrt{(20 - x)^2 - x^2} = \sqrt{400 - 40x + x^2 - x^2} = \sqrt{400 - 40x}$.

$O_{\Delta APD} = \frac{1}{2} \cdot AD \cdot AP = \frac{1}{2} \cdot \sqrt{400 - 40x} \cdot x = \frac{1}{2}x\sqrt{400 - 40x}$.

28b $O = \frac{1}{2}x\sqrt{400 - 40x} \Rightarrow \frac{dO}{dx} = \frac{1}{2} \cdot \sqrt{400 - 40x} + \frac{1}{2}x \cdot \frac{1}{2\sqrt{400 - 40x}} \cdot -40 = \frac{\sqrt{400 - 40x}}{2} - \frac{10x}{\sqrt{400 - 40x}}$.

$\frac{dO}{dx} = 0 \Rightarrow \frac{\sqrt{400 - 40x}}{2} - \frac{10x}{\sqrt{400 - 40x}} = 0 \Rightarrow \frac{\sqrt{400 - 40x}}{2} = \frac{10x}{\sqrt{400 - 40x}} \Rightarrow 20x = 400 - 40x \Rightarrow 60x = 400 \Rightarrow x = \frac{400}{60} = \frac{20}{3}$.

$O_{\text{max}} = O(\frac{20}{3}) = \frac{1}{2} \cdot \frac{20}{3} \cdot \sqrt{400 - 40 \cdot \frac{20}{3}} \approx 38{,}49$ (cm^2).

29a $x_P = r\cos(ct)$ met $r = 3$ en de periode is 5, dus $c = \frac{2\pi}{5} = \frac{2}{5}\pi$.

$y_P = r\sin(ct)$ met $r = 3$ en de periode is 5, dus $c = \frac{2\pi}{5} = \frac{2}{5}\pi$.

29b $y_{P'} = y_P \Rightarrow y_{P'} = 3\sin(\frac{2}{5}\pi t)$.

30a De punten P en Q hebben beide frequentie $f = \frac{c}{2\pi} = \frac{40\pi}{2\pi} = \frac{40}{2} = 20$ Hz.

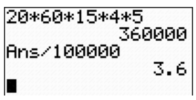
30b De trillingstijd van P en Q is $T = \frac{1}{f} = \frac{1}{20} = \frac{5}{100} = 0{,}05$ seconde.

$u_Q = 5\sin\left(40\pi t - \frac{3}{5}\pi\right) = 5\sin(40\pi(t - 0{,}015))$. Het faseverschil is $\frac{0{,}015}{T} = \frac{0{,}015}{0{,}05} = \frac{15}{50} = \frac{3}{10}$.

30c $u_P = 5\sin(40\pi t) \xrightarrow{\text{translatie } (0{,}015;\, 0)} u_Q = 5\sin(40\pi(t - 0{,}015))$.

30d P legt per kwartier 20(freq./sec.) $\cdot$ 60(seconden in 1 minuut) $\cdot$ 15(minuten) $\cdot\ 4 \cdot$ 5(amplitude) $= 360\,000$ cm af. Dit is 3,6 km.

(elke periode wordt de amplitude 4 keer (omhoog en terug, omlaag en terug) afgelegd)



31a $\left.\begin{array}{l} u_P = b\sin(ct) \\ \text{amplitude} = 10 \Rightarrow b = 10 \\ f = \frac{c}{2\pi} = 3 \Rightarrow c = 3 \cdot 2\pi = 6\pi \end{array}\right\} \Rightarrow u_P = 10\sin(6\pi t)$ (t in seconden en u_P in cm).

$\left.\begin{array}{l} u_Q = 10\sin(6\pi(t - d)) \\ \text{faseachterstand } \frac{1}{10} \text{ en trillingstijd } T = \frac{1}{f} = \frac{1}{3} \text{ geeft } d = \frac{1}{10} \cdot \frac{1}{3} = \frac{1}{30} \end{array}\right\} \Rightarrow u_Q = 10\sin\left(6\pi(t - \frac{1}{30})\right)$ (t in sec. en u_Q in cm).

31b $u_P = u_Q$ geeft $10\sin(6\pi t) = 10\sin\left(6\pi(t - \frac{1}{30})\right)$

$\sin(6\pi t) = \sin\left(6\pi t - \frac{1}{5}\pi\right)$

$6\pi t = 6\pi t - \frac{1}{5}\pi + k\cdot 2\pi \ \vee\ 6\pi t = \pi - (6\pi t - \frac{1}{5}\pi) + k\cdot 2\pi$

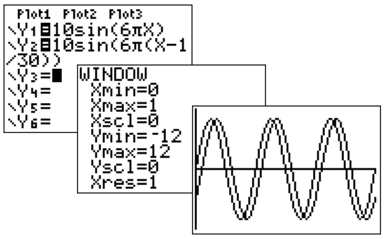
$0 = -\frac{1}{5}\pi + k\cdot 2\pi \ \vee\ 6\pi t = \pi - 6\pi t + \frac{1}{5}\pi + k\cdot 2\pi$

geen oplossing $\ \vee\ 12\pi t = \frac{6}{5}\pi + k\cdot 2\pi$

$t = \frac{1}{10} + k\cdot\frac{1}{6}.$

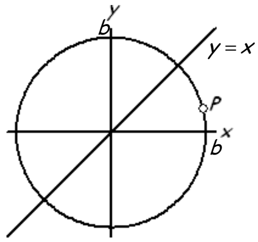
t op $\langle 0,1\rangle \Rightarrow t = \frac{6}{60} \vee t = \frac{16}{60} \vee t = \frac{26}{60} \vee t = \frac{36}{60} \vee t = \frac{46}{60} \vee t = \frac{56}{60}.$

Dus $t = \frac{1}{10} \vee t = \frac{4}{15} \vee t = \frac{13}{30} \vee t = \frac{3}{5} \vee t = \frac{23}{30} \vee t = \frac{14}{15}.$



Op $t = \frac{4}{15}$, $t = \frac{3}{5}$, $t = \frac{14}{15}$.
(bestudeer de grafieken hierboven en gebruik 31b)

32a $x_{P''} = x_P = b\cos(ct) = b\sin(ct + \frac{1}{2}\pi) = b\sin(c(t + \frac{\frac{1}{2}\pi}{c})) = b\sin(c(t + \frac{1}{2c}\pi)),$
dus P'' voert een harmonische trilling uit.



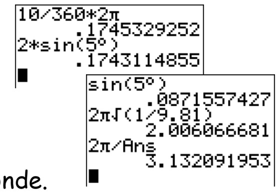
32b Door de figuur een achtste slag te draaien, zie je dat de projectie van P op de lijn $y = x$ op hetzelfde neerkomt als de projectie van een eenparige cirkelbeweging op de y-as.
Dus de projectie van P op de lijn $y = x$ voert ook een harmonische trilling uit.

33a De omtrek van de cirkel met middelpunt P en straal $r = 1{,}00$ (m) is $2\pi \cdot 1 = 2\pi$ (m).
De lengte van boog BC is $\frac{10}{360}\cdot 2\pi \approx 0{,}1745$ (m).

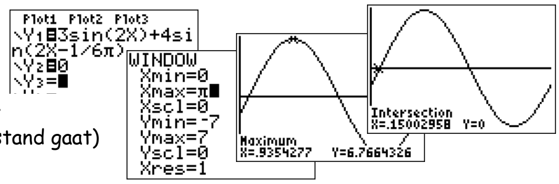
33b $\sin(5^\circ) = \frac{A'B}{BP} = \frac{A'B}{1} \Rightarrow A'B = \sin(5^\circ)$. Dus lijnstuk BC is $2A'B = 2\sin(5^\circ) \approx 0{,}1743$ (m).

33c $b = A'B = \sin(5^\circ) \approx 0{,}09.$ en $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9{,}81}} \approx 2{,}01$ (sec) $\Rightarrow c = \frac{2\pi}{T} \approx \frac{2\pi}{2{,}01} \approx 3{,}13.$

33d $T \approx 2{,}01$ (sec) $\Rightarrow$ de klok geeft 2 tikken per 2,01 seconde dus ongeveer 1 tik per seconde.



34 $u = 3\sin(2t) + 4\sin(2t - \frac{1}{6}\pi)$ (optie maximum) $\Rightarrow b \approx 6{,}77.$
$u = 3\sin(2t) + 4\sin(2t - \frac{1}{6}\pi) = 0$ (intersect of zero) $\Rightarrow t = d \approx 0{,}15.$
(je zoekt de t-waarde waar de grafiek van u stijgend door de evenwichtsstand gaat)



35a $\begin{cases} t + u = a \\ t - u = b \end{cases}$ +
$2t = a + b$
$t = \frac{1}{2}(a + b)$

$\begin{cases} t + u = a \\ t - u = b \end{cases}$ −
$2u = a - b$
$u = \frac{1}{2}(a - b).$

Je kent al de somformules:

$\sin(t + u) = \sin(t)\cos(u) + \cos(t)\sin(u)$
$\sin(t - u) = \sin(t)\cos(u) - \cos(t)\sin(u)$
$\cos(t + u) = \cos(t)\cos(u) - \sin(t)\sin(u)$
$\cos(t - u) = \cos(t)\cos(u) + \sin(t)\sin(u)$

35b $\sin(a) + \sin(b) = \sin(t + u) + \sin(t - u)$
$= \sin(t)\cos(u) + \cos(t)\sin(u) + \sin(t)\cos(u) - \cos(t)\sin(u)$
$= 2\sin(t)\cos(u) = 2\sin\left(\frac{1}{2}(a + b)\right)\cos\left(\frac{1}{2}(a - b)\right).$

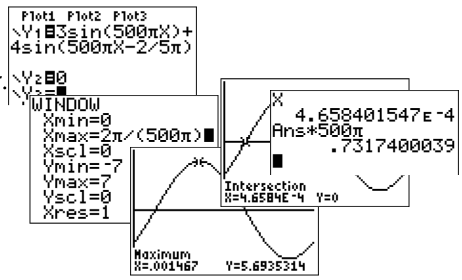
36 $\sin(a) - \sin(b) = \sin(a) + \sin(-b) = 2\sin\left(\frac{1}{2}(a + (-b))\right)\cos\left(\frac{1}{2}(a - (-b))\right) = 2\sin\left(\frac{1}{2}(a - b)\right)\cos\left(\frac{1}{2}(a + b)\right).$

$a = t + u$ en $b = t - u \Rightarrow t = \frac{1}{2}(a + b)$ en $u = \frac{1}{2}(a - b)$ (zie de afleiding in35a)

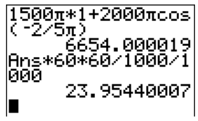
$\cos(a) + \cos(b) = \cos(t + u) + \cos(t - u)$
$= \cos(t)\cos(u) - \sin(t)\sin(u) + \cos(t)\cos(u) + \sin(t)\sin(u)$
$= 2\cos(t)\cos(u) = 2\cos\left(\frac{1}{2}(a + b)\right)\cos\left(\frac{1}{2}(a - b)\right).$

$\cos(a) - \cos(b) = \cos(t + u) - \cos(t - u)$
$= \cos(t)\cos(u) - \sin(t)\sin(u) - (\cos(t)\cos(u) + \sin(t)\sin(u))$
$= \cos(t)\cos(u) - \sin(t)\sin(u) - \cos(t)\cos(u) - \sin(t)\sin(u)$
$= -2\sin(t)\sin(u) = -2\sin\left(\frac{1}{2}(a + b)\right)\sin\left(\frac{1}{2}(a - b)\right).$

37a $3\sin(500\pi t)$ en $4\sin(500\pi t - \frac{2}{5}\pi)$ hebben dezelfde frequentie $\Rightarrow c = 500\pi.$
$u = 3\sin(500\pi t) + 4\sin(500\pi t - \frac{2}{5}\pi)$ (optie maximum) $\Rightarrow b \approx 5{,}69.$
$u = 3\sin(500\pi t) + 4\sin(500\pi t - \frac{2}{5}\pi) = 0$ (intersect of zero) $\Rightarrow t \approx 0{,}000466.$
(je zoekt de t-waarde waar de grafiek van u stijgend door de evenwichtsstand gaat)
Dus $u = 5{,}69\sin(500\pi(t - \text{Ans})) \approx 5{,}69\sin(500\pi t - 0{,}73) \Rightarrow d \approx 0{,}73.$



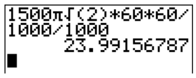
37b De frequentie $f = \frac{c}{2\pi} = \frac{500\pi}{2\pi} = 250$ Hz.
Het punt legt in 1 seconde $250 \cdot 4 \cdot 5{,}69 = 1000 \cdot 5{,}69 = 5\,690$ mm af. Dus 5,69 m.



37c $u = 3\sin(500\pi t) + 4\sin(500\pi t - \frac{2}{5}\pi) \Rightarrow \frac{du}{dt} = 3\cos(500\pi t)\cdot 500\pi + 4\cos(500\pi t - \frac{2}{5}\pi)\cdot 500\pi$
De snelheid op $t = 0$ is $\left[\frac{du}{dt}\right]_{t=0} = 1500\pi\cos(0) + 2\,000\pi\cos(-\frac{2}{5}\pi)$ mm/s. Dat is (ongeveer) 24 km/u.

38a $u = 3\sin(500\pi t) + 3\sin(500\pi t - \frac{1}{2}\pi) = 3\left(\sin(500\pi t) + \sin(500\pi t - \frac{1}{2}\pi)\right)$
$= 3 \cdot 2\sin(\frac{1}{2}(500\pi t + 500\pi t - \frac{1}{2}\pi)) \cdot \cos(\frac{1}{2}(500\pi t - 500\pi t + \frac{1}{2}\pi))$
$= 6\sin(500\pi t - \frac{1}{4}\pi)\cdot\cos(\frac{1}{4}\pi) = 6\sin(500\pi t - \frac{1}{4}\pi)\cdot\frac{1}{2}\sqrt{2} = 3\sqrt{2}\sin(500\pi t - \frac{1}{4}\pi).$

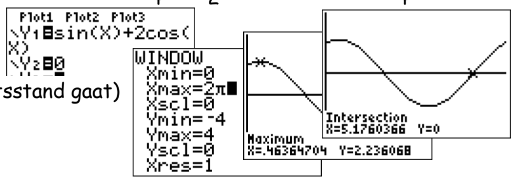
38b $u = 3\sqrt{2}\sin(500\pi t - \frac{1}{4}\pi) \Rightarrow \frac{du}{dt} = 3\sqrt{2}\cos(500\pi t - \frac{1}{4}\pi)\cdot 500\pi = 1500\pi\sqrt{2}\cos(500\pi t - \frac{1}{4}\pi)$
De maximale snelheid is $\left[\frac{du}{dt}\right]_{max} = 1500\pi\sqrt{2}\cdot 1$ mm/s. Dat is (ongeveer) 24 km/u.



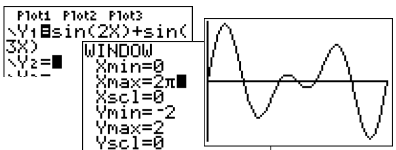
39 $u = p\sin(ct) + p\sin(ct - d) = p\left(\sin(ct) + \sin(ct - d)\right) = p\cdot 2\sin(\frac{1}{2}(ct + ct - d))\cdot\cos(\frac{1}{2}(ct - ct + d))$
$= 2p\sin(\frac{1}{2}(2ct - d))\cdot\cos(\frac{1}{2}d) = 2p\sin(ct - \frac{1}{2}d)\cdot\cos(\frac{1}{2}d) = b\sin(ct - \frac{1}{2}d)$ met $b = 2p\cos(\frac{1}{2}d)$.

40a $u_1 = \sin(t) + \cos(t) = \sin(t) + \sin(t + \frac{1}{2}\pi) = 2\sin(\frac{1}{2}(t + t + \frac{1}{2}\pi))\cos(\frac{1}{2}(t - t - \frac{1}{2}\pi))$
$= 2\sin(\frac{1}{2}(2t + \frac{1}{2}\pi))\cos(\frac{1}{2}(-\frac{1}{2}\pi)) = 2\sin(t + \frac{1}{4}\pi)\cos(-\frac{1}{4}\pi) = 2\sin(t + \frac{1}{4}\pi)\cdot\frac{1}{2}\sqrt{2} = \sqrt{2}\sin(t + \frac{1}{4}\pi).$

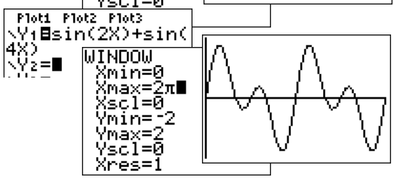
40b $u_2 = \sin(t) + 2\cos(t)$ (optie maximum) $\Rightarrow b \approx 2{,}24.$
$u_2 = \sin(t) + 2\cos(t) = 0$ (intersect of zero) $\Rightarrow t = d \approx 5{,}18.$
(je zoekt de t-waarde waar de grafiek stijgend door de evenwichtsstand gaat)
Dus $u_2 = \sin(t) + 2\cos(t) \approx 2{,}24\sin(t - 5{,}18).$



41a Zie de plot van u_1 hiernaast.
De periode van $u = \sin(2t)$ is $\frac{2\pi}{2} = \pi$ en de periode van $u = \sin(3t)$ is $\frac{2\pi}{3} = \frac{2}{3}\pi.$
Het kleinste getal waar een geheel aantal keer π en $\frac{2}{3}\pi$ in past is 2π.
Dus de periode van $u_1 = \sin(2t) + \sin(3t)$ is 2π.



41b Zie de plot van u_2 hiernaast.
De periode van $u = \sin(2t)$ is $\frac{2\pi}{2} = \pi$ en de periode van $u = \sin(4t)$ is $\frac{2\pi}{4} = \frac{1}{2}\pi.$
Het kleinste getal waar een geheel aantal keer π en $\frac{1}{2}\pi$ in past is π.
Dus de periode van $u_2 = \sin(2t) + \sin(4t)$ is π.



42a $u = \sin(100\pi t) + \sin(101\pi t)$ heeft periode 2 sec.
(zie de uitleg hieronder)

	$u_1 = \sin(100\pi t)$	$u_2 = \sin(101\pi t)$
in $[0, 2\pi]$	100π periodes	101π periodes
in $[0, 2]$	100 periodes	101 periodes

42b $u = \sin(100t) + \sin(101t)$ heeft periode 2π sec.
(zie de uitleg hieronder)

	$u_1 = \sin(100t)$	$u_2 = \sin(101t)$
in $[0, 2\pi]$	100 periodes	101 periodes

42c $u = 5\sin(100\pi t) + \sin(105\pi t)$ heeft periode $\frac{2}{5}$ sec.
(zie de uitleg hieronder)

	$u_1 = 5\sin(100\pi t)$	$u_2 = \sin(105\pi t)$
in $[0, 2\pi]$	100π periodes	105π periodes
in $\left[0, \frac{2}{5}\right]$	20 periodes	21 periodes

42d $u = 3\sin(\frac{1}{4}\pi t) + 6\sin(\frac{1}{5}\pi t)$ heeft periode 40 sec.
(zie de uitleg hieronder)

	$u_1 = 3\sin(\frac{1}{4}\pi t)$	$u_2 = 6\sin(\frac{1}{5}\pi t)$
in $[0, 2\pi]$	$\frac{1}{4}\pi$ periodes	$\frac{1}{5}\pi$ periodes
in $[0, 40]$	5 periodes	4 periodes

43 De periode van de zweving $u = \sin(660\pi t) + \sin(661\pi t)$ is 2 sec.
(zie de uitleg hieronder)

	$u_1 = \sin(660\pi t)$	$u_2 = \sin(661\pi t)$
in $[0, 2\pi]$	660π periodes	661π periodes
in $[0, 2]$	660 periodes	661 periodes

44a De boventoon $u_2 = 0{,}2\sin(1400\pi t)$ heeft frequentie $f = \frac{1400\pi}{2\pi} = 700$ Hz.
De boventoon $u_3 = 0{,}3\sin(2100\pi t)$ heeft frequentie $f = \frac{2100\pi}{2\pi} = 1050$ Hz.
De boventoon $u_4 = 0{,}1\sin(2800\pi t)$ heeft frequentie $f = \frac{2800\pi}{2\pi} = 1400$ Hz.

44b De periode van $u = 1{,}5\sin(700\pi t) + 0{,}2\sin(1400\pi t) + 0{,}3\sin(2100\pi t) + 0{,}1\sin(2800\pi t)$ is $\frac{1}{350}$ sec. (zie hieronder).

	$u_1 = 1{,}5\sin(700\pi t)$	$u_2 = 0{,}2\sin(1400\pi t)$	$u_3 = 0{,}3\sin(2100\pi t)$	$u_4 = 0{,}1\sin(2800\pi t)$
in $[0, 2\pi]$	700π periodes	1400π periodes	2100π periodes	2800π periodes
in $\left[0, \frac{1}{350}\right]$	1 periode	2 periodes	3 periodes	4 periodes

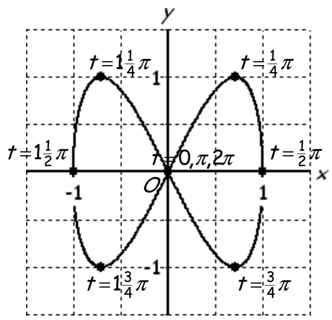
45a De periode van $u_4 = u_1 + u_2 = 0{,}6\sin(500\pi t) + 0{,}6\sin(550\pi t)$ is $\frac{1}{25}$ sec. (zie de uitleg hieronder)

	$u_1 = 0{,}6\sin(500\pi t)$	$u_2 = 0{,}6\sin(550\pi t)$
in $[0, 2\pi]$	500π periodes	550π periodes
in $\left[0, \frac{1}{25}\right]$	10 periodes	11 periodes

45b Op het GR-scherm zie je $1\frac{1}{2}$ periode van de zweving $\Rightarrow$ Xmax $= 1\frac{1}{2} \cdot \frac{1}{25} = 1\frac{1}{2} \cdot 0{,}04 = 0{,}06$.

45c $u_5 = u_1 + u_3 = 0{,}6\sin(500\pi t) + 0{,}6\sin(500\pi t - \frac{1}{2}\pi) = 0{,}6\left(\sin(500\pi t) + \sin(500\pi t - \frac{1}{2}\pi)\right)$
$= 0{,}6 \cdot 2\sin(\frac{1}{2}(500\pi t + 500\pi t - \frac{1}{2}\pi)) \cdot \cos(\frac{1}{2}(500\pi t - 500\pi t + \frac{1}{2}\pi))$
$= 1{,}2\sin(500\pi t - \frac{1}{4}\pi) \cdot \cos(\frac{1}{4}\pi) = 1{,}2\sin(500\pi t - \frac{1}{4}\pi) \cdot \frac{1}{2}\sqrt{2} = 0{,}6\sqrt{2}\sin(500\pi t - \frac{1}{4}\pi)$.

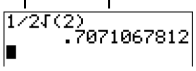
46a $t = \frac{1}{4}\pi$ geeft $\begin{cases} x = \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} \\ y = \sin(2 \cdot \frac{1}{4}\pi) = \sin(\frac{1}{2}\pi) = 1 \end{cases} \Rightarrow$ het punt $(\frac{1}{2}\sqrt{2}, 1)$ ligt op de grafiek van kromme K.



46b Zie de grafieken in figuur 15.30.
Op $t = 0$ is $x = 0$ en $y = 0$ $\Rightarrow$ het punt (0, 0) ligt op de grafiek van kromme K.
Op $t = \frac{1}{2}\pi$ is $x = 1$ en $y = 0$ $\Rightarrow$ het punt (1, 0) ligt op de grafiek van kromme K.

46c

t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\frac{1}{2}\sqrt{2}$	1	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	-1	$-\frac{1}{2}\sqrt{2}$	0
y	0	1	0	-1	0	1	0	-1	0

46d Zie de grafiek van de kromme K in de figuur hiernaast.


47a In de x-richting wordt begonnen in de evenwichtsstand $x = 0$ (de y-as), daarna gaat de kromme naar het maximum $x = 1$, terug door de evenwichtsstand $x = 0$ en door naar het minimum $x = -1$ om te eindigen in de evenwichtsstand $x = 0$.
In de y-richting worden de maxima $y = 1$ en de minima $y = -1$ vier keer bereikt (begin- en eindpunt zijn $y = 0$).

47b Dan in de x-richting worden 2 periodes doorlopen $\Rightarrow a = 2$ en in de y-richting 8 periodes $\Rightarrow b = 8$.

47c Dan in de x-richting worden 3 periodes doorlopen $\Rightarrow a = 3$ en in de y-richting 12 periodes $\Rightarrow b = 12$.

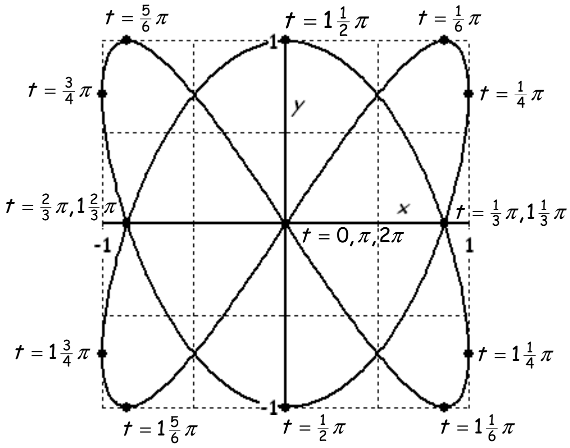
48 In de y-richting wordt 1 periode doorlopen en in de x-richting 2 periodes,
dus de frequentie van x is twee keer zo groot als die van $y \Rightarrow c = 2 \cdot 1 = 2$.

49 In de x-richting worden 2 periodes doorlopen $\Rightarrow a = 2$ en in de y-richting 5 periodes $\Rightarrow b = 5$.

50a Dan in de x-richting worden 2 periodes doorlopen $\Rightarrow a = 2$ en in de y-richting 3 periodes $\Rightarrow b = 3$.

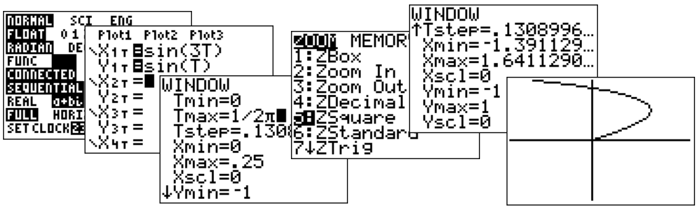
50b $x = \sin(2t)$ is maximaal voor ($2t = \frac{1}{2}\pi + k \cdot 2\pi \Rightarrow$) $t = \frac{1}{4}\pi$ en $t = 1\frac{1}{4}\pi$.
$x = \sin(2t)$ is minimaal voor ($2t = 1\frac{1}{2}\pi + k \cdot 2\pi \Rightarrow$) $t = \frac{3}{4}\pi$ en $t = 1\frac{3}{4}\pi$.
$x = \sin(2t) = 0$ voor ($2t = k \cdot \pi \Rightarrow$) $t = 0$, $t = \frac{1}{2}\pi$, $t = \pi$, $t = 1\frac{1}{2}\pi$ en $t = 2\pi$.

$y = \sin(3t)$ is maximaal voor ($3t = \frac{1}{2}\pi + k \cdot 2\pi \Rightarrow$) $t = \frac{1}{6}\pi,\ t = \frac{5}{6}\pi$ en $t = 1\frac{1}{2}\pi$.
$y = \sin(3t)$ is minimaal voor ($3t = 1\frac{1}{2}\pi + k \cdot 2\pi \Rightarrow$) $t = \frac{1}{2}\pi,\ t = 1\frac{1}{6}\pi$ en $t = 1\frac{5}{6}\pi$.
$y = \sin(3t) = 0$ voor ($3t = k \cdot \pi \Rightarrow$) $t = 0,\ t = \frac{1}{3}\pi,\ t = \frac{2}{3}\pi,\ t = \pi,\ t = 1\frac{1}{3}\pi,\ t = 1\frac{2}{3}\pi$ en $t = 2\pi$.
Zie de t-waarden in de figuur hiernaast.

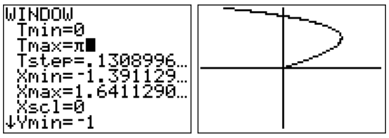


✱✱✱ **Neem GR - practicum 12 door.** (uitwerkingen aan het eind)

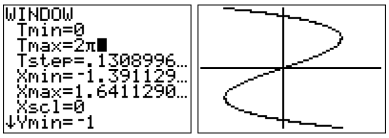
51a



51b



51c

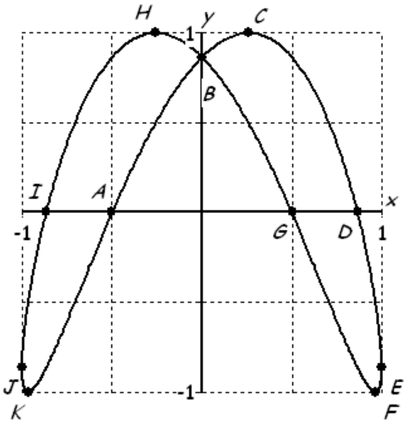


De figuur wordt twee keer doorlopen.

51d Voor t op $\left[\frac{1}{2}\pi, 1\frac{1}{2}\pi\right]$ wordt de kromme één keer doorlopen.

52a In de x-richting wordt $\frac{1}{2}$ periode doorlopen
en in de y-richting $2\frac{1}{2}$ periode $\Rightarrow c = 5$.

52b Voor t op $\left[\frac{1}{2}\pi, 1\frac{1}{2}\pi\right]$ wordt de figuur in de omgekeerde richting doorlopen (van rechtsboven naar linksonder).

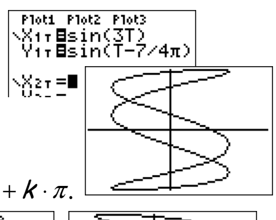


53a De periode van x is 2π.
$x = 0$ voor $t = \frac{1}{6}\pi$ en $t = \frac{1}{6}\pi + \frac{1}{2} \cdot 2\pi = 1\frac{1}{6}\pi$ $\Rightarrow B(0,\ \frac{1}{2}\sqrt{3})$.
$x = 1$ voor $t = \frac{1}{6}\pi + \frac{1}{4} \cdot 2\pi = \frac{2}{3}\pi$ $\Rightarrow E(1,\ -\frac{1}{2}\sqrt{3})$.
$x = -1$ voor $t = \frac{1}{6}\pi + \frac{3}{4} \cdot 2\pi = 1\frac{2}{3}\pi$ $\Rightarrow J(-1,\ -\frac{1}{2}\sqrt{3})$.
De periode van y is π.
$y = 0$ voor $t = 0,\ t = \frac{1}{2}\pi,\ t = \pi,\ t = 1\frac{1}{2}\pi$ en $t = 2\pi$ $\Rightarrow A(-\frac{1}{2}, 0),\ D(\frac{1}{2}\sqrt{3}, 0),\ G(\frac{1}{2}, 0)$ en $I(-\frac{1}{2}\sqrt{3}, 0)$.
$y = 1$ voor $t = \frac{1}{4}\pi$ en $t = 1\frac{1}{4}\pi$ $\Rightarrow C(\sin(\frac{1}{12}\pi), 1)$ en $H(\sin(1\frac{1}{12}\pi), 1)$.
$y = -1$ voor $t = \frac{3}{4}\pi$ en $t = 1\frac{3}{4}\pi$ $\Rightarrow F(\sin(\frac{7}{12}\pi),\ -1)$ en $K(\sin(\frac{7}{12}\pi),\ -1)$.

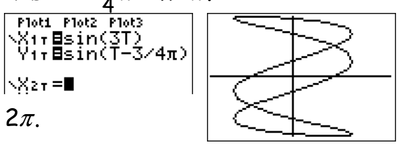
53b $x = \frac{1}{2} \Rightarrow \sin(t - \frac{1}{6}\pi) = \frac{1}{2}$
$t - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi \ \vee\ t - \frac{1}{6}\pi = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
$t = \frac{1}{3}\pi + k \cdot 2\pi \ \vee\ t = \pi + k \cdot 2\pi$.
t op $[0, 2\pi] \Rightarrow t = \frac{1}{3}\pi \ \vee\ t = \pi$.
$t = \frac{1}{3}\pi$ geeft $y = \sin(2 \cdot \frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$
en $t = \pi$ geeft $y = \sin(2 \cdot \pi) = 0$.
De lengte van het lijnstuk is $\frac{1}{2}\sqrt{3}$.

53c $y = x \Rightarrow \sin(t - \frac{1}{6}\pi) = \sin(2t)$
$t - \frac{1}{6}\pi = 2t + k \cdot 2\pi \ \vee\ t - \frac{1}{6}\pi = \pi - 2t + k \cdot 2\pi$
$-t = \frac{1}{6}\pi + k \cdot 2\pi \ \vee\ 3t = 1\frac{1}{6}\pi + k \cdot 2\pi$
$t = -\frac{1}{6}\pi + k \cdot 2\pi \ \vee\ t = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$.
t op $[0, 2\pi] \Rightarrow t = 1\frac{5}{6}\pi \ \vee\ t = \frac{7}{18}\pi \ \vee\ t = 1\frac{1}{18}\pi \ \vee\ t = 1\frac{13}{18}\pi$.

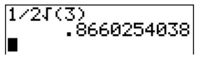
54 In de x-richting worden 3 periodes doorlopen $\Rightarrow a = 3$.
Voor $t = 1\frac{3}{4}\pi$ en $t = \frac{3}{4}\pi$ is $y = 0 \Rightarrow \sin(1\frac{3}{4}\pi + b) = 0$ én $\sin(\frac{3}{4}\pi + b) = 0$
$1\frac{3}{4}\pi + b = k \cdot \pi$ én $\frac{3}{4}\pi + b = k \cdot \pi$
$b = -1\frac{3}{4}\pi + k \cdot \pi$ én $b = -\frac{3}{4}\pi + k \cdot \pi \Rightarrow b = -1\frac{3}{4}\pi + k \cdot \pi$.



Bij $b = -1\frac{3}{4}\pi + k \cdot 2\pi$ hoort de kromme van figuur 15.38.
Bij $b = -\frac{3}{4}\pi + k \cdot 2\pi$ hoort de kromme die het spiegelbeeld van de kromme van figuur 15.38 is bij spiegelen in de x-as. Dus $b = -1\frac{3}{4}\pi + k \cdot 2\pi$.



55a $x = 0$ (de y-as) $\Rightarrow \sin(2t) = 0 \Rightarrow 2t = k \cdot \pi \Rightarrow t = k \cdot \frac{1}{2}\pi$.

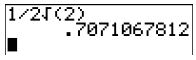
$t = 0$ geeft $y = \sin(\frac{1}{3}\pi) = \sin(2\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3} \approx 0{,}866$. Dus bij $t = 0$ hoort $A(0,\ \frac{1}{2}\sqrt{3})$. 

$t = \frac{1}{2}\pi$ geeft $y = \sin(\frac{1}{2}\pi + \frac{1}{3}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$. Dus bij $t = \frac{1}{2}\pi$ hoort $B(0,\ \frac{1}{2})$.

$t = \pi$ geeft $y = \sin(\pi + \frac{1}{3}\pi) = \sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3} \approx -0{,}866$. Dus bij $t = \pi$ hoort $D(0,\ -\frac{1}{2}\sqrt{3})$.

$t = 1\frac{1}{2}\pi$ geeft $y = \sin(1\frac{1}{2}\pi + \frac{1}{3}\pi) = \sin(1\frac{5}{6}\pi) = -\frac{1}{2}$. Dus bij $t = 1\frac{1}{2}\pi$ hoort $C(0,\ -\frac{1}{2})$.

55b $x = -\frac{1}{2} \Rightarrow \sin(2t) = -\frac{1}{2} \Rightarrow 2t = -\frac{1}{6}\pi + k \cdot 2\pi \ \vee\ 2t = \pi - -\frac{1}{6}\pi + k \cdot 2\pi \Rightarrow t = -\frac{1}{12}\pi + k \cdot \pi \ \vee\ t = \frac{7}{12}\pi + k \cdot \pi$.

$t = -\frac{1}{12}\pi$ geeft $y = \sin(-\frac{1}{12}\pi + \frac{1}{3}\pi) = \sin(\frac{1}{4}\pi) = \frac{1}{2}\sqrt{2} \approx 0{,}707$. Dus bij $t = -\frac{1}{12}\pi$ hoort $E(-\frac{1}{2},\ \frac{1}{2}\sqrt{2})$. 

$t = \frac{11}{12}\pi$ geeft $y = \sin(\frac{11}{12}\pi + \frac{1}{3}\pi) = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$. Dus bij $t = \frac{11}{12}\pi$ hoort $H(-\frac{1}{2},\ -\frac{1}{2}\sqrt{2})$.

$EH = \frac{1}{2}\sqrt{2} - -\frac{1}{2}\sqrt{2} = \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2} = \sqrt{2}$.

55c $t = a$ geeft $x = \sin(2a)$ en $y = \sin(a + \frac{1}{3}\pi)$. Dus $T(\sin(2a),\ \sin(a + \frac{1}{3}\pi))$.

$t = a + \pi$ geeft $x = \sin(2a + 2\pi) = \sin(2a)$ en $y = \sin(a + \pi + \frac{1}{3}\pi) = \sin(a + 1\frac{1}{3}\pi)$. Dus $U(\sin(2a),\ \sin(a + 1\frac{1}{3}\pi))$.

$TU = \left|\sin(a + \frac{1}{3}\pi) - \sin(a + 1\frac{1}{3}\pi)\right| = \left|\sin(a + \frac{1}{3}\pi) - \sin(a + \frac{1}{3}\pi + \pi)\right| = \left|\sin(a + \frac{1}{3}\pi) - -\sin(a + \frac{1}{3}\pi)\right| = \left|2\sin(a + \frac{1}{3}\pi)\right|$.

56a Een cirkel kun je opvatten als de baan van een punt dat deelneemt aan twee harmonische trillingen in twee verschillende (loodrechte) richtingen (met gelijke trillingstijd). Dus een cirkel is een Lissajous-figuur.

56b Een parametervoorstelling van de eenheidscirkel is bijvoorbeeld $\begin{cases} x(t) = \cos(t) \\ y(t) = \sin(t) \end{cases}$ dus $\begin{cases} x(t) = \sin(t + \frac{1}{2}\pi) \\ y(t) = \sin(t) \end{cases}$

57 Op de grafiek in figuur 15.40 ligt het punt $(0,\ -1) \Rightarrow -1 = p \cdot 0^2 + q \Rightarrow -1 = 0 + q \Rightarrow q = -1$.

Dus $y = px^2 - 1$ waaraan ook het punt $(1, 1)$ moet voldoen $\Rightarrow 1 = p \cdot 1^2 - 1 \Rightarrow 2 = p$.

58 Substitutie van een willekeurig punt P met $x_P = \sin(t - \frac{1}{4}\pi)$ en $y_P = \sin(2t)$ in de formule $y = -2x^2 + 1$ geeft

$\sin(2t) = -2\sin^2(t - \frac{1}{4}\pi) + 1$

$\sin(2t) = 1 - 2\sin^2(t - \frac{1}{4}\pi)$

$\sin(2t) = \cos(2(t - \frac{1}{4}\pi))$

$\sin(2t) = \cos(2t - \frac{1}{2}\pi)$

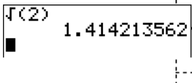
$\sin(2t) = \sin(2t - \frac{1}{2}\pi + \frac{1}{2}\pi)$

$\sin(2t) = \sin(2t)$ (klopt voor elke t).

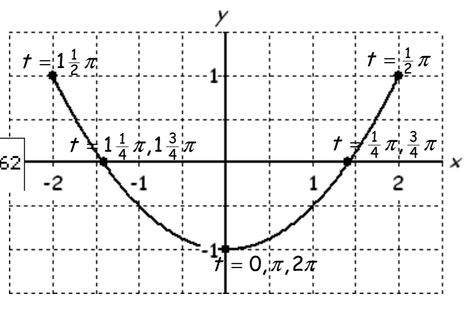
Dus elk punt van de Lissajous-figuur voldoet aan de formule $y = -2x^2 + 1$. Omdat $x = \sin(t - \frac{1}{4}\pi)$ met $-1 \le \sin(t - \frac{1}{4}\pi) \le 1$ hoort bij de parametervoorstelling de formule $y = -2x^2 + 1$ met $-1 \le x \le 1$.

59a

t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
y	-1	0	1	0	-1	0	1	0	-1

Zie de grafiek van de kromme K in de figuur hiernaast. 

De keerpunten zijn $(-2, 1)$ en $(2, 1)$.



59b $y = ax^2 + b$ door $(0,\ -1) \Rightarrow -1 = a \cdot 0^2 + b \Rightarrow -1 = 0 + b \Rightarrow b = -1$.

Dus $y = ax^2 - 1$ door $(2, 1) \Rightarrow 1 = a \cdot 2^2 - 1 \Rightarrow 2 = 4a \Rightarrow a = \frac{1}{2}$.

Vermoedelijk hoort bij K de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \le x \le 2$.

Substitutie van een willekeurig punt P met $x_P = 2\sin(t)$ en $y_P = \sin(2t - \frac{1}{2}\pi)$ in de formule $y = \frac{1}{2}x^2 - 1$ geeft

$\sin(2t - \frac{1}{2}\pi) = \frac{1}{2} \cdot (2\sin(t))^2 - 1$

$\cos(2t - \frac{1}{2}\pi - \frac{1}{2}\pi) = \frac{1}{2} \cdot 4\sin^2(t) - 1$

$\cos(2t - \pi) = 2\sin^2(t) - 1$

$\cos(2t + \pi) = -1 + 2\sin^2(t)$

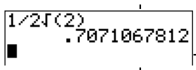
$\cos(2t + \pi) = -(1 - 2\sin^2(t))$

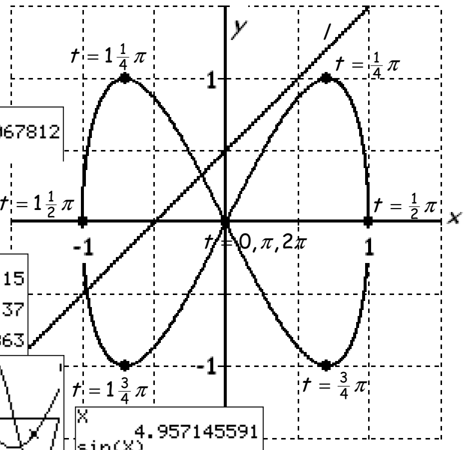
$-\cos(2t) = -\cos(2t)$ (klopt voor elke t).

Elk punt van de Lissajous-figuur voldoet aan de formule $y = \frac{1}{2}x^2 - 1$. Omdat $x = 2\sin(t)$ met $-2 \le 2\sin(t)) \le 2$ hoort bij de parametervoorstelling de formule $y = \frac{1}{2}x^2 - 1$ met $-2 \le x \le 2$.

60a Lijn l: $y = x + \frac{1}{2}$ gaat door $(0, \frac{1}{2})$ en $(-\frac{1}{2}, 0)$.

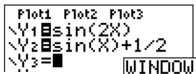
t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\frac{1}{2}\sqrt{2}$	1	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	-1	$-\frac{1}{2}\sqrt{2}$	0
y	0	1	0	-1	0	1	0	-1	0

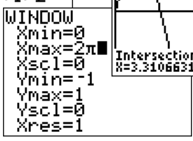


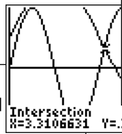


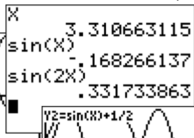
Zie de baan van P in figuur hiernaast.

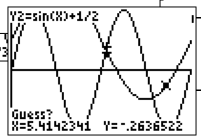
60b $x = \sin(t)$ en $y = \sin(2t)$ invullen in $y = x + \frac{1}{2}$ geeft
$\sin(2t) = \sin(t) + \frac{1}{2}$ (intersect)
$t \approx 3{,}31 \ \vee \ t \approx 4{,}96.$
$t \approx 3{,}31$ geeft $x \approx -0{,}17$ en $y \approx 0{,}33$.
$t \approx 4{,}96$ geeft $x \approx -0{,}97$ en $y \approx -0{,}47$.
Dus l snijdt de baan van P in de punten
$(-0{,}17;\ 0{,}33)$ en $(-0{,}97;\ -0{,}47)$.

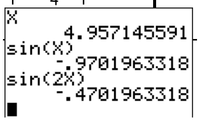












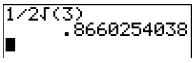
60c Substitutie van $x = \sin(t)$ en $y = \sin(2t)$ in de formule $y^2 = 4x^2 - 4x^4$ geeft
$\sin^2(2t) = 4 \cdot \sin^2(t) - 4 \cdot \sin^4(t)$
$(2\sin(t)\cos(t))^2 = 4\sin^2(t) \cdot \left(1 - \sin^2(t)\right)$
$4\sin^2(t)\cos^2(t) = 4\sin^2(t)\cos^2(t)$
(klopt voor elke t).

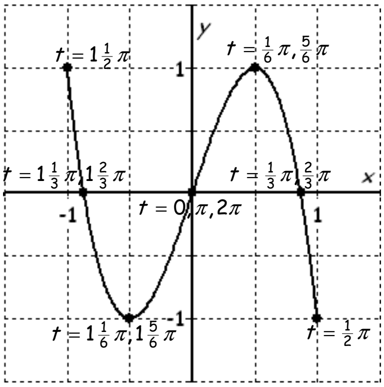
Elk punt van de Lissajous-figuur voldoet aan de formule $y^2 = 4x^2 - 4x^4$.
Omdat $x = \sin(t)$ met $-1 \le \sin(t)) \le 1$ hoort bij de parametervoorstelling
de formule $y^2 = 4x^2 - 4x^4$ met $-1 \le x \le 1$.

61a

t	0	$\frac{1}{6}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$1\frac{1}{6}\pi$	$1\frac{1}{3}\pi$	$1\frac{1}{2}\pi$	$1\frac{2}{3}\pi$	$1\frac{5}{6}\pi$	2π
x	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{3}$	-1	$-\frac{1}{2}\sqrt{3}$	$-\frac{1}{2}$	0
y	0	1	0	-1	0	1	0	-1	0	1	0	-1	0

Zie de grafiek van K in de figuur hiernaast.
De keerpunten zijn $(-1, 1)$ en $(1, -1)$.

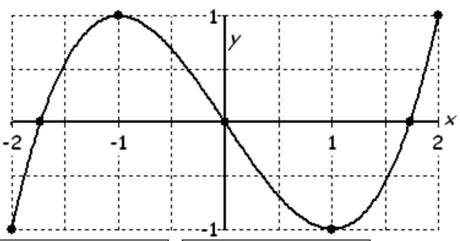




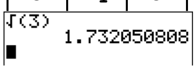
61b Substitutie van $x = \sin(t)$ en $y = \sin(3t)$ in de formule $y = 3x - 4x^3$ geeft
$\sin(3t) = 3\sin(t) - 4\sin^3(t)$
$\sin(t + 2t) = 3\sin(t) - 4\sin^3(t)$
$\sin(t)\cos(2t) + \cos(t)\sin(2t) = 3\sin(t) - 4\sin^3(t)$
$\sin(t) \cdot \left(1 - 2\sin^2(t)\right) + \cos(t) \cdot 2\sin(t)\cos(t) = 3\sin(t) - 4\sin^3(t)$
$\sin(t) - 2\sin^3(t) + 2\sin(t)\cos^2(t) = 3\sin(t) - 4\sin^3(t)$
$\sin(t) - 2\sin^3(t) + 2\sin(t) \cdot \left(1 - \sin^2(t)\right) = 3\sin(t) - 4\sin^3(t)$
$\sin(t) - 2\sin^3(t) + 2\sin(t) - 2\sin^3(t) = 3\sin(t) - 4\sin^3(t)$
$3\sin(t) - 4\sin^3(t) = 3\sin(t) - 4\sin^3(t)$ (klopt voor elke t). Dus alle punten van K liggen op de grafiek van $y = 3x - 4x^3$.

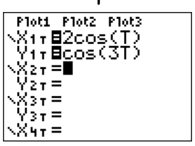
62a

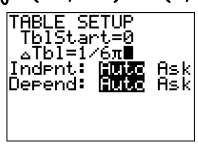
t	0	$\frac{1}{6}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{5}{6}\pi$	π	$1\frac{1}{6}\pi$	$1\frac{1}{3}\pi$	$1\frac{1}{2}\pi$	$1\frac{2}{3}\pi$	$1\frac{5}{6}\pi$	2π
x	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	2
y	1	0	-1	0	1	0	-1	0	1	0	-1	0	1

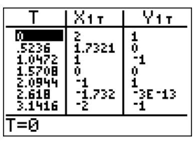


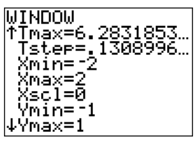
Zie de grafiek van K in de figuur hiernaast.
(de tabel kan ook met TABLE op de GR opgevraagd worden)
De keerpunten zijn $(-2, -1)$ en $(2, 1)$.

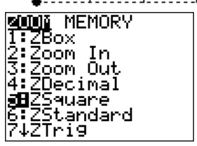


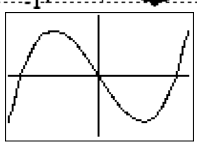












62b $y = ax^3 + bx$ door $(2, 1) \Rightarrow 1 = a \cdot 2^3 + b \cdot 2 \Rightarrow 1 = 8a + 2b$.
$y = ax^3 + bx$ door $(1, -1) \Rightarrow -1 = a \cdot 1^3 + b \cdot 1 \Rightarrow -1 = a + b$.

$$\begin{cases} 1 = 8a + 2b \ ❶ & |\times 1 \\ -1 = a + b \ ❷ & |\times 2 \end{cases} \Rightarrow \begin{cases} 1 = 8a + 2b \\ -2 = 2a + 2b \end{cases} -$$

$3 = 6a \Rightarrow a = \frac{3}{6} = \frac{1}{2}$ in❷ $\Rightarrow -1 = \frac{1}{2} + b \Rightarrow b = -1\frac{1}{2}$.

$$\left.\begin{matrix} x = 2\cos(t) \\ -2 \le 2\cos(t) \le 2 \end{matrix}\right\} \Rightarrow -2 \le x \le 2.$$

Dus $c = -2$ en $d = 2$.

Substitutie van $x = 2\cos(t)$ en $y = \cos(3t)$ in de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ geeft

$\cos(3t) = \frac{1}{2} \cdot (2\cos(t))^3 - 1\frac{1}{2} \cdot 2\cos(t)$

$\cos(2t + t) = 4\cos^3(t) - 3\cos(t)$

$\cos(2t)\cos(t) - \sin(2t)\sin(t) = 4\cos^3(t) - 3\cos(t)$

$\left(2\cos^2(t) - 1\right) \cdot \cos(t) - 2\sin(t)\cos(t) \cdot \sin(t) = 4\cos^3(t) - 3\cos(t)$

$2\cos^3(t) - \cos(t) - 2\sin^2(t)\cos(t) = 4\cos^3(t) - 3\cos(t)$

$2\cos^3(t) - \cos(t) - 2 \cdot \left(1 - \cos^2(t)\right) \cdot \cos(t) = 4\cos^3(t) - 3\cos(t)$

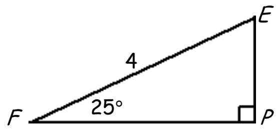
$2\cos^3(t) - \cos(t) - 2\cos(t) + 2\cos^3(t) = 4\cos^3(t) - 3\cos(t)$

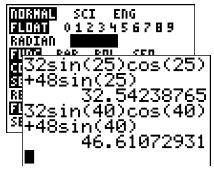
$4\cos^3(t) - 3\cos(t) = 4\cos^3(t) - 3\cos(t)$ (klopt voor elke t).

Dus bij de baan van P hoort de formule $y = \frac{1}{2}x^3 - 1\frac{1}{2}x$ met $-2 \le x \le 2$.

63a $\sin 25° = \frac{EP}{EF} = \frac{EP}{4} \Rightarrow EP = 4 \cdot \sin 25°$.

$\cos 25° = \frac{FP}{EF} = \frac{FP}{4} \Rightarrow FP = 4 \cdot \cos 25°$.

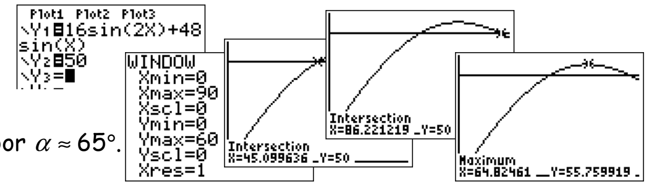




63b $O_{ABCDEF} = 4 \cdot O_{\Delta FPE} + O_{ABDE} = 4 \cdot \frac{1}{2} \cdot FP \cdot PE + AB \cdot AE$

$= 2 \cdot 4\cos 25° \cdot 4\sin 25° + 6 \cdot 2 \cdot 4\sin 25° = 32\sin 25° \cos 25° + 48\sin 25° \approx 32{,}54$.

63c $O_{ABCDEF} = 32\sin 40° \cos 40° + 48\sin 40° \approx 46{,}61$.



64a Zie de eerste twee GR-schermen hiernaast.

64b $16\sin(2\alpha) + 48\sin(\alpha) = 50$ (intersect) $\Rightarrow \alpha \approx 45° \vee \alpha \approx 86°$.

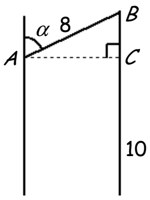
64c $O = 16\sin(2\alpha) + 48\sin(\alpha)$ (optie maximum) $\Rightarrow O_{max} \approx 55{,}76$ voor $\alpha \approx 65°$.

65a $\angle ABC = \alpha$ (Z-hoeken)

$\cos \angle ABC = \frac{BC}{AB}$

$\cos(\alpha) = \frac{BC}{8} \Rightarrow BC = 8\cos(\alpha)$

$h = 10 + BC = 10 + 8\cos(\alpha)$.

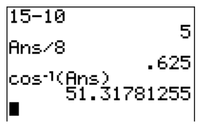


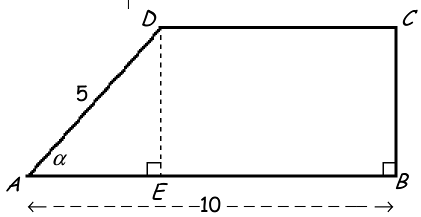
65b $h = 10 + 8\cos(\alpha) = 15$ (intersect of)

$8\cos(\alpha) = 5$

$\cos(\alpha) = \frac{5}{8}$

$\alpha \approx 51°$.





66a $\sin(\alpha) = \frac{DE}{AD} = \frac{DE}{5} \Rightarrow DE = 5\sin(\alpha)$.

$\cos(\alpha) = \frac{AE}{AD} = \frac{AE}{5} \Rightarrow AE = 5\cos(\alpha)$ en $EB = 10 - AE = 10 - 5\cos(\alpha)$.

$O_{ABCD} = O_{\Delta AED} + O_{EBCD} = \frac{1}{2} \cdot AE \cdot DE + EB \cdot DE$

$= \frac{1}{2} \cdot 5\cos(\alpha) \cdot 5\sin(\alpha) + (10 - 5\cos(\alpha)) \cdot 5\sin(\alpha)$

$= 12\frac{1}{2}\sin(\alpha)\cos(\alpha) + 50\sin(\alpha) - 25\sin(\alpha)\cos(\alpha) = 50\sin(\alpha) - 12\frac{1}{2}\sin(\alpha)\cos(\alpha)$.

66b $\left.\begin{array}{l}\sin(180° - \alpha) = \frac{DE}{AD} = \frac{DE}{5} \\ \sin(180° - \alpha) = \sin(\alpha)\end{array}\right\} \Rightarrow \sin(\alpha) = \frac{DE}{5} \Rightarrow DE = 5\sin(\alpha)$.

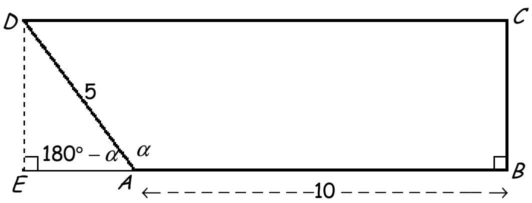
$\left.\begin{array}{l}\cos(180° - \alpha) = \frac{AE}{AD} = \frac{AE}{5} \\ \cos(180° - \alpha) = -\cos(\alpha)\end{array}\right\} \Rightarrow -\cos(\alpha) = \frac{AE}{5} \Rightarrow AE = -5\cos(\alpha)$.

$EB = 10 + AE = 10 - 5\cos(\alpha)$.

$O_{ABCD} = O_{EBCD} - O_{\Delta AED} = EB \cdot DE - \frac{1}{2} \cdot AE \cdot DE$

$= (10 - 5\cos(\alpha)) \cdot 5\sin(\alpha) - \frac{1}{2} \cdot -5\cos(\alpha) \cdot 5\sin(\alpha)$

$= 50\sin(\alpha) - 25\sin(\alpha)\cos(\alpha) + 12\frac{1}{2}\sin(\alpha)\cos(\alpha) = 50\sin(\alpha) - 12\frac{1}{2}\sin(\alpha)\cos(\alpha)$.



66c $\alpha = 90° \Rightarrow O_{ABCD} = 50\sin 90° - 12\frac{1}{2}\sin 90° \cos 90° = 50 \cdot 1 - 12\frac{1}{2} \cdot 1 \cdot 0 = 50$.

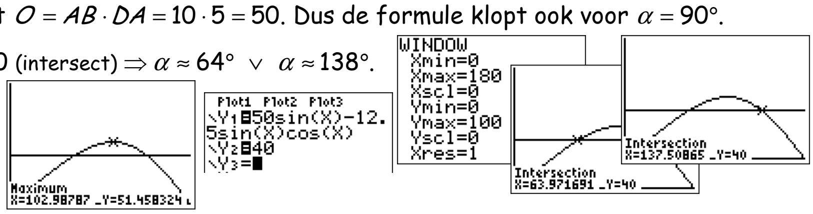
Voor $\alpha = 90°$ is $ABCD$ een rechthoek met $O = AB \cdot DA = 10 \cdot 5 = 50$. Dus de formule klopt ook voor $\alpha = 90°$.

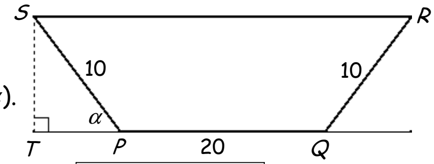
66d $O_{ABCD} = 50\sin(\alpha) - 12\frac{1}{2}\sin(\alpha)\cos(\alpha) = 40$ (intersect) $\Rightarrow \alpha \approx 64° \vee \alpha \approx 138°$.

$O_{ABCD} > 40$ (zie de plot) $\Rightarrow 64° < \alpha < 138°$

66e $O_{ABCD} = 50\sin(\alpha) - 12\frac{1}{2}\sin(\alpha)\cos(\alpha)$

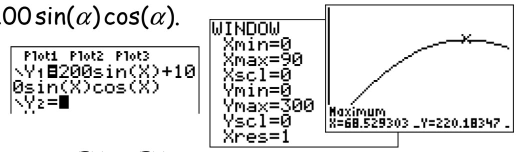
(optie maximum) O maximaal voor $\alpha \approx 103°$.



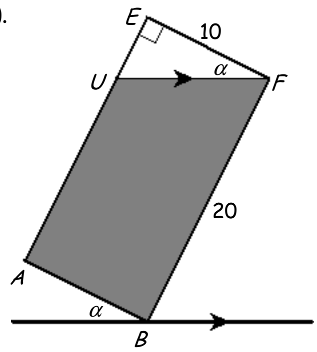


67a $\sin(\alpha)=\frac{ST}{SP}=\frac{ST}{10}\Rightarrow ST=10\sin(\alpha)$.
$\cos(\alpha)=\frac{PT}{SP}=\frac{PT}{10}\Rightarrow PT=10\cos(\alpha)$ en $RS=20+2\cdot 10\cos(\alpha)=20+20\cos(\alpha)$.
$A=O_{PQRS}=\frac{1}{2}\cdot(PQ+RS)\cdot ST=\frac{1}{2}\cdot(20+20+20\cos(\alpha))\cdot 10\sin(\alpha)$
$=5\sin(\alpha)\cdot(40+20\cos(\alpha))=200\sin(\alpha)+100\sin(\alpha)\cos(\alpha)$.

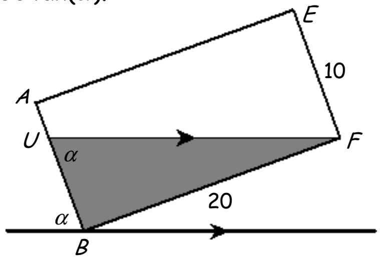
67b $O=200\sin(\alpha)+100\sin(\alpha)\cos(\alpha)$ (optie maximum)
$O_{max}\approx 220\text{ cm}^2$ voor $\alpha\approx 69°$.



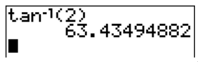
68a $\angle UFE=\alpha$ (de benen van de hoeken lopen evenwijdig) en $\tan(\alpha)=\frac{EU}{EF}=\frac{EU}{10}\Rightarrow EU=10\tan(\alpha)$.
$V=G\cdot h=(O_{ABFE}-O_{\Delta EUF})\cdot FG=(AB\cdot BF-\frac{1}{2}\cdot EU\cdot EF)\cdot FG$
$=(10\cdot 20-\frac{1}{2}\cdot 10\tan(\alpha)\cdot 10)\cdot 10=2\,000-500\tan(\alpha)$.



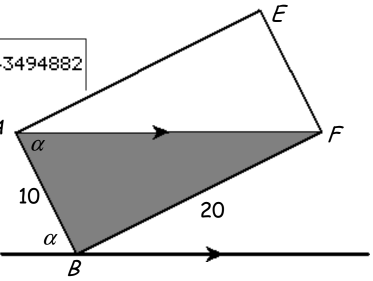
68b In 15.48d is het deel van de bak zonder water geen prisma meer met als grondvlak een driehoek.
$\angle BUF=\alpha$ (Z-hoeken) en
$\tan(\alpha)=\frac{BF}{BU}=\frac{20}{BU}\Rightarrow BU=\frac{20}{\tan(\alpha)}$.
$V=G\cdot h=O_{\Delta BUF}\cdot FG=\frac{1}{2}\cdot BF\cdot BU\cdot FG$
$=\frac{1}{2}\cdot 20\cdot\frac{20}{\tan(\alpha)}\cdot 10=\frac{2000}{\tan(\alpha)}$.

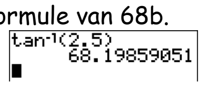


68c In deze situatie is $U=A$ (het wateroppervlak is rechthoek $AFGD$).
$\angle BAF=\alpha$ (Z-hoeken) en $\tan(\alpha)=\tan\angle BAF=\frac{BF}{AB}=\frac{20}{10}=2\Rightarrow\alpha\approx 63{,}4°$.



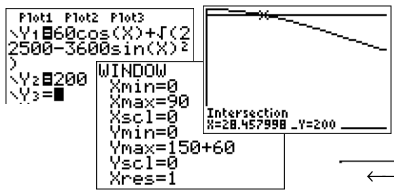
68d Er is 1200 liter uit de bak weggestroomd $\Rightarrow V=2\,000-1200=800$.
Dit is minder dan de helft van de inhoud van de bak $\Rightarrow$ de formule van 68b.
$\frac{2000}{\tan(\alpha)}=800$ (intersect of) $\Rightarrow\tan(\alpha)=\frac{2000}{800}=2\frac{1}{2}\Rightarrow\alpha\approx 68{,}2°$.



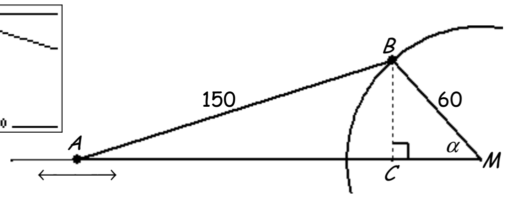


69a $\sin(\alpha)=\frac{BC}{BM}=\frac{BC}{60}\Rightarrow BC=60\sin(\alpha)$ en $\cos(\alpha)=\frac{CM}{BM}=\frac{CM}{60}\Rightarrow CM=60\cos(\alpha)$.
Pyth. in ΔABC: $AC=\sqrt{AB^2-BC^2}=\sqrt{150^2-(60\sin(\alpha))^2}=\sqrt{22\,500-3\,600\sin^2(\alpha)}$.

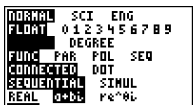
$L=AC+CM$
$=\sqrt{22\,500-3\,600\sin^2(\alpha)}+60\cos(\alpha)$
$=60\cos(\alpha)+\sqrt{22\,500-3\,600\sin^2(\alpha)}$.

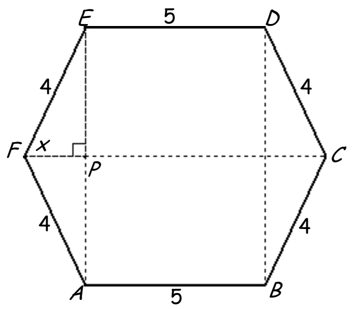


69b $L=200$ (intersect) $\Rightarrow\alpha\approx 28°$.
$L>200$ (zie de plot) $\Rightarrow 0°<\alpha<28°$.



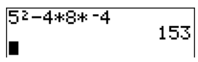
70a $\sin(x)=\frac{EP}{EF}=\frac{EP}{4}\Rightarrow EP=4\sin(x)$.
$\cos(x)=\frac{FP}{EF}=\frac{FP}{4}\Rightarrow FP=4\cos(x)$.
$O_{ABCDEF}=4\cdot O_{\Delta FPE}+O_{ABDE}=4\cdot\frac{1}{2}\cdot FP\cdot PE+AB\cdot AE$
$=2\cdot 4\cos(x)\cdot 4\sin(x)+5\cdot 8\sin(x)=32\sin(x)\cos(x)+40\sin(x)$.



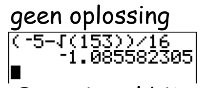


70b $O=16\cdot 2\sin(x)\cos(x)+40\sin(x)=16\sin(2x)+40\sin(x)$.
$\frac{dO}{dx}=16\cos(2x)\cdot 2+40\cos(x)=32\cos(2x)+40\cos(x)$.
$\frac{dO}{dx}=0\Rightarrow 32\cos(2x)+40\cos(x)=0$
$32\left(2\cos^2(x)-1\right)+40\cos(x)=0$
$64\cos^2(x)-32+40\cos(x)=0$
$64\cos^2(x)+40\cos(x)-32=0$
$8\cos^2(x)+5\cos(x)-4=0$ (stel $\cos(x)=t$)

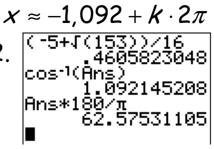
$8t^2+5t-4=0$ met $D=5^2-4\cdot 8\cdot -4=153$



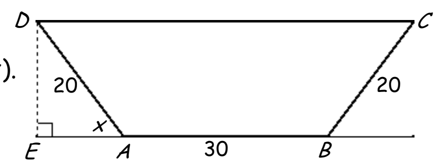
$t=\cos(x)=\frac{-5-\sqrt{153}}{16}\ \vee\ t=\cos(x)=\frac{-5+\sqrt{153}}{16}$
geen oplossing $\quad x\approx 1{,}092+k\cdot 2\pi\ \vee\ x\approx -1{,}092+k\cdot 2\pi$



x op $\left[0,\frac{1}{2}\pi\right]\Rightarrow x\approx 1{,}092$.
O maximaal bij een hoek van Ans$\times\frac{180°}{\pi}\approx 63°$.



71 $\sin(x)=\frac{DE}{AD}=\frac{DE}{20}\Rightarrow DE=20\sin(x)$.
$\cos(x)=\frac{EA}{AD}=\frac{EA}{20}\Rightarrow EA=20\cos(x)$ en $CD=30+2\cdot 20\cos(x)=30+40\cos(x)$.
$O_{ABCD}=\frac{1}{2}\cdot(AB+CD)\cdot DE=\frac{1}{2}\cdot(30+30+40\cos(x))\cdot 20\sin(x)$
$=10\sin(x)\cdot(60+40\cos(x))=600\sin(x)+400\sin(x)\cos(x)$.



$O_{ABCD} = 600\sin(x) + 200 \cdot 2\sin(x)\cos(x) = 600\sin(x) + 200\sin(2x)$.

$\frac{dO}{dx} = 600\cos(x) + 200\cos(2x)\cdot 2 = 600\cos(x) + 400\cos(2x)$.

$\frac{dO}{dx} = 0 \Rightarrow 600\cos(x) + 400\cos(2x) = 0$

$600\cos(x) + 400\left(2\cos^2(x) - 1\right) = 0$

$600\cos(x) + 800\cos^2(x) - 400 = 0$

$800\cos^2(x) + 600\cos(x) - 400 = 0$

$4\cos^2(x) + 3\cos(x) - 2 = 0$ (stel $\cos(x) = t$)

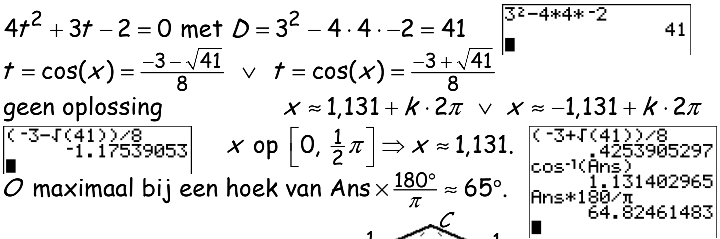
$4t^2 + 3t - 2 = 0$ met $D = 3^2 - 4\cdot 4 \cdot -2 = 41$

$t = \cos(x) = \frac{-3-\sqrt{41}}{8} \vee t = \cos(x) = \frac{-3+\sqrt{41}}{8}$

geen oplossing $\quad x \approx 1{,}131 + k\cdot 2\pi \vee x \approx -1{,}131 + k\cdot 2\pi$

x op $\left[0, \frac{1}{2}\pi\right] \Rightarrow x \approx 1{,}131$.

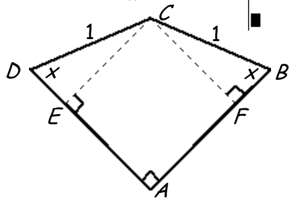
O maximaal bij een hoek van $\text{Ans} \times \frac{180°}{\pi} \approx 65°$.



72a $\sin(x) = \frac{EC}{CD} = \frac{EC}{1} \Rightarrow EC = \sin(x) = EA$ en $\cos(x) = \frac{DE}{CD} = \frac{DE}{1} \Rightarrow DE = \cos(x)$.

$O_{ABCD} = O_{AECF} + 2\cdot O_{\Delta DEC} = EA \cdot EC + 2 \cdot \frac{1}{2} \cdot DE \cdot EC$

$= \sin(x)\cdot\sin(x) + \cos(x)\cdot\sin(x) = \sin^2(x) + \sin(x)\cdot\cos(x)$



72b $O = \sin^2(x) + \sin(x)\cdot\cos(x)$

$\frac{dO}{dx} = 2\sin(x)\cdot\cos(x) + \cos(x)\cdot\cos(x) + \sin(x)\cdot -\sin(x) = \sin(2x) + \cos^2(x) - \sin^2(x) = \sin(2x) + \cos(2x)$.

$\frac{dO}{dx} = 0 \Rightarrow \sin(2x) + \cos(2x) = 0 \Rightarrow \sin(2x) = -\cos(2x) \Rightarrow \cos(2x - \frac{1}{2}\pi) = \cos(2x + \pi)$

$2x - \frac{1}{2}\pi = 2x + \pi + k\cdot 2\pi \vee 2x - \frac{1}{2}\pi = -2x - \pi + k\cdot 2\pi$

geen oplossing $\quad 4x = -\frac{1}{2}\pi + k\cdot 2\pi \Rightarrow x = -\frac{1}{8}\pi + k\cdot\frac{1}{2}\pi$. Er moet gelden: x op $\left[0, \frac{1}{2}\pi\right] \Rightarrow x = \frac{3}{8}\pi$.

73a $\sin(x) = \frac{NP}{MN} = \frac{NP}{12} \Rightarrow NP = 12\sin(x)$ en $\cos(x) = \frac{MP}{MN} = \frac{MP}{12} \Rightarrow MP = 12\cos(x)$.

Verder is $BP = BN + NP = 5 + 12\sin(x)$ en $AP = AM + MP = 7 + 12\cos(x)$.

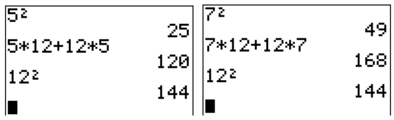
Pyth. in ΔABP: $L = AB = \sqrt{BP^2 + AP^2} = \sqrt{(5+12\sin(x))^2 + (7+12\cos(x))^2}$

$= \sqrt{25 + 120\sin(x) + 144\sin^2(x) + 49 + 168\cos(x) + 144\cos^2(x)}$

$= \sqrt{144\left(\sin^2(x) + \cos^2(x)\right) + 120\sin(x) + 168\cos(x) + 74}$

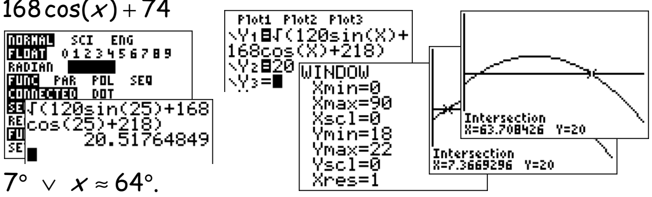
$= \sqrt{144\cdot 1 + 120\sin(x) + 168\cos(x) + 74}$

$= \sqrt{120\sin(x) + 168\cos(x) + 218}$.



73b $L = \sqrt{120\sin(25°) + 168\cos(25°) + 218} \approx 20{,}5$ dm.

73c $L = \sqrt{120\sin(x) + 168\cos(x) + 218} = 20$ (intersect) $\Rightarrow x \approx 7° \vee x \approx 64°$.

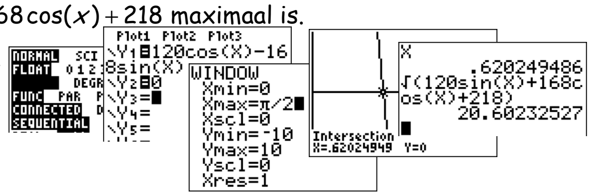


73d $L = \sqrt{120\sin(x) + 168\cos(x) + 218}$ is maximaal als $120\sin(x) + 168\cos(x) + 218$ maximaal is.

$y = 120\sin(x) + 168\cos(x) + 218 \Rightarrow \frac{dy}{dx} = 120\cos(x) - 168\sin(x)$.

$\frac{dy}{dx} = 0 \Rightarrow 120\cos(x) - 168\sin(x) = 0$ (intersect) $\Rightarrow x \approx 0{,}62$ (rad).

$L_{max} = \sqrt{120\sin(\text{Ans}) + 168\cos(\text{Ans}) + 218} \approx 20{,}6$ dm.



74a De gemiddelde snelheid gedurende de eerste zes seconden is $\frac{v(0)+v(6)}{2} = \frac{0+10}{2} = 5$ m/s.

Gedurende de eerste zes seconden wordt $5\cdot 6 = 30$ meter afgelgd.

74b $s(t)$ (de afstandsfunctie) is een primitieve van $v(t)$ (de snelheidsfunctie).

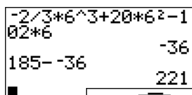
$O_{\Delta OAB} = \int_0^6 v(t)\,dt = \left[s(t)\right]_0^6 = s(6) - s(0)$ is de afgelegde afstand gedurende de eerste zes seconden.

74c De afgelegde afstand op het interval $[0, 15]$ is O(gehele driehoek) $= \frac{1}{2}\cdot 15 \cdot 10 = 75$ m.

75 $N(0) + \int_0^6 (s^2 + 4s + 6)\,ds = N(6)$. (dus de berekening van Julia komt op hetzelfde neer als de berekening in het voorbeeld)

76a $N(t) = N(6) + \int_6^t N'(s)\,ds = 185 + \int_6^t (-2s^2 + 40s - 102)\,ds = 185 + \left[-\frac{2}{3}s^3 + 20s^2 - 102s\right]_6^t$

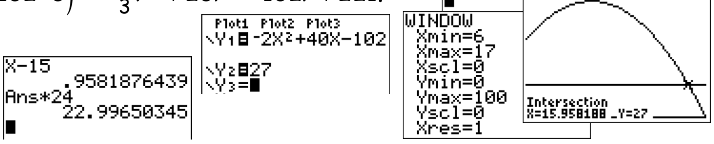
$= 185 - \frac{2}{3}t^3 + 20t^2 - 102t - \left(-\frac{2}{3}\cdot 6^3 + 20\cdot 6^2 - 102\cdot 6\right) = -\frac{2}{3}t^3 + 20t^2 - 102t + 221$.



76b $N'(3) = 3^2 + 4\cdot 3 + 6 = 9 + 12 + 6 = 27$.

$-2t^2 + 40t - 102 = 27$ (intersect) $\Rightarrow t \approx 15{,}96$ (dagen).

Dit is na 15 dagen en (afgerond) 23 uur.



77a $I(t) = I(0) + \int_0^t I'(s)\,ds = 1 + \int_0^t \left(10 - 10e^{-0,1s}\right) ds$ (I het aantal in honderdtallen en $0 \le t < 25$)

$= 1 + \left[10s - 10e^{-0,1s} \cdot \frac{1}{-0,1}\right]_0^t = 1 + \left[10s + 100e^{-0,1s}\right]_0^t = 1 + 10t + 100e^{-0,1t} - \left(0 + 100e^0\right) = 10t + 100e^{-0,1t} - 99.$

77b $I'(10) = 10 - 10e^{-0,1 \cdot 10} = 10 - 10e^{-1}.$

$10 - 10e^{0,1t-5,5} = 10 - 10e^{-1} \Rightarrow -10e^{0,1t-5,5} = -10e^{-1} \Rightarrow e^{0,1t-5,5} = e^{-1} \Rightarrow 0,1t - 5,5 = -1 \Rightarrow 0,1t = 4,5 \Rightarrow t = 45.$

77c Nee, het betekent dat (voor $25 \le t < 30$) het aantal insecten met constante snelheid $I'(t) = 10 - 10e^{-2,5}$ groeit.

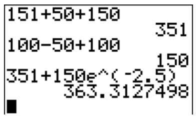
77d $I(55) = I(25) + \int_{25}^{30} I'(t)\,dt + \int_{30}^{55} I'(t)\,dt = I(25) + \int_{25}^{30}\left(10 - 10e^{-2,5}\right) dt + \int_{30}^{55}\left(10 - 10e^{0,1t-5,5}\right) dt$

$= 10 \cdot 25 + 100e^{-0,1 \cdot 25} - 99 + \left[\left(10 - 10e^{-2,5}\right) \cdot t\right]_{25}^{30} + \left[10t - 100e^{0,1t-5,5}\right]_{30}^{55}$

$= 151 + 100e^{-2,5} + \left(10 - 10e^{-2,5}\right) \cdot 30 - \left(10 - 10e^{-2,5}\right) \cdot 25 + 550 - 100e^0 - \left(300 - 100e^{-2,5}\right)$

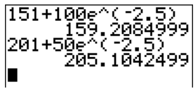
$= 151 + 100e^{-2,5} + \left(10 - 10e^{-2,5}\right) \cdot 5 + 550 - 100 - 300 + 100e^{-2,5}$

$= 151 + 100e^{-2,5} + 50 - 50e^{-2,5} + 150 + 100e^{-2,5} = 351 + 150e^{-2,5} \approx 363$ ($\times 100$).



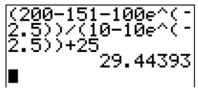
Dus op $t = 55$ zijn er ongeveer 36 300 insecten.

77e $I(25)$ (zie 77d) $= 151 + 100e^{-2,5} \approx 159$ ($\times 100$).



$I(30)$ (zie 77d) $= I(25) + \int_{25}^{30} I'(t)\,dt = 151 + 100e^{-2,5} + 50 - 50e^{-2,5} = 201 + 50e^{-2,5} \approx 205$ ($\times 100$).

Dus in fase 2 is er ergens een tijdstip waarop er 20 000 insecten zijn.



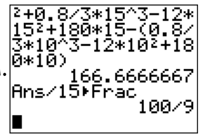
$I(t) = 200 \Rightarrow I(25) + \int_{25}^{t} I'(s)\,ds = 200 \Rightarrow 151 + 100e^{-2,5} + \left(10 - 10e^{-2,5}\right) \cdot (t - 25) = 200$ (intersect of) $\Rightarrow$

$\left(10 - 10e^{-2,5}\right) \cdot (t - 25) = 200 - 151 - 100e^{-2,5} \Rightarrow t - 25 = \frac{200 - 151 - 100e^{-2,5}}{10 - 10e^{-2,5}} \Rightarrow t = \frac{200 - 151 - 100e^{-2,5}}{10 - 10e^{-2,5}} + 25 \approx 29,4$ (dagen).

78a $s(15) = s(0) + \int_0^{10} v(t)\,dt + \int_{10}^{15} v(t)\,dt = 0 + \int_0^{10}\left(-0,2t^2 + 4t\right) dt + \int_{10}^{15}\left(0,8t^2 - 24t + 180\right) dt$

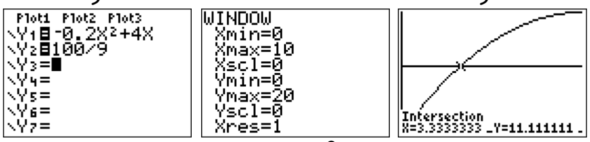
$= \left[-\frac{0,2}{3}t^3 + 2t^2\right]_0^{10} + \left[\frac{0,8}{3}t^3 - 12t^2 + 180t\right]_{10}^{15}$

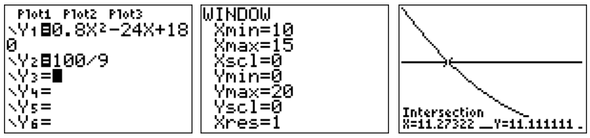
$= \left(-\frac{0,2}{3} \cdot 10^3 + 2 \cdot 10^2\right) - 0 + \left(\frac{0,8}{3} \cdot 15^3 - 12 \cdot 15^2 + 180 \cdot 15\right) - \left(\frac{0,8}{3} \cdot 10^3 - 12 \cdot 10^2 + 180 \cdot 10\right) = 166\frac{2}{3}$ m.



78b De gemiddelde snelheid gedurende de 15 seconden is $\frac{166\frac{2}{3}}{15} = \frac{100}{9}$ m/s.

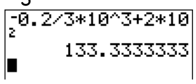
$v(t) = \frac{100}{9}$ (met $0 < t < 10$) $\Rightarrow -0,2t^2 + 4t = \frac{100}{9}$ (intersect of algebraïsch) $\Rightarrow t \approx 3,33.$





$v(t) = \frac{100}{9}$ ($10 < t < 15$) $\Rightarrow 0,8t^2 - 24t + 180 = \frac{100}{9}$ (intersect of algebraïsch) $\Rightarrow t \approx 11,27.$

78c $s(10) = s(0) + \int_0^{10} v(t)\,dt = 0 + \int_0^{10}\left(-0,2t^2 + 4t\right) dt = \left[-\frac{0,2}{3}t^3 + 2t^2\right]_0^{10} = -\frac{0,2}{3} \cdot 10^3 + 2 \cdot 10^2 - 0 = 133\frac{1}{3}$ (m).

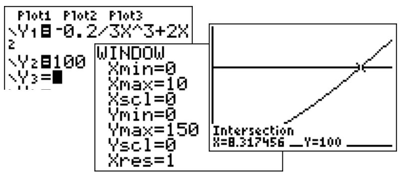


Dus binnen de eerste 10 seconden is 100 meter afgelgd.

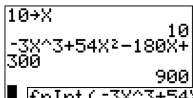
$s(t)$ ($0 \le t < 10$) $= s(0) + \int_0^t v(p)\,dp = 0 + \int_0^t \left(-0,2p^2 + 4p\right) dp$

$= \left[-\frac{0,2}{3}p^3 + 2p^2\right]_0^t = -\frac{0,2}{3}t^3 + 2t^2.$

$s(t) = 100 \Rightarrow -\frac{0,2}{3}t^3 + 2t^2 = 100$ (intersect) $\Rightarrow t \approx 8,3$ (sec).



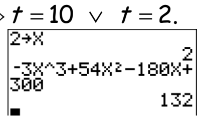
79a $F(t) = -3t^3 + 54t^2 - 180t + 300 \Rightarrow F'(t) = -9t^2 + 108t - 180.$

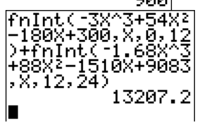


$F'(t) = 0 \Rightarrow -9t^2 + 108t - 180 = 0 \Rightarrow t^2 - 12t + 20 = 0 \Rightarrow (t - 10)(t - 2) = 0 \Rightarrow t = 10 \;\vee\; t = 2.$

$t = 2$ (in de vroege ochtend) geeft minimum (gegeven) $F(2) = 132$ (m^3/uur).

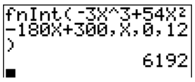
$t = 10$ (in de loop van de ochtend) geeft maximum (gegeven) $F(10) = 900$ (m^3/uur).



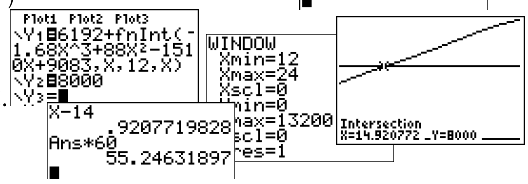


79b $\int_0^{12}\left(-3t^3 + 54t^2 - 180t + 300\right) dt + \int_{12}^{24}\left(-1,68t^3 + 88t^2 - 1510t + 9083\right) dt$ (fnInt) ≈ 13200 (m^3).

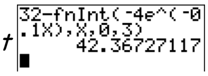
79c $\int_0^{12}\left(-3t^3 + 54t^2 - 180t + 300\right) dt$ (fnInt) $= 6192$ (m^3).

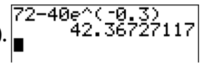


$6192 + \int_{12}^{t}\left(-1,68x^3 + 88x^2 - 1510x + 9083\right) dx$ (intersect) $\Rightarrow t \approx 14,92.$



Het is dan (ongeveer) 14:55.

80a $v(3) = v(0) + \int_0^3 a(t)\,dt \Rightarrow v(0) = 32 - \int_0^3 \left(-4e^{-0,1t}\right) dt$ 

$= 32 - \left[40e^{-0,1t}\right]_0^3 = 32 - 40e^{-0,3} + 40 = 72 - 40e^{-0,3} \approx 42,4$ (m/s). 

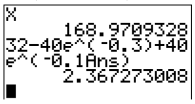
80b $v(t) = v(3) + \int_3^t a(p)\,dp = 32 + \int_3^t \left(-4e^{-0,1p}\right) dp = 32 + \left[40e^{-0,1p}\right]_3^t = 32 + 40e^{-0,1t} - 40e^{-0,3}$.

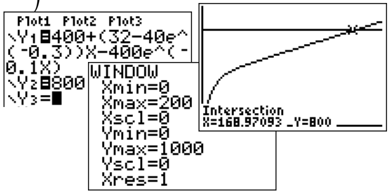
$s(t) = s(0) + \int_0^t v(p)\,dp = 0 + \int_0^t \left(32 - 40e^{-0,3} + 40e^{-0,1p}\right) dp = \left[\left(32 - 40e^{-0,3}\right) \cdot p - 400e^{-0,1p}\right]_0^t$

$= \left(32 - 40e^{-0,3}\right) \cdot t - 400e^{-0,1t} - (0 - 400) = 400 + \left(32 - 40e^{-0,3}\right) \cdot t - 400e^{-0,1t}$.

$s(t) = 800 \Rightarrow 400 + \left(32 - 40e^{-0,3}\right) \cdot t - 400e^{-0,1t} = 800$ (intersect) $\Rightarrow t \approx 168,97$ (sec).

$v(\text{Ans}) = 32 - 40e^{-0,3} + 40e^{-0,1\text{Ans}} \approx 2,4$ m/s.





81a $OZ : ZD = 2 : 1 \Rightarrow OZ = \frac{2}{3} OD$ met $D(1\frac{1}{2}, 3) \Rightarrow Z(1, 2)$.

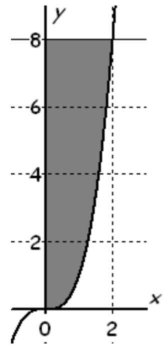
81b $\int_0^3 (x \cdot y)\,dx = \int_0^3 (x \cdot (-2x + 6))\,dx = \int_0^3 \left(-2x^2 + 6x\right) dx = \left[-\frac{2}{3}x^3 + 3x^2\right]_0^3 = -\frac{2}{3} \cdot 3^3 + 3 \cdot 3^2 - 0 = -2 \cdot 3^2 + 3^3 = 9$

en $\int_0^3 y\,dx = \int_0^3 (-2x + 6)\,dx = \left[-x^2 + 6x\right]_0^3 = -3^2 + 6 \cdot 3 - 0 = -9 + 18 = 9$. Dus $x_Z = \frac{9}{9} = 1$ (klopt).

$\int_0^6 (y \cdot x)\,dy = \int_0^6 \left(y \cdot (-\frac{1}{2}y + 3)\right) dy = \int_0^6 \left(-\frac{1}{2}y^2 + 3y\right) dy = \left[-\frac{1}{6}y^3 + \frac{3}{2}y^2\right]_0^6 = -\frac{1}{6} \cdot 6^3 + \frac{3}{2} \cdot 6^2 - 0 = -1 \cdot 6^2 + 1\frac{1}{2} \cdot 6^2 = 18$

en $\int_0^6 x\,dy = \int_0^6 \left(-\frac{1}{2}y + 3\right) dx = \left[-\frac{1}{4}y^2 + 3y\right]_0^6 = -\frac{1}{4} \cdot 6^2 + 3 \cdot 6 - 0 = -\frac{36}{4} + 18 = -9 + 18 = 9$. Dus $y_Z = \frac{18}{9} = 2$ (klopt).

82 $x^3 = 8 \Rightarrow x = 2$ en $y = x^3 \Rightarrow x = \sqrt[3]{y} = y^{\frac{1}{3}}$.

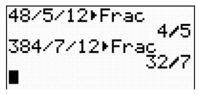


$O(V) = \int_0^2 \left(8 - x^3\right) dx = \left[8x - \frac{1}{4}x^4\right]_0^2 = 8 \cdot 2 - \frac{1}{4} \cdot 2^4 - 0 = 16 - 4 = 12$.

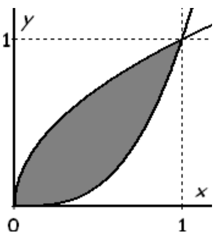
$\int_0^2 \left(x \cdot (8 - x^3)\right) dx = \int_0^2 \left(8x - x^4\right) dx = \left[4x^2 - \frac{1}{5}x^5\right]_0^2 = 4 \cdot 2^2 - \frac{1}{5} \cdot 2^5 - 0 = 16 - \frac{32}{5} = \frac{48}{5}$.

$\int_0^8 (y \cdot x)\,dy = \int_0^8 \left(y \cdot \sqrt[3]{y}\right) dy = \int_0^8 \left(y^{1\frac{1}{3}}\right) dy = \left[\frac{1}{2\frac{1}{3}} y^{2\frac{1}{3}} \cdot \frac{3}{3}\right]_0^8 = \left[\frac{3}{7} y^{2\frac{1}{3}}\right]_0^8 = \frac{3}{7} \cdot 8^2 \cdot \sqrt[3]{8} - 0 = \frac{3}{7} \cdot 64 \cdot 2 = \frac{384}{7}$.

$x_Z = \frac{\frac{48}{5}}{12} = \frac{48}{5} \cdot \frac{1}{12} = \frac{4}{5}$ en $y_Z = \frac{\frac{384}{7}}{12} = \frac{384}{7} \cdot \frac{1}{12} = \frac{32}{7}$. Dus $Z(\frac{4}{5}, \frac{32}{7})$.



83 $x^3 = \sqrt{x}$ (kwadrateren) $\Rightarrow x^6 = x \Rightarrow x^6 - x = 0 \Rightarrow x(x^5 - 1) = 0 \Rightarrow x = 0$ (vold.) $\vee$ $x = 1$ (vold.).



$O(V) = \int_0^1 \left(\sqrt{x} - x^3\right) dx = \int_0^1 \left(x^{\frac{1}{2}} - x^3\right) dx = \left[\frac{1}{1\frac{1}{2}} x^{1\frac{1}{2}} - \frac{1}{4}x^4\right]_0^1 = \frac{2}{3} \cdot 1 - \frac{1}{4} \cdot 1 - 0 = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.

$\int_0^1 \left(x \cdot (\sqrt{x} - x^3)\right) dx = \int_0^1 \left(x^{1\frac{1}{2}} - x^4\right) dx = \left[\frac{1}{2\frac{1}{2}} x^{2\frac{1}{2}} - \frac{1}{5}x^5\right]_0^1 = \frac{2}{5} \cdot 1 - \frac{1}{5} \cdot 1 - 0 = \frac{1}{5}$.

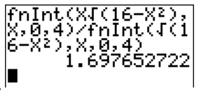
$y = x^3 \Rightarrow x = \sqrt[3]{y} = y^{\frac{1}{3}}$ en $y = \sqrt{x} \Rightarrow x = y^2$.

$\int_0^1 \left(y \cdot (\sqrt[3]{y} - y^2)\right) dy = \int_0^1 \left(y^{1\frac{1}{3}} - y^3\right) dy = \left[\frac{1}{2\frac{1}{3}} y^{2\frac{1}{3}} - \frac{1}{4}y^4\right]_0^1 = \frac{3}{7} \cdot 1 - \frac{1}{4} \cdot 1 - 0 = \frac{12}{28} - \frac{7}{28} = \frac{5}{28}$.

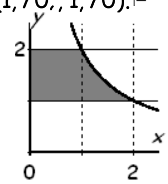
$x_Z = \frac{\frac{1}{5}}{\frac{5}{12}} = \frac{1}{5} \cdot \frac{12}{5} = \frac{12}{25}$ en $y_Z = \frac{\frac{5}{28}}{\frac{5}{12}} = \frac{5}{28} \cdot \frac{12}{5} = \frac{12}{28} = \frac{3}{7}$. Dus $Z(\frac{12}{25}, \frac{3}{7})$.

84 $x^2 + y^2 = 16 \Rightarrow y^2 = 16 - x^2$ $(y \geq 0) \Rightarrow y = \sqrt{16 - x^2}$.

$x_Z = \dfrac{\int_0^4 \left(x\sqrt{16 - x^2}\right) dx}{\int_0^4 \sqrt{16 - x^2}\,dx}$ (fnInt) $\approx 1,70$ en op grond van de symmetrie is $y_Z = x_Z$. Dus $Z(1,70;, 1,70)$.



85 $\frac{2}{x} = 2 \Rightarrow x = 1$ en $\frac{2}{x} = 1 \Rightarrow x = 2$. $y = \frac{2}{x} \Rightarrow xy = 2 \Rightarrow x = \frac{2}{y}$.



$O(V) = 1 \cdot 1 + \int_1^2 \left(\frac{2}{x} - 1\right) dx = 1 + \left[2\ln|x| - x\right]_1^2 = 1 + (2\ln(2) - 2) - (2\ln(1) - 1) = 2\ln(2)$.

$\int_0^1 (x \cdot (2 - 1))\,dx + \int_1^2 \left(x(\frac{2}{x} - 1)\right) dx = \int_0^1 x\,dx + \int_1^2 (2 - x))\,dx = \left[\frac{1}{2}x^2\right]_0^1 + \left[2x - \frac{1}{2}x^2\right]_1^2 = \frac{1}{2} - 0 + (4 - 2) - \left(2 - \frac{1}{2}\right) = 1$.

$\int_1^2 (y \cdot x)\,dy = \int_1^2 \left(y \cdot \frac{2}{y}\right) dy = \int_1^2 2\,dy = \left[2y\right]_1^2 = 4 - 2 = 2$. Dus $x_Z = \frac{1}{2\ln(2)} = 2\ln(2)$ en $y_Z = \frac{2}{2\ln(2)} = \frac{1}{\ln(2)}$.

86 $\ln(x) = 0 \Rightarrow x = e^0 = 1$ en $\ln(x) = 1 \Rightarrow x = e^1 = e$. $y = \ln(x) \Rightarrow x = e^y$.

$O(V) = 1 \cdot 1 + \int_1^e (1 - \ln(x))\, dx$ (fnInt) $\approx 1{,}718$.

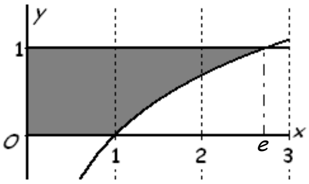
$\int_0^1 (x \cdot 1)\, dx + \int_1^e (x \cdot (1 - \ln(x)))\, dx$ (fnInt) $\approx 1{,}597 \Rightarrow x_Z \approx \frac{1{,}597}{1{,}718} \approx 0{,}93$.

$\int_0^1 \left(y \cdot e^y\right) dy$ (fnInt) $= 1 \Rightarrow y_Z \approx \frac{1}{1{,}718} \approx 0{,}58$.

```
1*1+fnInt(1-ln(X
),X,1,e)→A
      1.718281828
fnInt(X,X,0,1)+f
nInt(X(1-ln(X)),
X,1,e)→B
      1.597264025
```

```
B/A
     .9295704571
```

```
fnInt(X(e^(X)),X
,0,1)→C
                1
Ans/A
     .5819767069
```



87 $I = \int_3^6 \pi y^2\, dx = \int_3^6 \left(\pi(36 - x^2)\right) dx = \left[\pi(36x - \frac{1}{3}x^3)\right]_3^6 = \pi(36 \cdot 6 - \frac{1}{3} \cdot 6^3) - \pi(36 \cdot 3 - \frac{1}{3} \cdot 3^3) = 45\pi$.

$\int_3^6 \pi x y^2\, dx = \int_3^6 \left(\pi(36x - x^3)\right) dx = \left[\pi(18x^2 - \frac{1}{4}x^4)\right]_3^6 = \pi(18 \cdot 6^2 - \frac{1}{4} \cdot 6^4) - \pi(18 \cdot 3^2 - \frac{1}{4} \cdot 3^4) = 182{,}25\pi$.

Dus $x_Z \approx \frac{182{,}25\pi}{45\pi} = 4{,}05$.

```
36*6-1/3*6^3-(36
*3-1/3*3^3)
               45
18*6²-1/4*6^4-(1
8*3²-1/4*3^4)
           182.25
```

```
182.25/45
             4.05
```

88 $I = \int_{-2}^4 \pi y^2\, dx = \int_{-2}^4 \left(\pi(36 - x^2)\right) dx = \left[\pi(36x - \frac{1}{3}x^3)\right]_{-2}^4 = \pi(36 \cdot 4 - \frac{1}{3} \cdot 4^3) - \pi(36 \cdot -2 - \frac{1}{3} \cdot (-2)^3) = 192\pi$.

$\int_{-2}^4 \pi x y^2\, dx = \int_{-2}^4 \left(\pi(36x - x^3)\right) dx = \left[\pi(18x^2 - \frac{1}{4}x^4)\right]_{-2}^4 = \pi(18 \cdot 4^2 - \frac{1}{4} \cdot 4^4) - \pi(18 \cdot (-2)^2 - \frac{1}{4} \cdot (-2)^4) = 156\pi$.

Dus $x_Z \approx \frac{156\pi}{192\pi} = \frac{13}{16}$.

```
156/192
            .8125
Ans▸Frac
            13/16
```

```
36*4-1/3*4^3-(36
*-2-1/3*(-2)^3)
              192
18*4²-1/4*4^4-(1
8*(-2)²-1/4*(-2)
^4)
              156
```

89 Het afgeknotte deel ontstaat door het vlakdel V, ingesloten door de lijn l, de x-as, de lijn $x = 6$ en de y-as te wentelen om de x-as.

l: $y = ax + b$ met $a = \frac{-5}{10} = -\frac{1}{2}$ door $(0, 5) \Rightarrow b = 5$. Dus l: $y = -\frac{1}{2}x + 5$.

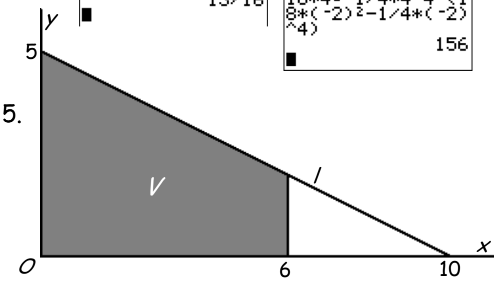
$I = \int_0^6 \pi y^2\, dx = \int_0^6 \left(\pi(-\frac{1}{2}x + 5)^2\right) dx = \int_0^6 \left(\pi(\frac{1}{4}x^2 - 5x + 25)\right) dx$

$= \left[\pi(\frac{1}{12}x^3 - \frac{5}{2}x^2 + 25x)\right]_0^6 = \pi(\frac{1}{12} \cdot 6^3 - \frac{5}{2} \cdot 6^2 + 25 \cdot 6) - 0 = 78\pi$.

$\int_0^6 \pi x y^2\, dx = \int_0^6 \left(\pi x(-\frac{1}{2}x + 5)^2\right) dx = \int_0^6 \left(\pi(\frac{1}{4}x^3 - 5x^2 + 25x)\right) dx$

$= \left[\pi(\frac{1}{16}x^4 - \frac{5}{3}x^3 + \frac{25}{2}x^2)\right]_0^6 = \pi(\frac{1}{16} \cdot 6^4 - \frac{5}{3} \cdot 6^3 + \frac{25}{2} \cdot 6^2) - 0 = 171\pi$.

$x_Z \approx \frac{171\pi}{78\pi} = \frac{57}{26}$. Dus het zwaartepunt ligt $\frac{57}{26}$ boven het grondvlak van de afgeknotte kegel.

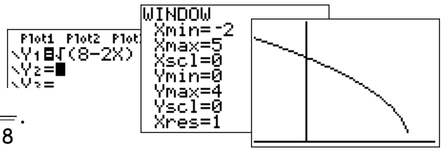



```
1/12*6^3-5/2*6²+
25*6
               78
1/16*6^4-5/3*6^3
+25/2*6²
              171
```

```
171/78▸Frac
            57/26
```

Diagnostische toets

D1a $L = OP = \sqrt{(x_P)^2 + (y_P)^2} = \sqrt{p^2 + \sqrt{8-2p}^2} = \sqrt{p^2 + 8 - 2p} = \sqrt{p^2 - 2p + 8}.$



D1b $L = \sqrt{p^2 - 2p + 8} \Rightarrow \frac{dL}{dp} = \frac{1}{2 \cdot \sqrt{p^2 - 2p + 8}} \cdot (2p - 2) = \frac{2p - 2}{2 \cdot \sqrt{p^2 - 2p + 8}} = \frac{p - 1}{\sqrt{p^2 - 2p + 8}}.$

$\frac{dL}{dp} = 0 \Rightarrow \frac{p-1}{\sqrt{p^2 - 2p + 8}} = 0$ (teller = 0) $\Rightarrow p - 1 = 0 \Rightarrow p = 1$. Hieruit volgt dan: $L_{min} = L(1) = \sqrt{1^2 - 2 \cdot 1 + 8} = \sqrt{7}.$

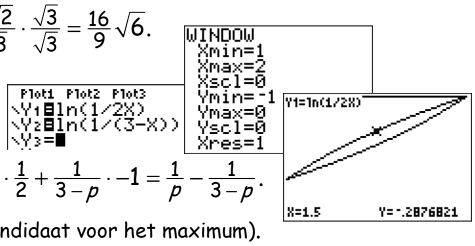
D1c $A = O_{OQPR} = x_P \cdot y_P = p \cdot \sqrt{8-2p}.$

$A = p \cdot \sqrt{8-2p} \Rightarrow \frac{dA}{dp} = 1 \cdot \sqrt{8-2p} + p \cdot \frac{1}{2 \cdot \sqrt{8-2p}} \cdot -2 = \sqrt{8-2p} - \frac{p}{\sqrt{8-2p}}.$

$\frac{dA}{dp} = 0 \Rightarrow \sqrt{8-2p} - \frac{p}{\sqrt{8-2p}} = 0 \Rightarrow \sqrt{8-2p} = \frac{p}{\sqrt{8-2p}} \Rightarrow 8 - 2p = p \Rightarrow 8 = 3p \Rightarrow p = \frac{8}{3}.$

Hieruit volgt dan: $A_{max} = A(\frac{8}{3}) = \frac{8}{3} \cdot \sqrt{8 - 2 \cdot \frac{8}{3}} = \frac{8}{3} \cdot \sqrt{\frac{24}{3} - \frac{16}{3}} = \frac{8}{3} \cdot \sqrt{\frac{8}{3}} = \frac{8}{3} \cdot \frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16}{9}\sqrt{6}.$

D2 $L = AB = f(p) - g(p) = \ln\left(\frac{1}{2}p\right) - \ln\left(\frac{1}{3-p}\right).$

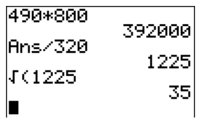


$L = \ln\left(\frac{1}{2}p\right) - \ln\left(\frac{1}{3-p}\right) = \ln\left(\frac{1}{2}p\right) - \ln\left((3-p)^{-1}\right) = \ln\left(\frac{1}{2}p\right) + \ln(3-p) \Rightarrow \frac{dL}{dp} = \frac{1}{\frac{1}{2}p} \cdot \frac{1}{2} + \frac{1}{3-p} \cdot -1 = \frac{1}{p} - \frac{1}{3-p}.$

$\frac{dL}{dp} = 0 \Rightarrow \frac{1}{p} - \frac{1}{3-p} = 0 \Rightarrow \frac{1}{p} = \frac{1}{3-p} \Rightarrow p = 3 - p \Rightarrow 2p = 3 \Rightarrow p = \frac{3}{2}$ (de enige kandidaat voor het maximum).

D3a $K = (2x + 2y) \cdot 160 + 2y \cdot 85 = 320x + 320y + 170y = 320x + 490y.$

$\left.\begin{array}{l} O = l \cdot b = x \cdot y = xy \\ O = 800 \end{array}\right\} \Rightarrow xy = 800 \Rightarrow y = \frac{800}{x}$
$K = 320x + 490y$
$\Rightarrow K = 320x + 490 \cdot \frac{800}{x} = 320x + \frac{392000}{x}.$

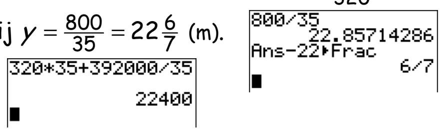


D3b $K = 320x + \frac{392000}{x} = 320x + 392\,000x^{-1} \Rightarrow \frac{dK}{dx} = 320 - 392\,000x^{-2} = 320 - \frac{392\,000}{x^2}.$

$\frac{dK}{dx} = 0 \Rightarrow 320 - \frac{392\,000}{x^2} = 0 \Rightarrow 320 = \frac{392\,000}{x^2} \Rightarrow 320x^2 = 392\,000 \Rightarrow x^2 = \frac{392000}{320} = 1225$ $(x > 0) \Rightarrow x = 35.$

K is minimaal met de afmetingen $x = 35$ (m) bij $y = \frac{800}{35} = 22\frac{6}{7}$ (m).

$K_{min} = K(35) = 320 \cdot 35 + \frac{392000}{35} = 22\,400$ (€).



D4a $AB = x$ en $AB + BC = 6 \Rightarrow BC = 6 - x.$

Pythagoras in ΔABC: $AC = \sqrt{BC^2 - AB^2} = \sqrt{(6-x)^2 - x^2} = \sqrt{36 - 12x + x^2 - x^2} = \sqrt{36 - 12x}.$

$O_{\Delta ABC} = \frac{1}{2} \cdot AB \cdot AC = \frac{1}{2} \cdot x \cdot \sqrt{36 - 12x} = \frac{1}{2}x\sqrt{36 - 12x}.$

D4b $O = \frac{1}{2}x\sqrt{36 - 12x} \Rightarrow \frac{dK}{dx} = \frac{1}{2} \cdot \sqrt{36 - 12x} + \frac{1}{2}x \cdot \frac{1}{2\sqrt{36 - 12x}} \cdot -12 = \frac{\sqrt{36 - 12x}}{2} - \frac{3x}{\sqrt{36 - 12x}}.$

$\frac{dK}{dx} = 0 \Rightarrow \frac{\sqrt{36 - 12x}}{2} - \frac{3x}{\sqrt{36 - 12x}} = 0 \Rightarrow \frac{\sqrt{36 - 12x}}{2} = \frac{3x}{\sqrt{36 - 12x}} \Rightarrow 6x = 36 - 12x \Rightarrow 18x = 36 \Rightarrow x = \frac{36}{18} = 2.$

$O_{max} = O(2) = \frac{1}{2} \cdot 2 \cdot \sqrt{36 - 12 \cdot 2} = \sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}.$

D5a De trillingstijd is $T = \frac{1}{f} = \frac{2\pi}{c} = \frac{2\pi}{\frac{1}{3}\pi} = \frac{6\pi}{\pi} = 6$ seconden, dus het faseverschil is $\frac{2}{6} = \frac{1}{3}.$

D5b $u_Q = 3\sin(\frac{1}{3}\pi(t-2))$ met u_Q in dm en t in seconden.

D5c $\frac{1}{f} = 6$ seconden $\Rightarrow f = \frac{1}{6}$ Hz.

Het punt P legt in 1 minuut $\frac{1}{6} \cdot 60 \cdot 4 \cdot 3 = 120$ dm af. Dat is 12 meter.

D5d $u_P = u_Q \Rightarrow 3\sin\left(\frac{1}{3}\pi t\right) = 3\sin\left(\frac{1}{3}\pi(t-2)\right) \Rightarrow \sin\left(\frac{1}{3}\pi t\right) = \sin\left(\frac{1}{3}\pi t - \frac{2}{3}\pi\right) \Rightarrow$

$\frac{1}{3}\pi t = \frac{1}{3}\pi t - \frac{2}{3}\pi + k \cdot 2\pi$ (geen oplossing) $\vee$ $\frac{1}{3}\pi t = \pi - (\frac{1}{3}\pi t - \frac{2}{3}\pi) + k \cdot 2\pi \Rightarrow$

$\frac{1}{3}\pi t = \pi - \frac{1}{3}\pi t + \frac{2}{3}\pi + k \cdot 2\pi \Rightarrow \frac{2}{3}\pi t = \frac{5}{3}\pi + k \cdot 2\pi \Rightarrow t = \frac{5}{2} + k \cdot 3$. We zoeken $t = \frac{5}{2} = 2\frac{1}{2}.$

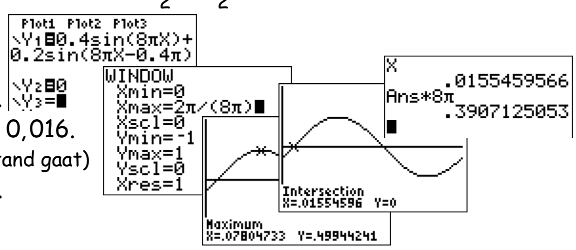
D6a u_1 en u_2 hebben dezelfde frequentie $\Rightarrow c = 8\pi.$

$u_3 = 0{,}4\sin(8\pi t) + 0{,}2\sin(8\pi t - 0{,}4\pi)$ (optie maximum) $\Rightarrow b \approx 0{,}50.$

$u_3 = 0{,}4\sin(8\pi t) + 0{,}2\sin(8\pi t - 0{,}4\pi) = 0$ (intersect of zero) $\Rightarrow t \approx 0{,}016.$

(je zoekt de t-waarde waar de grafiek van u stijgend door de evenwichtsstand gaat)

Dus $u_3 = 0{,}50\sin(8\pi(t - \text{Ans})) \approx 0{,}50\sin(8\pi t - 0{,}39) \Rightarrow d \approx 0{,}39.$



D6b $u_4 = u_1 + 2u_2 = 0{,}4\sin(8\pi t) + 0{,}4\sin(8\pi t - 0{,}4\pi) = 0{,}4(\sin(8\pi t) + \sin(8\pi t - 0{,}4\pi))$
$= 0{,}4 \cdot 2\sin(\frac{1}{2}(8\pi t + 8\pi t - 0{,}4\pi)) \cdot \cos(\frac{1}{2}(8\pi t - 8\pi t + 0{,}4\pi))$
$= 0{,}8\sin(8\pi t - 0{,}2\pi) \cdot \cos(0{,}2\pi) \approx 0{,}65\sqrt{2}\sin(8\pi t - 0{,}2\pi)$.

D7a $u = \sin(10t) + \sin(15t)$ heeft periode $\frac{2}{5}\pi$ sec.
(zie de uitleg hieronder)

	$u_1 = \sin(10t)$	$u_2 = \sin(15t)$
in $[0, 2\pi]$	10 periodes	15 periodes
in $\left[0, \frac{2}{5}\pi\right]$	2 periodes	3 periodes

D7b $u = 2\sin(450\pi t) + \sin(400\pi t)$ heeft periode $\frac{1}{25}$ sec.
(zie de uitleg hieronder)

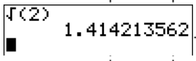
	$u_1 = 2\sin(450\pi t)$	$u_2 = \sin(400\pi t)$
in $[0, 2\pi]$	450π periodes	400π periodes
in $\left[0, \frac{1}{25}\right]$	9 periodes	8 periodes

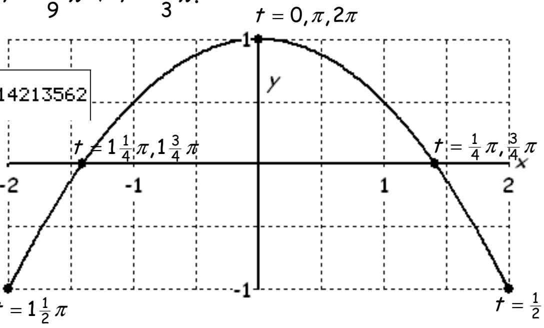
D8a In de x-richting worden 2 periodes doorlopen $\Rightarrow$ periode is $\frac{\frac{4\pi}{3}}{2} = \frac{2}{3}\pi \Rightarrow a = \frac{2\pi}{\frac{2}{3}\pi} = \frac{6\pi}{2\pi} = 3$.
In de y-richting worden 3 periodes doorlopen $\Rightarrow$ periode is $\frac{\frac{4\pi}{3}}{3} = \frac{4}{9}\pi \Rightarrow b = \frac{2\pi}{\frac{4}{9}\pi} = \frac{18\pi}{4\pi} = 4\frac{1}{2}$.

D8b $x = 0 \Rightarrow \sin(3t) = 0 \Rightarrow 3t = k \cdot \pi \Rightarrow t = k \cdot \frac{1}{3}\pi$.
t op $\left[0, 1\frac{1}{3}\pi\right]$: $t = 0 \vee t = \frac{1}{3}\pi \vee t = \frac{2}{3}\pi \vee t = \pi \vee t = 1\frac{1}{3}\pi$.
$y = 0 \Rightarrow \sin(4\frac{1}{2}t) = 0 \Rightarrow 4\frac{1}{2}t = k \cdot \pi \Rightarrow t = k \cdot \frac{2}{9}\pi$.
t op $\left[0, 1\frac{1}{3}\pi\right]$: $t = 0 \vee t = \frac{2}{9}\pi \vee t = \frac{4}{9}\pi \vee t = \frac{2}{3}\pi \vee t = \frac{8}{9}\pi \vee t = \frac{10}{9}\pi \vee t = \frac{4}{3}\pi$.

D9a

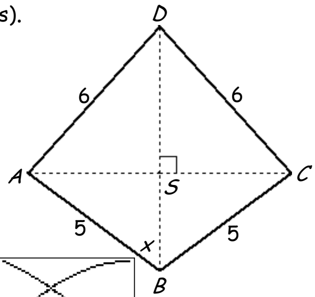
t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$1\frac{1}{4}\pi$	$1\frac{1}{2}\pi$	$1\frac{3}{4}\pi$	2π
x	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
y	1	0	-1	0	1	0	-1	0	1





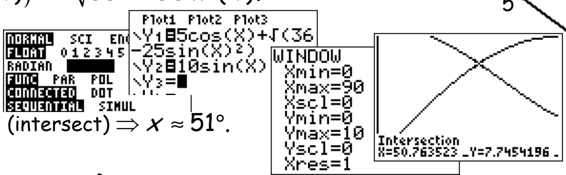
Zie de grafiek van K in de figuur hiernaast.
(de tabel kan ook met TABLE op de GR opgevraagd worden)
De keerpunten zijn $(-2, -1)$ en $(2, -1)$.

D9b $y = ax^2 + b$ door $(0, 1) \Rightarrow 1 = a \cdot 0^2 + b \Rightarrow 1 = b$.
$y = ax^2 + 1$ door $(2, -1) \Rightarrow -1 = a \cdot 2^2 + 1 \Rightarrow -1 = 4a + 1 \Rightarrow -2 = 4a \Rightarrow a = -\frac{2}{4} = -\frac{1}{2}$.
Vermoedelijk hoort de formule $y = -\frac{1}{2}x^2 + 1$ bij K.
$-\frac{1}{2}x^2 + 1 = -\frac{1}{2} \cdot (2\sin(t))^2 + 1 = -2\sin^2(t) + 1 = 1 - 2\sin^2(t) = \cos(2t) = y$ (wat te bewijzen was).
Bij K hoort de formule $y = -\frac{1}{2}x^2 + 1$ met $-2 \le x \le 2$ (want $-2 \le 2\sin(t) \le 2$).

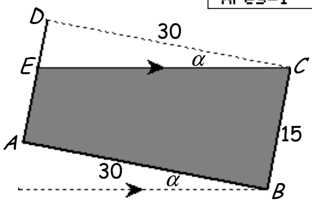


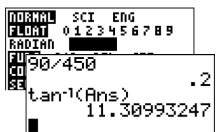
D10a Noem S het snijpunt van BD en AC. Omdat $AB = BC$ en $AD = CD$ is $AC \perp BD$.
$\sin(x) = \frac{AS}{AB} = \frac{AS}{5} \Rightarrow AS = 5\sin(x)$ en $\cos(x) = \frac{BS}{AB} = \frac{BS}{5} \Rightarrow BS = 5\cos(x)$.
Pyth. in ΔADS: $SD = \sqrt{AD^2 - AS^2} = \sqrt{6^2 - (5\sin(x))^2} = \sqrt{36 - 25\sin^2(x)}$.
$BD = BS + SD = 5\cos(x) + \sqrt{36 - 25\sin^2(x)}$.

D10b $AS = 5\sin(x) \Rightarrow AC = 10\sin(x)$.
$BD = AC \Rightarrow 5\cos(x) + \sqrt{36 - 25\sin^2(x)} = 10\sin(x)$ (intersect) $\Rightarrow x \approx 51°$.

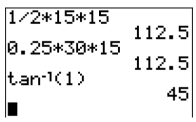


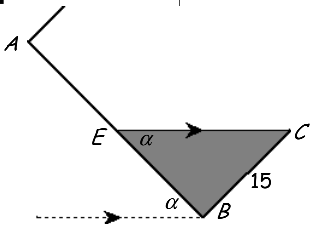
D11a $O_{ABCE} = 0{,}8 \cdot O_{ABCE}$ of $O_{\Delta EDC} = 0{,}2 \cdot O_{ABCE}$.
$\angle ECD = \alpha$ en $\tan(\alpha) = \frac{DE}{CD} = \frac{DE}{30} \Rightarrow DE = 30\tan(\alpha)$.
$O_{\Delta EDC} = \frac{1}{2} \cdot DE \cdot DC = \frac{1}{2} \cdot 30\tan(\alpha) \cdot 30 = 450\tan(\alpha)$
$O_{\Delta EDC} = 0{,}2 \cdot O_{ABCE} = 0{,}2 \cdot 30 \cdot 15 = 6 \cdot 15 = 90$ $\Rightarrow$
$450\tan(\alpha) = 90 \Rightarrow \tan(\alpha) = \frac{90}{450} = \frac{1}{5} \Rightarrow a \approx 11°$.





D11b $\angle BEC = \alpha$ (Z-hoeken) en $\tan(\alpha) = \frac{BC}{EB} = \frac{15}{EB} \Rightarrow EB = \frac{15}{\tan(\alpha)}$.
$O_{\Delta EBC} = \frac{1}{2} \cdot EB \cdot BC = \frac{1}{2} \cdot \frac{15}{\tan(\alpha)} \cdot 15 = 112{,}5\tan(\alpha)$
$O_{\Delta EBC} = 0{,}25 \cdot O_{ABCE} = 0{,}25 \cdot 30 \cdot 15 = 112{,}5$ $\Rightarrow$
$112{,}5\tan(\alpha) = 112{,}5 \Rightarrow \tan(\alpha) = 1 \Rightarrow a = 45°$.





D12a Stel $a(t) = mt + n$ met $m = \frac{68-8}{60} = \frac{60}{60} = 1$ en $a(0) = 8 \Rightarrow a(t) = t + 8$.

$v(t) = \frac{1}{2}t^2 + 8t + v(0)$ met $v(0) = 0 \Rightarrow v(t) = \frac{1}{2}t^2 + 8t$.

$s(t) = \frac{1}{6}t^3 + 4t^2 + s(0)$ met $s(0) = 0 \Rightarrow s(t) = \frac{1}{6}t^3 + 4t^2$.

$s(60) = \frac{1}{6} \cdot 60^3 + 4 \cdot 60^2 = 50\,400$ m.

```
1/6*60^3+4*60²
          50400
```

D12b Vanaf $t = 60$ is $a(t) = -10$.

```
1/2*60²+8*60
           2280
```

$v(t) = v(60) + \int_{60}^{t} a(p)\,dp = \frac{1}{2} \cdot 60^2 + 8 \cdot 60 + \int_{60}^{t} -10\,dp = 2\,280 + \left[-10p\right]_{60}^{t} = 2\,280 - 10t + 600 = 2\,880 - 10t$.

De hoogte is maximaal als de snelheid $v(t) = 0 \Rightarrow 2\,880 - 10t = 0 \Rightarrow 2\,880 = 10t \Rightarrow t = 288$.

$s(288) = s(60) + \int_{60}^{288} (v(t))\,dt = 50\,400 + \int_{60}^{288} (2\,880 - 10t)\,dt = 50\,400 + \left[2\,880t - 5t^2\right]_{60}^{288}$

$= 50\,400 + 2\,880 \cdot 288 - 5 \cdot 288^2 - (2\,880 \cdot 60 - 5 \cdot 60^2) = 310\,320$ m.

```
50400+2880*288-5
*288²-(2880*60-5
*60²)
          310320
```

D13 $y = 3^x \Rightarrow x = {}^3\log(y)$.

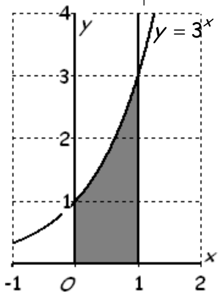
$O(V) = \int_0^1 3^x\,dx$ (fnInt) $\approx 1{,}820$
$\int_0^1 \left(x \cdot 3^x\right) dx$ (fnInt) $\approx 1{,}074$
$\Rightarrow x_Z \approx \frac{1{,}074}{1{,}820} \approx 0{,}59$.

$\int_0^1 (y \cdot 1)\,dy + \int_1^3 \left(y \cdot (1 - {}^3\log(y))\right) dy$ (fnInt) $\approx 1{,}820 \Rightarrow y_Z = \frac{Ans}{O(V)} = 1$.

```
fnInt(3^X,X,0,1)
→A
      1.820478453
fnInt(X*3^X,X,0,
1)→B
      1.073646781
```

```
B/A
       .5897607734
```

```
fnInt(X,X,0,1)+f
nInt(X(1-log(X)/
log(3)),X,1,3)
      1.820478453
Ans/A
                1
```



D14a $I(L) = \int_0^2 \left(\pi \cdot y^2\right) dx = \int_0^2 \left(\pi(4 - x^2)^2\right) dx = \int_0^2 \left(\pi(16 - 8x^2 + x^4)\right) dx$

$= \left[\pi(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5)\right]_0^2 = \pi(16 \cdot 2 - \frac{8}{3} \cdot 2^3 + \frac{1}{5} \cdot 2^5) - 0 = \frac{256}{15}\pi$.

```
16*2-8/3*2^3+1/5
*2^5
      17.06666667
Ans▸Frac
           256/15
```

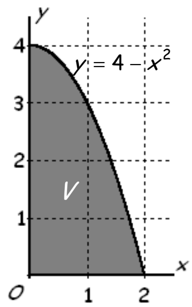
$\int_0^2 \left(\pi x \cdot (4 - x^2)^2\right) dx = \int_0^2 \left(\pi(16x - 8x^3 + x^5)\right) dx$

$= \left[\pi(8x^2 - 2x^4 + \frac{1}{6}x^6)\right]_0^2 = \pi(8 \cdot 2^2 - 2 \cdot 2^4 + \frac{1}{6} \cdot 2^6) - 0 = \frac{32}{3}\pi$.

```
8*2²-2*2^4+1/6*2
^6
      10.66666667
Ans▸Frac
             32/3
```

De x-coördinaat van het zwaartepunt van L is $\frac{\frac{32}{3}\pi}{\frac{256}{15}\pi} = \frac{5}{8}$.

```
(32/3)/(256/15)
             .625
Ans▸Frac
              5/8
```



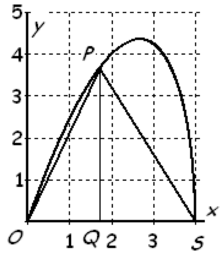
D14b $y = 4 - x^2 \Rightarrow x^2 = 4 - y$ (met $x \geq 0$) $\Rightarrow x = \sqrt{4 - y}$

$I(M) = \int_0^4 \left(\pi \cdot x^2\right) dy = \int_0^4 (\pi(4 - y))\,dy = \left[\pi(4y - \frac{1}{2}y^2)\right]_0^4 = \pi(4 \cdot 4 - \frac{1}{2} \cdot 4^2) - 0 = \pi(16 - 8) = 8\pi$.

$= \int_0^4 \left(\pi y x^2\right) dy = \int_0^4 \left(\pi(4y - y^2)\right) dy = \left[\pi(2y^2 - \frac{1}{3}y^3)\right]_0^4 = \pi(2 \cdot 4^2 - \frac{1}{3} \cdot 4^3) - 0 = \pi(32 - \frac{64}{3}) = \frac{32}{3}\pi$.

```
2*4²-1/3*4^3
      10.66666667
Ans▸Frac
             32/3
Ans/8▸Frac
              4/3
```

De y-coördinaat van het zwaartepunt van M is $\frac{\frac{32}{3}\pi}{8\pi} = \frac{4}{3}$.



Gemengde opgaven 15. Toepassingen

G39a $f(x)=0 \Rightarrow x\cdot\sqrt{8-2x}=0 \Rightarrow x=0 \ \vee\ 8-2x=0 \Rightarrow x=0 \ \vee\ 8=2x \Rightarrow x=0 \ \vee\ x=4.$

$y_P=p$ (met $0<p<4$) $\Rightarrow y_P=p\cdot\sqrt{8-2p}.$

$O_{\Delta OSP}=\frac12\cdot OS\cdot y_P=\frac12\cdot4\cdot p\sqrt{8-2p}=2p\sqrt{8-2p}.$

$\frac{dO}{dp}=2\cdot\sqrt{8-2p}+2p\cdot\frac{1}{2\sqrt{8-2p}}\cdot-2=2\cdot\sqrt{8-2p}-\frac{2p}{\sqrt{8-2p}}.$

$\frac{dO}{dp}=0\Rightarrow 2\cdot\sqrt{8-2p}-\frac{2p}{\sqrt{8-2p}}=0\Rightarrow 2\cdot\sqrt{8-2p}=\frac{2p}{\sqrt{8-2p}}\Rightarrow 2p=2(8-2p)\Rightarrow p=8-2p\Rightarrow 3p=8\Rightarrow p=\frac83.$

$O_{max}=O(\frac83)=2\cdot\frac83\cdot\sqrt{8-2\cdot\frac83}=\frac{16}{3}\cdot\sqrt{\frac{24}{3}-\frac{16}{3}}=\frac{16}{3}\cdot\sqrt{\frac83}=\frac{16}{3}\cdot\frac{\sqrt{4\cdot2}}{\sqrt3}=\frac{32}{3}\cdot\frac{\sqrt2}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3}=\frac{32}{9}\sqrt6.$

G39b $O_{\Delta QSP}=\frac12\cdot QS\cdot y_P=\frac12\cdot(4-p)\cdot p\sqrt{8-2p}=(2p-\frac12p^2)\cdot\sqrt{8-2p}.$

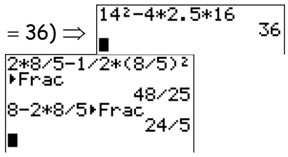
$\frac{dO}{dp}=(2-p)\cdot\sqrt{8-2p}+(2p-\frac12p^2)\cdot\frac{1}{2\sqrt{8-2p}}\cdot-2=(2-p)\cdot\sqrt{8-2p}-\frac{2p-\frac12p^2}{\sqrt{8-2p}}.$

$\frac{dO}{dp}=0\Rightarrow(2-p)\cdot\sqrt{8-2p}-\frac{2p-\frac12p^2}{\sqrt{8-2p}}=0\Rightarrow\frac{(2-p)\cdot\sqrt{8-2p}}{1}=\frac{2p-\frac12p^2}{\sqrt{8-2p}}\Rightarrow(2-p)(8-2p)=(2p-\frac12p^2)\Rightarrow$

$16-4p-8p+2p^2=2p-\frac12p^2\Rightarrow2\frac12p^2-14p+16=0$ (met $D=(-14)^2-4\cdot2\frac12\cdot16=36$) $\Rightarrow$

$p=\frac{14+6}{5}=\frac{20}{5}=4$ (voldoet niet) $\vee$ $p=\frac{14-6}{5}=\frac85$ (de enige kandidaat).

$O_{max}=O(\frac85)=(2\cdot\frac85-\frac12\cdot(\frac85)^2)\cdot\sqrt{8-2\cdot\frac85}=\frac{48}{25}\cdot\sqrt{\frac{24}{5}}=\frac{48}{25}\cdot\frac{\sqrt{4\cdot6}}{\sqrt5}=\frac{96}{25}\cdot\frac{\sqrt6}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}=\frac{96}{125}\sqrt{30}.$



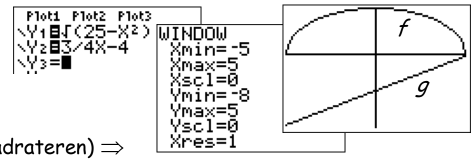
G40a $L=AB=f(p)-g(p)=\sqrt{25-p^2}-\left(\frac34p-4\right)=\sqrt{25-p^2}-\frac34p+4.$

$\frac{dL}{dp}=\frac{1}{2\sqrt{25-p^2}}\cdot-2p-\frac34=\frac{-p}{\sqrt{25-p^2}}-\frac34.$

$\frac{dO}{dp}=0\Rightarrow\frac{-p}{\sqrt{25-p^2}}-\frac34=0\Rightarrow\frac{-p}{\sqrt{25-p^2}}=\frac34\Rightarrow-4p=3\cdot\sqrt{25-p^2}$ (* kwadrateren) $\Rightarrow$

$16p^2=9\cdot(25-p^2)\Rightarrow16p^2=9\cdot25-9p^2\Rightarrow25p^2=9\cdot25\Rightarrow p^2=9\Rightarrow p=-3$ (vold.) $\vee$ $p=3$ (vold. niet aan *).

$L_{max}=L(-3)=\sqrt{25-(-3)^2}-\frac34\cdot-3+4=\sqrt{25-9}+\frac94+4=\sqrt{16}+2\frac14+4=4+2\frac14+4=10\frac14.$



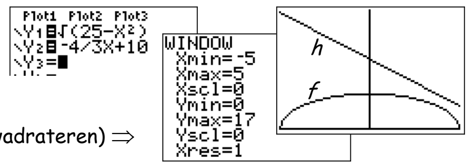
G40b $L=AC=h(p)-f(p)=-\frac43p+10-\sqrt{25-p^2}.$

$\frac{dL}{dp}=-\frac43-\frac{1}{2\sqrt{25-p^2}}\cdot-2p=-\frac43+\frac{p}{\sqrt{25-p^2}}.$

$\frac{dO}{dp}=0\Rightarrow-\frac43+\frac{p}{\sqrt{25-p^2}}=0\Rightarrow\frac{p}{\sqrt{25-p^2}}=\frac43\Rightarrow3p=4\cdot\sqrt{25-p^2}$ (* kwadrateren) $\Rightarrow$

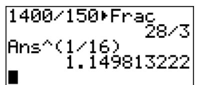
$9p^2=16\cdot(25-p^2)\Rightarrow9p^2=16\cdot25-16p^2\Rightarrow25p^2=16\cdot25\Rightarrow p^2=16\Rightarrow p=-4$ (vold. niet aan *) $\vee$ $p=4$ (vold.).

$L_{min}=L(4)=-\frac43\cdot4+10-\sqrt{25-16}=-\frac{16}{3}+10-\sqrt9=-5\frac13+10-3=10-8\frac13=1\frac23.$



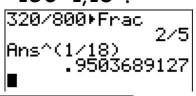
G41a Rechte lijn op (enkelvoudig) logaritmisch papier $\Rightarrow$ exponentiële functie $\Rightarrow N_A=b\cdot g^t.$

Lijn door (0, 150) en (16, 1 400) $\Rightarrow g_{16\text{ jaar}}=\frac{1400}{150}=\frac{140}{15}=9\frac13\Rightarrow g_{\text{jaar}}=(9\frac13)^{\frac{1}{16}}\approx1{,}15.$ Dus $N_A\approx150\cdot1{,}15^t.$



Rechte lijn op (enkelvoudig) logaritmisch papier $\Rightarrow$ exponentiële functie $\Rightarrow N_B=b\cdot g^t.$

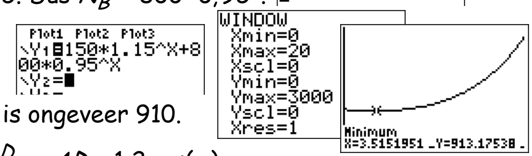
Door (0, 800) en (18, 320) $\Rightarrow g_{18\text{ jaar}}=\frac{320}{800}=\frac25\Rightarrow g_{\text{jaar}}=(\frac25)^{\frac{1}{18}}\approx0{,}95.$ Dus $N_B\approx800\cdot0{,}95^t.$



G41b $N_A+N_B\approx150\cdot1{,}15^t+800\cdot0{,}95^t$ (optie minimum geeft)

$t\approx3{,}5$ en $(N_A+N_B)_{min}\approx913.$

Dus het minimale aantal van deze plantenfamilie in dit natuurgebied is ongeveer 910.



G42a In ΔADR: is $\sin(\alpha)=\frac{DR}{AR}=\frac{DR}{1{,}2}\Rightarrow DR=1{,}2\sin(\alpha)$ en $\cos(\alpha)=\frac{AD}{AR}=\frac{AD}{1{,}2}\Rightarrow AD=1{,}2\cos(\alpha).$

In ΔAEQ: is $\cos(\alpha)=\frac{AE}{AQ}=\frac{AE}{0{,}3}\Rightarrow AE=0{,}3\cos(\alpha)\Rightarrow AP=0{,}6\cos(\alpha).$

$BD=AB-AD=AP+PB-AD=0{,}6\cos(\alpha)+0{,}6-1{,}2\cos(\alpha)=0{,}6-0{,}6\cos(\alpha).$

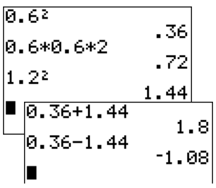
Pythagoras in ΔADR: $BR=\sqrt{BD^2+DR^2}=\sqrt{(0{,}6-0{,}6\cos(\alpha))^2+(1{,}2\sin(\alpha))^2}$

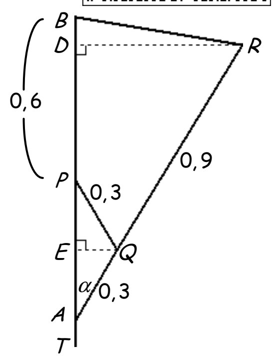
$=\sqrt{0{,}36-0{,}72\cos(\alpha)+0{,}36\cos^2(\alpha)+1{,}44\sin^2(\alpha)}$

$=\sqrt{0{,}36-0{,}72\cos(\alpha)+0{,}36\cos^2(\alpha)+1{,}44\left(1-\cos^2(\alpha)\right)}$

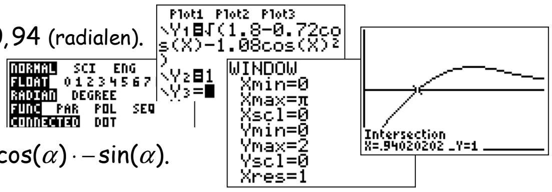
$=\sqrt{0{,}36-0{,}72\cos(\alpha)+0{,}36\cos^2(\alpha)+1{,}44-1{,}44\cos^2(\alpha)}$

$=\sqrt{1{,}8-0{,}72\cos(\alpha)-1{,}08\cos^2(\alpha)}.$





G42b $BR = \sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)} = 1$ (intersect) $\Rightarrow \alpha \approx 0{,}94$ (radialen).
$BR > 1$ (zie een plot) $\Rightarrow \alpha > 0{,}94$ (radialen).



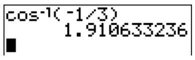
G42c $L = \sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}$ (met $0 \le \alpha \le \pi$) $\Rightarrow$

$\frac{dL}{d\alpha} = \frac{1}{2\sqrt{1{,}8 - 0{,}72\cos(\alpha) - 1{,}08\cos^2(\alpha)}} \cdot -0{,}72 \cdot -\sin(\alpha) - 1{,}08 \cdot 2\cos(\alpha) \cdot -\sin(\alpha)$.

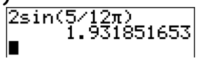
$\frac{dL}{d\alpha} = 0$ (teller = 0) $\Rightarrow 0{,}72\sin(\alpha) + 2{,}16\cos(\alpha)\sin(\alpha) = 0 \Rightarrow 0{,}72\sin(\alpha)\cdot(1 + 3\cos(\alpha)) = 0 \Rightarrow$

$\sin(\alpha) = 0 \ \vee\ 1 + 3\cos(\alpha) = 0 \Rightarrow \alpha = k\cdot\pi \ \vee\ 3\cos(\alpha) = -1 \Rightarrow \alpha = k\cdot\pi \ \vee\ \cos(\alpha) = -\frac{1}{3}$ (met $0 \le \alpha \le \pi$) $\Rightarrow$

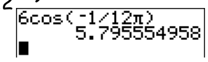
Dus BR is maximaal (zie plot) bij een hoek van (ongeveer) 1,91 radialen.



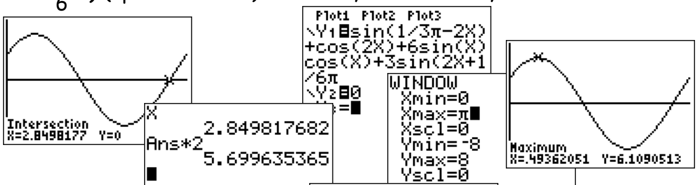
G43a $f(x) = \sin(\frac{1}{3}\pi - 2x) + \cos(2x)$
$= \sin(\frac{1}{3}\pi - 2x) + \sin(2x + \frac{1}{2}\pi)$
$= 2\sin(\frac{1}{2}(\frac{1}{3}\pi + \frac{1}{2}\pi)) \cdot \cos(\frac{1}{2}(\frac{1}{3}\pi - 2x - (2x + \frac{1}{2}\pi)))$
$= 2\sin(\frac{1}{2}(\frac{5}{6}\pi)) \cdot \cos(\frac{1}{2}(-\frac{1}{6}\pi - 4x))$
$= 2\sin(\frac{5}{12}\pi) \cdot \cos(-\frac{1}{12}\pi - 2x)$
$= 2\sin(\frac{5}{12}\pi) \cdot \sin(-\frac{1}{12}\pi - 2x + \frac{1}{2}\pi)$
$\approx 1{,}93\sin(\frac{5}{12}\pi - 2x)$.



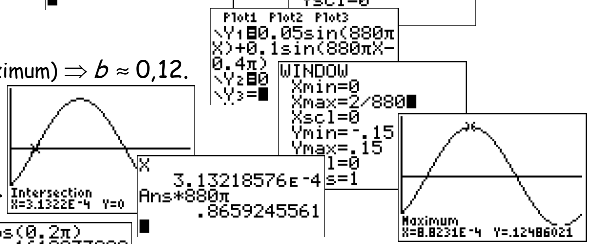
$g(x) = 6\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$
$= 3\cdot 2\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$
$= 3\sin(2x) + 3\sin(2x + \frac{1}{6}\pi)$
$= 3\cdot\left(\sin(2x) + \sin(2x + \frac{1}{6}\pi)\right)$
$= 3\cdot 2\sin(\frac{1}{2}(4x + \frac{1}{6}\pi)) \cdot \cos(\frac{1}{2}(-\frac{1}{6}\pi))$
$= 6\cos(-\frac{1}{12}\pi) \cdot \sin(2x + \frac{1}{12}\pi)$
$\approx 5{,}80\sin(2x + \frac{1}{12}\pi)$.



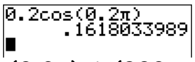
G43b De grafieken van f en g hebben beide dezelfde periode dus $h(x)$ is te schrijven in de vorm $y = b\sin(cx - d)$.
De periode van de grafiek van f en de grafiek van g is $\frac{2\pi}{2} = \pi \Rightarrow c = \frac{2\pi}{\pi} = 2$.
$h(x) = \sin(\frac{1}{3}\pi - 2x) + \cos(2x) + 6\sin(x)\cos(x) + 3\sin(2x + \frac{1}{6}\pi)$ (optie maximum) $\Rightarrow x \approx 0{,}49$ en $b \approx 6{,}11$.
$h(x) = 0$ (intersect of zero) $\Rightarrow x \approx 2{,}850$
(je zoekt de x-waarde waar de grafiek stijgend door de evenwichtsstand gaat).
Dus $h(x) = 6{,}11\sin(2(x - 2{,}850)) = 6{,}11\sin(2x - 5{,}70)$.
Dus $b \approx 6{,}11$, $c = 2$ en $d \approx 5{,}70$.



G44a u_1 en u_2 hebben beide dezelfde frequentie $\Rightarrow c = 880\pi$
$u_4 = u_1 + u_2 = 0{,}05\sin(880\pi t) + 0{,}1\sin(880\pi t - 0{,}4\pi)$ (optie maximum) $\Rightarrow b \approx 0{,}12$.
$u_4 = 0$ (intersect of zero) $\Rightarrow t \approx 0{,}000313$
(je zoekt de t-waarde waar de grafiek stijgend door de evenwichtsstand gaat).
Dus $u_4 = 0{,}12\sin(880\pi(t - 0{,}000313)) = 0{,}12\sin(880\pi t - 0{,}87)$.



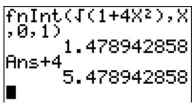
G44b $u_5 = 2u_1 + u_2 = 2\cdot 0{,}05\sin(880\pi t) + 0{,}1\sin(880\pi t - 0{,}4\pi)$
$= 0{,}1\sin(880\pi t) + 0{,}1\sin(880\pi t - 0{,}4\pi)$
$= 0{,}1\cdot 2\sin(880\pi t - 0{,}2\pi)\cos(0{,}2\pi) = 0{,}2\cos(0{,}2\pi)\sin(880\pi t - 0{,}2\pi) \approx 0{,}16\sin(880\pi t - 0{,}2\pi)$.



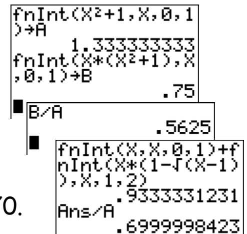
G44c De periodes van u_1, u_2 en u_3 zijn niet gelijk dus u is niet te schrijven in de vorm $u = b\sin(ct - d)$.
Dus de trilling is geen harmonische trilling maar vertoont wel een zweving.
De periode van de zweving $2u_1 + u_2 + u_3$ is $\frac{1}{2}$ sec. (zie de uitleg hieronder)

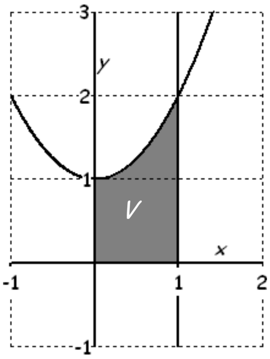
	$2u_1 = 0{,}1\sin(880\pi t)$	$u_2 = 0{,}1\sin(880\pi t - 0{,}4\pi)$	$u_3 = 0{,}2\sin(884\pi t)$
in $[0, 2\pi]$	880π periodes	880π periodes	884π periodes
in $\left[0, \frac{1}{2}\right]$	220 periodes	220 periodes	221 periodes

G45a Omtrek $= f(0) + 1 + f(1) + \int_0^1 \sqrt{1 + (f'(x))^2}\, dx$
$= 1 + 1 + 2 + \int_0^1 \sqrt{1 + (2x)^2}\, dx$ (fnInt) $\approx 5{,}48$.



G45b $O(V) = \int_0^1 \left(x^2 + 1\right) dx$ (fnInt) $\approx 1{,}333$
$\int_0^1 \left(x \cdot (x^2 + 1)\right) dx$ (fnInt) $= 0{,}75$
$\Rightarrow x_Z = \frac{0{,}75}{O(V)} = 0{,}5625 \approx 0{,}56$.
$y = x^2 + 1 \Rightarrow x^2 = y - 1$ (met $x \ge 0$) $\Rightarrow x = \sqrt{y - 1}$.
$\int_0^1 (y \cdot 1)\, dy + \int_1^2 \left(y \cdot (1 - \sqrt{y - 1}\right) dy$ (fnInt) $\approx 0{,}933 \Rightarrow y_Z = \frac{Ans}{O(V)} \approx 0{,}70$.





G45c $f(x) = x^2 + 1 \xrightarrow{\text{translatie } (-1,\,0)} g(x) = (x + 1)^2 + 1$. (een zelfde lichaam L als je de grafiek van g wentelt om de y-as)

$y=(x+1)^2+1 \Rightarrow (x+1)^2=y-1\ (x+1\ge 0) \Rightarrow x+1=\sqrt{y-1} \Rightarrow x=-1+\sqrt{y-1}.$

$I(L)=I(\text{cilinder})+\int_1^2 \pi x^2\,\mathrm{d}y=\pi\cdot 1^2\cdot 1+\int_1^2 \pi(-1+\sqrt{y-1})^2\,\mathrm{d}y=\pi+\int_1^2 \pi(1-\sqrt{y-1}-\sqrt{y-1}+y-1)\,\mathrm{d}y$

$=\pi+\int_1^2 \pi(y-2(y-1)^{\frac{1}{2}})\,\mathrm{d}y=\pi+\left[\pi(\tfrac{1}{2}y^2-\tfrac{2}{1\frac{1}{2}}(y-1)^{1\frac{1}{2}})\right]_1^2=\pi+\pi(\tfrac{1}{2}\cdot 4-\tfrac{4}{3}\cdot 1)-\pi(\tfrac{1}{2}\cdot 1)=\pi+\tfrac{2}{3}\pi-\tfrac{1}{2}\pi=1\tfrac{1}{6}\pi.$

G45d $I(L)=\int_0^1 \pi y^2\,\mathrm{d}x=\int_0^1 \pi(x^2+1)^2\,\mathrm{d}x=\int_0^1 \pi(x^4+2x^2+1)\,\mathrm{d}x=\left[\pi(\tfrac{1}{5}x^5+\tfrac{2}{3}x^3+x)\right]_0^1=\pi(\tfrac{1}{5}+\tfrac{2}{3}+1)-\pi\cdot 0=1\tfrac{13}{15}\pi.$

$\int_0^1 \pi x(x^2+1)^2\,\mathrm{d}x=\int_0^1 \pi(x^5+2x^3+x)\,\mathrm{d}x=\left[\pi(\tfrac{1}{6}x^6+\tfrac{2}{4}x^4+\tfrac{1}{2}x^2)\right]_0^1=\pi(\tfrac{1}{6}+\tfrac{1}{2}+\tfrac{1}{2})-\pi\cdot 0=1\tfrac{1}{6}\pi.$

De x-coördinaat van het zwaartepunt is $x_Z=\frac{1\frac{1}{6}\pi}{1\frac{13}{15}\pi}=\frac{\frac{7}{6}}{\frac{28}{15}}=\frac{7}{6}\cdot\frac{15}{28}=\frac{5}{8}.$

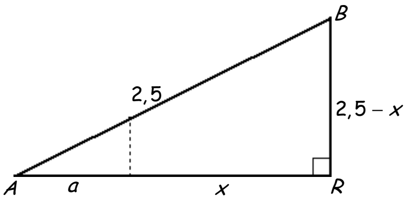
G45e $y=x^2+1 \Rightarrow x^2=y-1$

$I(L)=\int_0^1 \pi\cdot 1^2\,\mathrm{d}y+\int_1^2 \pi x^2\,\mathrm{d}y=\int_0^1 \pi\,\mathrm{d}y+\int_1^2 \pi(y-1)\,\mathrm{d}y=[\pi y]_0^1+\left[\pi(\tfrac{1}{2}y^2-y\right]_1^2=\pi\cdot 1-0+\pi(2-2)-\pi(\tfrac{1}{2}-1)=1\tfrac{1}{2}\pi.$

$\int_0^1 \pi y\,\mathrm{d}y+\int_1^2 \pi y(y-1)\,\mathrm{d}y=\left[\pi\cdot\tfrac{1}{2}y^2\right]_0^1+\left[\pi(\tfrac{1}{3}y^3-\tfrac{1}{2}y^2\right]_1^2=\pi\cdot\tfrac{1}{2}-0+\pi(\tfrac{8}{3}-2)-\pi(\tfrac{1}{3}-\tfrac{1}{2})=\tfrac{1}{2}\pi+\tfrac{2}{3}\pi+\tfrac{1}{6}\pi=\tfrac{8}{6}\pi=1\tfrac{1}{3}\pi.$

De y-coördinaat van het zwaartepunt is $y_Z=\frac{1\frac{1}{3}\pi}{1\frac{1}{2}\pi}=\frac{\frac{4}{3}}{\frac{3}{2}}=\frac{4}{3}\cdot\frac{2}{3}=\frac{8}{9}.$

G46 $AQ+BR=2{,}5 \Rightarrow x+BR=2{,}5 \Rightarrow BR=2{,}5-x.$



Pyth. in ΔARB: $AR^2+BR^2=AB^2$

$(a+x)^2+(2{,}5-x)^2=2{,}5^2$

$(a+x)^2+6{,}25-5x+x^2=6{,}25$

$(a+x)^2=5x-x^2 \Rightarrow a+x=\sqrt{5x-x^2} \Rightarrow a=-x+\sqrt{5x-x^2}.$

$a=-x+\sqrt{5x-x^2} \Rightarrow \frac{\mathrm{d}a}{\mathrm{d}x}=-1+\frac{1}{2\sqrt{5x-x^2}}\cdot(5-2x)=-1+\frac{5-2x}{2\sqrt{5x-x^2}}.$

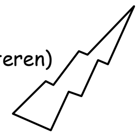
$\frac{\mathrm{d}a}{\mathrm{d}x}=0 \Rightarrow -1+\frac{5-2x}{2\sqrt{5x-x^2}}=0$

$\frac{5-2x}{2\sqrt{5x-x^2}}=\frac{1}{1}$

$5-2x=2\sqrt{5x-x^2}$ (* kwadrateren)

$(5-2x)^2=4(5x-x^2)$

$25-20x+4x^2=20x-4x^2$

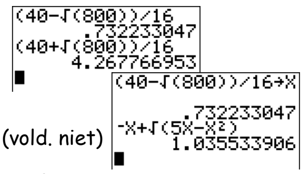


$8x^2-40x+25=0$

$D=(-40)^2-4\cdot 8\cdot 25=1600-800=800$

$x=\frac{40-\sqrt{800}}{16}$ (voldoet aan *) $\vee$ $x=\frac{40+\sqrt{800}}{16}$ (vold. niet)

$x=\frac{40-\sqrt{800}}{16}$ invullen in $a=-x+\sqrt{5x-x^2}$ geeft $a_{max}\approx 1{,}04$ (m).



G47a $x+AC=2{,}5 \Rightarrow AC=2{,}5-x.$

Pyth. in ΔARB: $AC^2+BC^2=AB^2$

$(2{,}5-x)^2+BC^2=2{,}5^2$

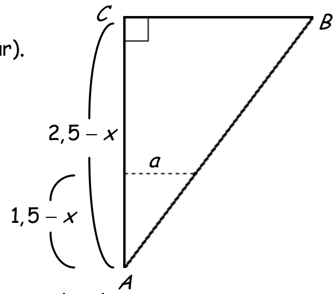
$6{,}25-5x+x^2+BC^2=6{,}25$

$BC^2=5x-x^2 \Rightarrow BC=\sqrt{5x-x^2}.$

$\Delta AED \backsim \Delta ACB$ (snavelfiguur).

$\frac{1{,}5-x}{2{,}5-x}=\frac{a}{\sqrt{5x-x^2}}$

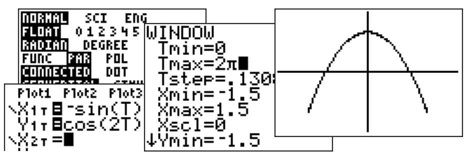
$a=\frac{(1{,}5-x)\sqrt{5x-x^2}}{2{,}5-x}.$



G47b $a=\frac{(1{,}5-x)\sqrt{5x-x^2}}{2{,}5-x}$ (optie maximum) $\Rightarrow a\approx 0{,}77$ (m).

G47c Het dak van een maximaal 1,5 m hoge auto moet minimaal 77 cm van de garagedeur verwijderd staan.

G48a De grafieken van x en y hebben voor $t=\frac{1}{2}\pi$ en $t=1\frac{1}{2}\pi$ een extreem en de grafieken van x en y hebben $t=\frac{1}{2}\pi$ en $t=1\frac{1}{2}\pi$ als symmetrieas. Dus de kleinste positieve waarden waarvoor de krommen precies één keer wordt doorlopen zijn $a=\frac{1}{2}\pi$ en $b=1\frac{1}{2}\pi$.



G48b $y=x$ geeft $\cos(2t)=-\sin(t) \Rightarrow \sin(2t+\frac{1}{2}\pi)=\sin(t+\pi)$

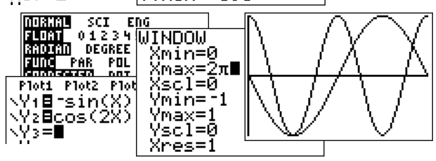
$2t+\frac{1}{2}\pi=t+\pi+k\cdot 2\pi \ \vee\ 2t+\frac{1}{2}\pi=\pi-(t+\pi)+k\cdot 2\pi$

$t=\frac{1}{2}\pi+k\cdot 2\pi \ \vee\ 2t+\frac{1}{2}\pi=\pi-t-\pi+k\cdot 2\pi$

$t=\frac{1}{2}\pi+k\cdot 2\pi \ \vee\ 3t=-\frac{1}{2}\pi+k\cdot 2\pi$

$t=\frac{1}{2}\pi+k\cdot 2\pi \ \vee\ t=-\frac{1}{6}\pi+k\cdot\frac{2}{3}\pi.$

t op $\left[\frac{1}{2}\pi,1\frac{1}{2}\pi\right]$ geeft: $t=\frac{1}{2}\pi \ \vee\ t=1\frac{1}{6}\pi.$



$t=\frac{1}{2}\pi$ geeft $x=-\sin(\frac{1}{2}\pi)=-1$ en $y=x=-1.$

$t=1\frac{1}{6}\pi$ geeft $x=-\sin(1\frac{1}{6}\pi)=\frac{1}{2}$ en $y=x=\frac{1}{2}.$

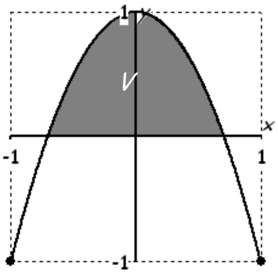
De snijpunten zijn $(-1,\ -1)$ en $(\frac{1}{2},\ \frac{1}{2})$.

G48c K gaat door de punten $(-1,\ -1)$ en $(0,1)$.

$\left.\begin{array}{l} y=ax^2+b \text{ door } (0,1)\Rightarrow 1=a\cdot 0+b\Rightarrow b=1 \\ y=ax^2+1 \text{ door } (-1,\ -1)\Rightarrow -1=a\cdot 1+1\Rightarrow a=-2 \end{array}\right\}\Rightarrow$ Vermoedelijk hoort de formule $y=-2x^2+1$ bij K.

$-2x^2+1=-2\cdot\left(-\sin(t)\right)^2+1=-2\sin^2(t)+1=\cos(2t)=y$ (wat te bewijzen was).

Bij K hoort de formule $y=-2x^2+1$ met $-1\le x\le 1$ (want $-1\le \sin(t)\le 1\Rightarrow -1\le -\sin(t)\le 1$).



G48d $y=0\Rightarrow -2x^2+1=0\Rightarrow -2x^2=-1\Rightarrow x^2=\frac{1}{2}\Rightarrow x=-\sqrt{\frac{1}{2}}\ \vee\ x=\sqrt{\frac{1}{2}}$.

Omtrek $=\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{2}+\int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}}\sqrt{1+(f'(x))^2}\,\mathrm{d}x=\sqrt{2}+\int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}}\sqrt{1+(-4x)^2}\,\mathrm{d}x$ (fnInt) $\approx 3{,}98$.

```
√(2)+fnInt(√(1+1
6X²),X,-1/2√(2),
1/2√(2))
        3.976220699
```

G48e $O(V)=\int_{-\frac{1}{2}\sqrt{2}}^{\frac{1}{2}\sqrt{2}}\left(-2x^2+1\right)\mathrm{d}x$ (fnInt) $\approx 0{,}94$.

```
fnInt(-2X²+1,X,-
1/2√(2),1/2√(2))
       .9428090416
```

G49a $y=\frac{1}{2}\sqrt{3}$ geeft $\cos(2t)=\frac{1}{2}\sqrt{3}$

$2t=\frac{1}{6}\pi+k\cdot 2\pi\ \vee\ 2t=-\frac{1}{6}\pi+k\cdot 2\pi$

$t=\frac{1}{12}\pi+k\cdot\pi\ \vee\ t=-\frac{1}{12}\pi+k\cdot\pi$

t op $\left[\frac{1}{2}\pi, 1\frac{1}{2}\pi\right]$ geeft: $t=\frac{11}{12}\pi\ \vee\ t=1\frac{1}{12}\pi$.

$t=\frac{11}{12}\pi$ geeft $x=\sin(3\cdot\frac{11}{12}\pi)=\sin(2\frac{3}{4}\pi)=\frac{1}{2}\sqrt{2}\Rightarrow A(\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{3})$.

$t=1\frac{1}{12}\pi$ geeft $x=\sin(3\cdot 1\frac{1}{12}\pi)=\sin(3\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{2}\Rightarrow B(-\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{3})$.

Dus $AB=\frac{1}{2}\sqrt{2}--\frac{1}{2}\sqrt{2}=\sqrt{2}$,

G49b $t=\frac{1}{2}\pi+a$ geeft $x=\sin(3\cdot(\frac{1}{2}\pi+a))=\sin(1\frac{1}{2}\pi+3a)=\cos(1\frac{1}{2}\pi+3a-\frac{1}{2}\pi)=\cos(3a+\pi)=-\cos(3a)$

en $y=\cos(2\cdot(\frac{1}{2}\pi+a))=\cos(\pi+2a)=-\cos(2a)$.

$t=1\frac{1}{2}\pi-a$ geeft $x=\sin(3\cdot(1\frac{1}{2}\pi-a))=\sin(4\frac{1}{2}\pi-3a)=\cos(4\frac{1}{2}\pi-3a-\frac{1}{2}\pi)=\cos(-3a+4\pi)=\cos(-3a)=\cos(3a)$

en $y=\cos(2\cdot(1\frac{1}{2}\pi-a))=\cos(3\pi-2a)=\cos(\pi-2a)=-\cos(-2a)=-\cos(2a)$.

Dus $S(-\cos(3a),\ -\cos(2a))$ en $T(\cos(3a),\ -\cos(2a))\Rightarrow ST=\left|\cos(3a)--\cos(3a)\right|=\left|2\cos(3a)\right|$.

G50a $x(t)=\cos(15t)+\cos(2t)=2\cos(\frac{1}{2}(15t+2t))\cos(\frac{1}{2}(15t-2t))=2\cos(8\frac{1}{2}t)\cos(6\frac{1}{2}t)=r(t)\cos(8\frac{1}{2}t)$.

$y(t)=\sin(15t)+\sin(2t)=2\sin(\frac{1}{2}(15t+2t))\cos(\frac{1}{2}(15t-2t))=2\sin(8\frac{1}{2}t)\cos(6\frac{1}{2}t)=r(t)\sin(8\frac{1}{2}t)$.

G50b $x(t)=0\Rightarrow 2\cos(6\frac{1}{2}t)\cos(8\frac{1}{2}t)=0$

$\cos(6\frac{1}{2}t)=0\ \vee\ \cos(8\frac{1}{2}t)=0$

$6\frac{1}{2}t=\frac{1}{2}\pi+k\cdot\pi\ \vee\ 8\frac{1}{2}t=\frac{1}{2}\pi+k\cdot\pi$

$t=\frac{1}{13}\pi+k\cdot\frac{2}{13}\pi\ \vee\ t=\frac{1}{17}\pi+k\cdot\frac{2}{17}\pi$

$y(t)=0\Rightarrow 2\cos(6\frac{1}{2}t)\cos(8\frac{1}{2}t)=0$

$\cos(6\frac{1}{2}t)=0\ \vee\ \sin(8\frac{1}{2}t)=0$

$6\frac{1}{2}t=\frac{1}{2}\pi+k\cdot\pi\ \vee\ 8\frac{1}{2}t=k\cdot\pi$

$t=\frac{1}{13}\pi+k\cdot\frac{2}{13}\pi\ \vee\ t=k\cdot\frac{2}{17}\pi$

Dus $x(t)=0\ \wedge\ y(t)=0$ voor $t=\frac{1}{13}\pi+k\cdot\frac{2}{13}\pi$. Dus P passeert 13 keer ($k=0, 1, 2, 3, \ldots, 12$) het punt $(0, 0)$.

G51a $y(t)=0\Rightarrow\sin(2t+\frac{1}{3}\pi)=0\Rightarrow 2t+\frac{1}{3}\pi=k\cdot\pi\Rightarrow 2t=-\frac{1}{3}\pi+k\cdot\pi\Rightarrow t=-\frac{1}{6}\pi+k\cdot\frac{1}{2}\pi$.

De punten zijn $(\sin(-\frac{1}{6}\pi), 0)=(-\frac{1}{2}, 0)$, $(\sin(\frac{1}{3}\pi), 0)=(\frac{1}{2}\sqrt{3}, 0)$, $(\sin(\frac{5}{6}\pi), 0)=(\frac{1}{2}, 0)$ en $(\sin(1\frac{1}{3}\pi), 0)=(-\frac{1}{2}\sqrt{3}, 0)$.

G51b $AB=\sin(2a+\frac{1}{3}\pi)-\sin(2(\pi-a)+\frac{1}{3}\pi)=\sin(2a+\frac{1}{3}\pi)-\sin(2\frac{1}{3}\pi-2a)$

$=2\sin(\frac{1}{2}(2a+\frac{1}{3}\pi-2\frac{1}{3}\pi+2a))\cos(\frac{1}{2}(2a+\frac{1}{3}\pi+2\frac{1}{3}\pi-2a))=2\sin(\frac{1}{2}(4a-2\pi))\cos(\frac{1}{2}(2\frac{2}{3}\pi))$

$=2\sin(2a-\pi)\cos(1\frac{1}{3}\pi)=-2\sin(2a)\cdot-\frac{1}{2}=\sin(2a)$.

G52a $z'(18)=100\cdot e^{0,1(18-40)}=100\cdot e^{-2,2}$

$100\cdot e^{-0,2(t-100)}=100\cdot e^{-2,2}\Rightarrow e^{-0,2(t-100)}=e^{-2,2}\Rightarrow -0{,}2(t-100)=-2{,}2\Rightarrow t-100=11\Rightarrow t=111$. Dus $t_3=111$.

G52b $\left.\begin{array}{l} z(t)=a\cdot e^{0,1(t-40)}+b\Rightarrow z'(t)=a\cdot e^{0,1(t-40)}\cdot 0{,}1=0{,}1a\cdot e^{0,1(t-40)} \\ z'(t)=100\cdot e^{0,1(t-40)} \end{array}\right\}\Rightarrow 0{,}1a=100\Rightarrow a=1000$.

$\left.\begin{array}{l} z(t)=1000\cdot e^{0,1(t-40)}+b \\ z(0)=30 \end{array}\right\}\Rightarrow 1000\cdot e^{0,1(0-40)}+b=30\Rightarrow 1000\cdot e^{-4}+b=30\Rightarrow b=30-1000\cdot e^{-4}=30-\frac{1000}{e^4}$.

G52c $z(100)=z(0)+\int_0^{40}100\cdot e^{0,1(t-40)}\,\mathrm{d}t+\int_{40}^{100}100\,\mathrm{d}t$ (fnInt) $\approx 7\,011{,}68$.

```
30+fnInt(100e^(0
.1(X-40)),X,0,40
)+fnInt(100,X,40
,100)→B
        7011.684361
```

G52d $z(120)=z(100)+\int_{100}^{120}z'(t)\,\mathrm{d}t=\text{Ans}+\int_0^{120}100\cdot e^{-0,2(t-100)}\,\mathrm{d}t$ (fnInt) $\approx 7\,503$ (kg).

```
B+fnInt(100*e^(-
0.2(X-100)),X,10
0,120)
        7502.526542
```

G53a P haalt Q voor het eerst in als $\frac{11}{10}t=t+\frac{2}{3}\pi\Rightarrow\frac{1}{10}t=\frac{2}{3}\pi\Rightarrow t=\frac{20}{3}\pi\approx 21$. Dus na 21 seconden.

```
20/3π
        20.94395102
```

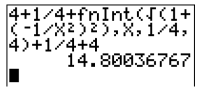
G53b $\frac{x_P+x_Q}{2}=\frac{5\cos(\frac{11}{10}t)+5\cos(t+\frac{2}{3}\pi)}{2}$
$=2\frac{1}{2}\cos(\frac{11}{10}t)+5\cos(t+\frac{2}{3}\pi)$
$=2\frac{1}{2}\cdot 2\cos(\frac{1}{2}(\frac{11}{10}t+t+\frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{11}{10}t-(t+\frac{2}{3}\pi)))$
$=5\cos(\frac{1}{2}(\frac{21}{10}t+\frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{1}{10}t-\frac{2}{3}\pi))$
$=5\cos(\frac{21}{20}t+\frac{1}{3}\pi)\cos(\frac{1}{20}t-\frac{1}{3}\pi).$

$\frac{y_P+y_Q}{2}=\frac{5\sin(\frac{11}{10}t)+5\sin(t+\frac{2}{3}\pi)}{2}$
$=2\frac{1}{2}\sin(\frac{11}{10}t)+5\sin(t+\frac{2}{3}\pi)$
$=2\frac{1}{2}\cdot 2\sin(\frac{1}{2}(\frac{11}{10}t+t+\frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{11}{10}t-(t+\frac{2}{3}\pi)))$
$=5\sin(\frac{1}{2}(\frac{21}{10}t+\frac{2}{3}\pi))\cos(\frac{1}{2}(\frac{1}{10}t-\frac{2}{3}\pi))$
$=5\sin(\frac{21}{20}t+\frac{1}{3}\pi)\cos(\frac{1}{20}t-\frac{1}{3}\pi).$

Dus $\begin{cases}x(t)=5\cos(\frac{21}{20}t+\frac{1}{3}\pi)\cos(\frac{1}{20}t-\frac{1}{3}\pi)\\ y(t)=5\sin(\frac{21}{20}t+\frac{1}{3}\pi)\cos(\frac{1}{20}t-\frac{1}{3}\pi)\end{cases}$ ofwel $\begin{cases}x(t)=5\cos(\frac{1}{20}t-\frac{1}{3}\pi)\cos(\frac{21}{20}t+\frac{1}{3}\pi)\\ y(t)=5\cos(\frac{1}{20}t-\frac{1}{3}\pi)\sin(\frac{21}{20}t+\frac{1}{3}\pi)\end{cases}\Rightarrow \phi(t)=5\cos(\frac{1}{20}t-\frac{1}{3}\pi).$

G54a $f(x)=\frac{1}{x}=x^{-1}\Rightarrow f'(x)=-x^{-2}=-\frac{1}{x^2}$. Dus $f(2)=\frac{1}{2}$ en $f'(2)=-\frac{1}{2^2}=-\frac{1}{4}$.
Raaklijn in $x=2$: $y=-\frac{1}{4}x+b$ door $(2,\frac{1}{2})\Rightarrow \frac{1}{2}=-\frac{1}{4}\cdot 2+b\Rightarrow \frac{1}{2}=-\frac{1}{2}+b\Rightarrow b=1$. Dus $y=-\frac{1}{4}x+1$.
$y=-\frac{1}{4}x+1$ gaat door ook $A(4,0)$, want $0=-\frac{1}{4}\cdot 4+1$.

G54b $f(x)=4\Rightarrow \frac{1}{x}=4\Rightarrow 4x=1\Rightarrow x=\frac{1}{4}\Rightarrow P(4,\frac{1}{4})$ en $Q(\frac{1}{4},4)$.
Omtrek $=4+\frac{1}{4}+\int_{\frac{1}{4}}^{4}\sqrt{1+(f'(x))^2}\,dx+\frac{1}{4}+4=4+\frac{1}{4}+\int_{\frac{1}{4}}^{4}\sqrt{1+\left(-\frac{1}{x^2}\right)^2}\,dx+\frac{1}{4}+4$ (fnInt) $\approx 14,80.$

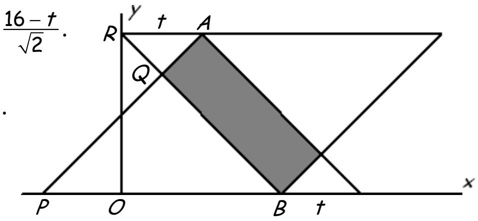


G54c $O(V)=\frac{1}{4}\cdot 4+\int_{\frac{1}{4}}^{4}\frac{1}{x}\,dx=1+\left[\ln|x|\right]_{\frac{1}{4}}^{4}=1+\ln(4)-\ln(\frac{1}{4})=1+\ln(4)-\ln(4^{-1})=1+\ln(4)+\ln(4)=1+2\cdot\ln(4).$

G54d $r_{AC}=-1\Rightarrow f'(x)=-1\Rightarrow -\frac{1}{x^2}=-1\Rightarrow -x^2=-1\Rightarrow x^2=1\Rightarrow x=1.$
Raaklijn in $x=1$ met $(f1)=\frac{1}{1}=1$: $y=-x+b$ door $(1,1)\Rightarrow 1=-1+b\Rightarrow b=2$. Dus $y=-x+2$.
$y=-x+2$ snijdt de y-as in $(0,2)\Rightarrow a=2.$

G55a $A(t,8)$ en $B(8,0)\Rightarrow AB=a(t)=\sqrt{(t-8)^2+8^2}=\sqrt{t^2-8t-8t+64+64}=\sqrt{128-16t+t^2}.$

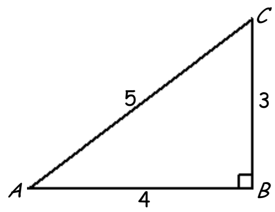
G55b ΔBPQ is een gelijkbenige rechthoekige driehoek met $BP=16-t\Rightarrow BQ=\frac{16-t}{\sqrt{2}}$.
(de zijden van een gelijkbenige rechthoekige driehoek verhouden zich als $1:1:\sqrt{2}$)
ΔAQR is een gelijkbenige rechthoekige driehoek met $AR=t\Rightarrow AQ=\frac{t}{\sqrt{2}}$.
$G(t)=AQ\cdot BQ=\frac{t}{\sqrt{2}}\cdot\frac{16-t}{\sqrt{2}}=\frac{16t-t^2}{2}=8t-\frac{1}{2}t^2=-\frac{1}{2}t^2+8t.$



G55c $G(t)=-\frac{1}{2}t^2+8t\Rightarrow G'(t)=-t+8$
$G'(t)=0\Rightarrow -t+8=0\Rightarrow -t=-8\Rightarrow t=8$
$a(t)=\sqrt{128-16t+t^2}\Rightarrow a'(t)=\frac{1}{2\sqrt{128-16t+t^2}}\cdot(-16+2t)=\frac{-16+2t}{\sqrt{128-16t+t^2}}$
$a'(t)=0\Rightarrow \frac{-16+2t}{\sqrt{128-16t+t^2}}=0$ (teller = 0) $\Rightarrow -16+2t=0\Rightarrow 2t=16\Rightarrow t=8$
$\Rightarrow G$ en a bereiken beide voor $t=8$ hun uiterste waarde.

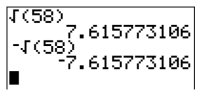
G55d $G=c-\frac{1}{2}a^2=c-\frac{1}{2}\sqrt{128-16t+t^2}^{\,2}=c-\frac{1}{2}(128-16t+t^2)=c-64+8t-\frac{1}{2}t^2$
$G(t)=-\frac{1}{2}t^2+8t$
$\Rightarrow c-64=0\Rightarrow c=64.$

G56a $O(ABC)=\frac{1}{2}\cdot 4\cdot 3=\frac{1}{2}\cdot 12=6$ en $s=\frac{1}{2}\cdot(3+4+5)=\frac{1}{2}\cdot 12=6.$
$H=\sqrt{6(6-3)(6-4)(6-5)}=\sqrt{6\cdot 3\cdot 2\cdot 1}=\sqrt{6\cdot 6}=6.$



G56b $s=\frac{1}{2}\cdot(3+7+x)=\frac{1}{2}\cdot(10+x)=5+\frac{1}{2}x.$
$H(x)=\sqrt{(5+\frac{1}{2}x)(5+\frac{1}{2}x-3)(5+\frac{1}{2}x-7)(5+\frac{1}{2}x-x)}$
$=\sqrt{(5+\frac{1}{2}x)(2+\frac{1}{2}x)(\frac{1}{2}x-2)(5-\frac{1}{2}x)}=\sqrt{(5+\frac{1}{2}x)(5-\frac{1}{2}x)(\frac{1}{2}x+2)(\frac{1}{2}x-2)}$
$=\sqrt{(25-2\frac{1}{2}x+2\frac{1}{2}x-\frac{1}{4}x^2)(\frac{1}{4}x^2-x+x-4)}=\sqrt{(25-\frac{1}{4}x^2)(\frac{1}{4}x^2-4)}.$

G56c $H(x)=\sqrt{(25-\frac{1}{4}x^2)(\frac{1}{4}x^2-4)}=\sqrt{\frac{25}{4}x^2-100-\frac{1}{16}x^4+x^2}=\sqrt{-100+\frac{29}{4}x^2-\frac{1}{16}x^4}.$
$H(x)=\sqrt{-100+\frac{29}{4}x^2-\frac{1}{16}x^4}\Rightarrow H'(x)=\frac{1}{2\cdot\sqrt{-100+\frac{29}{4}x^2-\frac{1}{16}x^4}}\cdot(\frac{58}{4}x-\frac{1}{4}x^3)=\frac{58x-x^3}{8\cdot\sqrt{-100+\frac{29}{4}x^2-\frac{1}{16}x^4}}.$

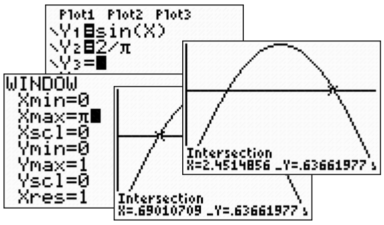


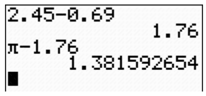
$H'(x)=0$ (teller = 0) $\Rightarrow 58x-x^3=0\Rightarrow x(58-x^2)=0\Rightarrow x=0\ \vee\ x^2=58$ (met $4<x<10$) $\Rightarrow x=\sqrt{58}.$

G57a $l(t) = A'B' = x_A - x_B = \cos(t - \frac{1}{6}\pi) - \cos(t + \frac{1}{6}\pi) = -2$
$= -2\sin(\frac{1}{2}(t - \frac{1}{6}\pi + t + \frac{1}{6}\pi))\sin(\frac{1}{2}(t - \frac{1}{6}\pi - (t + \frac{1}{6}\pi)))$
$= -2\sin(\frac{1}{2}\cdot 2t)\sin(\frac{1}{2}\cdot -\frac{1}{3}\pi) = -2\sin(t)\sin(-\frac{1}{6}\pi) = -2\sin(t)\cdot -\frac{1}{2} = \sin(t).$

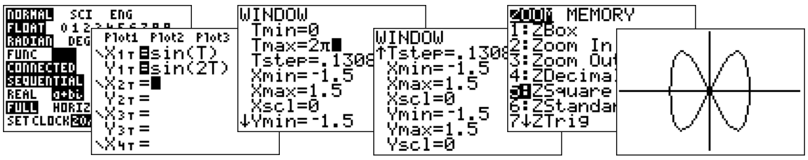
G57b $g(t) = \frac{1}{\pi}\cdot\int_0^{\pi}\sin(t)\,\mathrm{d}t = \frac{1}{\pi}\cdot\left[-\cos(t)\right]_0^{\pi} = \frac{1}{\pi}\cdot(-\cos(\pi) - -\cos(0)) = \frac{1}{\pi}\cdot(--1--1) = \frac{2}{\pi}.$

G57c $l(t) = \frac{2}{\pi} \Rightarrow \sin(t) = \frac{2}{\pi}$ (intersect) $\Rightarrow t \approx 0{,}69$ en $t \approx 2{,}45$.
de tijd dat $l(t) > \frac{2}{\pi}$ op $[0, \pi]$ is (ongeveer) $2{,}45 - 0{,}69 = 1{,}76$ (s).
de tijd dat $l(t) < \frac{2}{\pi}$ op $[0, \pi]$ is (ongeveer) $\pi - 1{,}76 \approx 1{,}38$ (s).
(dus de beide delen zijn niet even groot)





TI-84 12. Lissajous-figuren

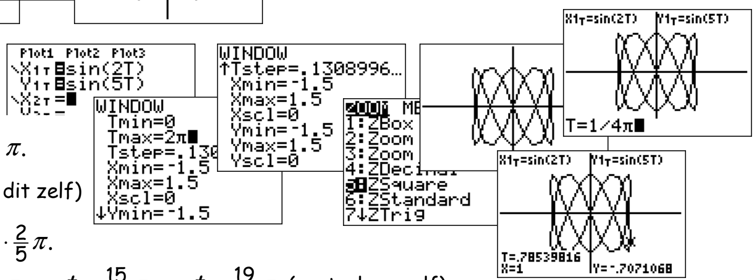


1a Zie de plot hiernaast.

1b $t = \frac{1}{4}\pi$ geeft het punt $(-1; -0{,}707)$.

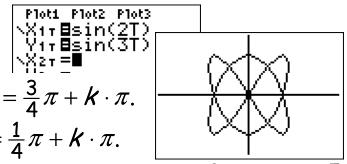
1c $x = \sin(2t) = -1 \Rightarrow 2t = \frac{3}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{3}{4}\pi + k\cdot\pi.$
$x = -1$ op $[0, 2\pi] \Rightarrow t = \frac{3}{4}\pi \ \vee\ t = \frac{7}{4}\pi$. (controleer dit zelf)

1d $y = \sin(5t) = -1 \Rightarrow 5t = \frac{3}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{3}{10}\pi + k\cdot\frac{2}{5}\pi.$
$y = -1$ op $[0, 2\pi] \Rightarrow t = \frac{3}{10}\pi \ \vee\ t = \frac{7}{10}\pi \ \vee\ t = \frac{11}{10}\pi \ \vee\ t = \frac{15}{10}\pi \ \vee\ t = \frac{19}{10}\pi$. (controleer zelf)



1e Uiteindelijk dezelfde kromme, alleen de volgorde waarin de punten worden geplot is anders.

1f Bijvoorbeeld $[0, \pi]$. (één keer van $x = 0$ via $x = 1$ terug naar $x = 0$ dan door naar $x = -1$ en weer terug naar $x = 0$)



2a Zie de plot hiernaast.

2b $x = \sin(2t) = -1 \Rightarrow 2t = \frac{3}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{3}{4}\pi + k\cdot\pi.$
$x = \sin(2t) = 1 \Rightarrow 2t = \frac{1}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{1}{4}\pi + k\cdot\pi.$
x heeft een extreme waarde op $[0, 2\pi] \Rightarrow t = \frac{1}{4}\pi \ \vee\ t = \frac{3}{4}\pi \ \vee\ t = \frac{5}{4}\pi \ \vee\ t = \frac{7}{4}\pi.$
$y = \sin(3t) = -1 \Rightarrow 3t = \frac{3}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{1}{2}\pi + k\cdot\frac{2}{3}\pi.$
$y = \sin(3t) = 1 \Rightarrow 3t = \frac{1}{2}\pi + k\cdot 2\pi \Rightarrow t = \frac{1}{6}\pi + k\cdot\frac{2}{3}\pi.$
y heeft een extreem op $[0, 2\pi] \Rightarrow t = \frac{1}{6}\pi \ \vee\ t = \frac{3}{6}\pi \ \vee\ t = \frac{5}{6}\pi \ \vee\ t = \frac{7}{6}\pi \ \vee\ t = \frac{9}{6}\pi \ \vee\ t = \frac{11}{6}\pi.$

2c Bijvoorbeeld $[-\frac{1}{2}\pi, \frac{1}{2}\pi]$. (van $(0, -1)$ via $(0, 0)$ naar eindpunt $(0, 1)$)