

1a  $(x^3 - 27)(x^2 - 4) = 0$   
 $x^3 - 27 = 0 \vee x^2 - 4 = 0$   
 $x^3 = 27 = 3^3 \vee x^2 = 4 = 2^2$   
 $x = 3 \vee x = 2 \vee x = -2.$

1b  $5x(x^2 - 4) = 15(x^2 - 4)$   
 $x^2 - 4 = 0 \vee 5x = 15$   
 $x^2 = 4 = 2^2 \vee x = 3$   
 $x = 2 \vee x = -2 \vee x = 3.$

1c  $(x^2 - 4)^2 = (x^2 - 8)^2$   
 $x^2 - 4 = x^2 - 8 \vee x^2 - 4 = -x^2 + 8$   
 $0 = -4 \vee 2x^2 = 12$   
 geen opl.  $\vee x^2 = 6$   
 $x = \sqrt{6} \vee x = -\sqrt{6}.$

2a In de opgave staat  $\ln(2x + 5)$  en dat is alleen gedefinieerd als  $2x + 5 > 0$ . Daarom moet bij de gevonden waarden gecontroleerd worden of ze aan deze voorwaarde voldoen.

2b  $(e^{2x} - 5) \cdot \ln(2x - 5) = 0$  ( $2x - 5 > 0 \Rightarrow x > 2,5$ )  
 $e^{2x} - 5 = 0 \vee \ln(2x - 5) = 0$   
 $e^{2x} = 5 \vee 2x - 5 = e^0 = 1$   
 $2x = \ln(5) \vee 2x = 6$   
 $x = \frac{1}{2}\ln(5)$  (voldoet niet)  $\vee x = 3$  (voldoet).  $\frac{1}{2}\ln(5)$   
0.8047189562

3a  $(3x^2 - 5)^2 = 4x^2$   
 $3x^2 - 5 = 2x \vee 3x^2 - 5 = -2x$   
 $3x^2 - 2x - 5 = 0 \vee 3x^2 + 2x - 5 = 0$   
 $D = (-2)^2 - 4 \cdot 3 \cdot -5 = 64 \quad D = 2^2 - 4 \cdot 3 \cdot -5 = 64$   
 $x = \frac{2-8}{2 \cdot 3} = \frac{-6}{6} = -1 \vee x = \frac{2+8}{2 \cdot 3} = \frac{10}{6} = \frac{5}{3} \vee x = \frac{-2-8}{2 \cdot 3} = \frac{-10}{6} = -\frac{5}{3} \vee x = \frac{-2+8}{2 \cdot 3} = \frac{6}{6} = 1.$

3b  $(4x - 1)^2 = (3x - 2)^2$   
 $4x - 1 = 3x - 2 \vee 4x - 1 = -3x + 2$   
 $x = -1 \vee 7x = 3$   
 $x = -1 \vee x = \frac{3}{7}.$

3c  $(3^x - 12) \cdot {}^3\log(2x + 1) = 0$  ( $2x + 1 > 0 \Rightarrow x > -0,5$ )  
 $3^x - 12 = 0 \vee {}^3\log(2x + 1) = 0$   
 $3^x = 12 \vee 2x + 1 = 3^0 = 1$   
 $x = {}^3\log(12)$  (voldoet)  $\vee 2x = 0$ .  $\frac{\ln(12)}{\ln(3)}$   
2.261859507  
 $\log(12)/\log(3)$   
2.261859507

3d  $(x - 1) \cdot \cos(2x + \frac{1}{4}\pi) = 0$   
 $x - 1 = 0 \vee \cos(2x + \frac{1}{4}\pi) = 0$   
 $x = 1 \vee 2x + \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot \pi$   
 $x = 1 \vee 2x = \frac{1}{4}\pi + k \cdot \pi$   
 $x = 1 \vee x = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi.$

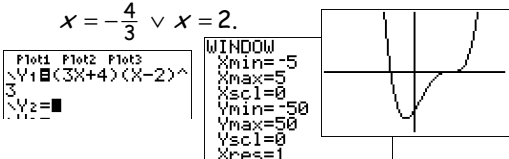
3e  $2^x \cdot \log(4x + 1) = 20 \cdot \log(4x + 1)$  ( $4x + 1 > 0 \Rightarrow x > -0,25$ )  
 $\log(4x + 1) = 0 \vee 2^x = 20$   
 $4x + 1 = 10^0 = 1 \vee x = {}^2\log(20)$  (voldoet)  $\frac{\log(20)}{\log(2)}$   
4.321928095  
 $4x = 0 \vee x = {}^2\log(20)$   
 $x = 0$  (voldoet)  $\vee x = {}^2\log(20).$

3f  $x^3 \cdot \sin(2x) = \sin(2x)$   
 $\sin(2x) = 0 \vee x^3 = 1$   
 $2x = k \cdot \pi \vee x = 1$   
 $x = k \cdot \frac{1}{2}\pi \vee x = 1.$

4a  $f(x) = 0 \Rightarrow (3x + 4)(x - 2)^3 = 0$   
 $3x + 4 = 0 \vee (x - 2)^3 = 0$   
 $3x = -4 \vee x - 2 = 0$   
 $x = -\frac{4}{3} \vee x = 2.$

4b  $f(x) = (3x + 4)(x - 2)^3 \Rightarrow f'(x) = 3 \cdot (x - 2)^3 + (3x + 4) \cdot 3(x - 2)^2 \cdot 1$   
 $= 3(x - 2) \cdot (x - 2)^2 + 3(3x + 4) \cdot (x - 2)^2$   
 $= (3x - 6 + 9x + 12) \cdot (x - 2)^2 = (12x + 6)(x - 2)^2$   
 $f'(x) = 0 \Rightarrow (12x + 6)(x - 2)^2 = 0$   
 $12x + 6 = 0 \vee (x - 2)^2 = 0$   
 $12x = -6 \vee x - 2 = 0$   
 $x = -\frac{1}{2} \vee x = 2.$

Minimum (zie plot)  $f(-\frac{1}{2}) = (3 \cdot -\frac{1}{2} + 4)(-\frac{1}{2} - 2)^3 = 2,5 \cdot (-2,5)^3 = -39 \frac{1}{16}.$



4c  $f(x) = 3x + 4 \Rightarrow (3x + 4)(x - 2)^3 = 3x + 4$   
 $3x + 4 = 0 \vee (x - 2)^3 = 1 = 1^3$   
 $3x = -4 \vee x - 2 = 1$   
 $x = -\frac{4}{3} \vee x = 3.$   
 $f(-\frac{4}{3}) = 0$  (zie 4a)  $\Rightarrow A(-\frac{4}{3}, 0)$  en  
 $x = 3 \Rightarrow y = 3 \cdot 3 + 4 = 13 \Rightarrow B(3, 13).$

4d  $f(x) = (3x + 4)(x - 2) \Rightarrow (3x + 4)(x - 2)^3 = (3x + 4)(x - 2)$   
 $(3x + 4)(x - 2) = 0 \vee (x - 2)^2 = 1 = 1^2$   
 $3x + 4 = 0 \vee x - 2 = 0 \vee (x - 2)^2 = 1 = 1^2$   
 $3x = -4 \vee x = 2 \vee x - 2 = 1 \vee x - 2 = -1$   
 $x = -\frac{4}{3} \vee x = 2 \vee x = 3 \vee x = 1.$   
 $(-\frac{4}{3}, 0)$  (zie 4a),  $(2, 0)$  (zie 4a),  $(3, 13)$  (zie 43c) en  $(1, -7).$

5a  $\frac{2x-1}{x-1} = 0$  (teller = 0 en noemer  $\neq 0$ )  $\Rightarrow 2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$  (voldoet).

5b  $\frac{2x-1}{x-1} = \frac{3x+1}{x+1}$  ( $x \neq 1$  en  $x \neq -1$ )  
 $(2x - 1)(x + 1) = (3x + 1)(x - 1)$   
 $2x^2 + 2x - x - 1 = 3x^2 - 3x + x - 1$   
 $-x^2 + 3x = 0$   
 $-x(x - 3) = 0$   
 $x = 0$  (voldoet)  $\vee x = 3$  (voldoet).

5c  $\frac{2x-1}{x-1} = \frac{2x-1}{2x+1}$  ( $x \neq 1$  en  $2x \neq -1 \Rightarrow x \neq -\frac{1}{2}$ )  
 $(2x - 1)(2x + 1) = (2x - 1)(x - 1)$   
 $2x - 1 = 0 \vee 2x + 1 = x - 1$   
 $2x = 1 \vee x = -2$   
 $x = \frac{1}{2}$  (voldoet)  $\vee x = -2$  (voldoet).

5d  $\frac{2x-1}{x-1} = \frac{x+1}{x-1} (x \neq 1) \Rightarrow 2x-1 = x+1 \Rightarrow x = 2$  (voldoet).

6 Neem  $C = 0$  en  $D \neq 0$  in  $\frac{A}{B} = \frac{C}{D}$  ( $B \neq 0$  en  $D \neq 0$ ), dan krijg je  $\frac{A}{B} = 0$  ( $B \neq 0$ )  $\Rightarrow A = 0$  ( $B \neq 0$ ).  
 Neem  $C = A$  in  $\frac{A}{B} = \frac{C}{D}$  ( $B \neq 0$  en  $D \neq 0$ ), dan krijg je  $\frac{A}{B} = \frac{A}{D}$  ( $B \neq 0$  en  $D \neq 0$ )  $\Rightarrow A = 0 \vee B = D$  ( $B \neq 0$  en  $D \neq 0$ ).  
 Neem  $D = B$  in  $\frac{A}{B} = \frac{C}{D}$  ( $B \neq 0$  en  $D \neq 0$ ), dan krijg je  $\frac{A}{B} = \frac{C}{B}$  ( $B \neq 0$ )  $\Rightarrow A = C$  ( $B \neq 0$ ).

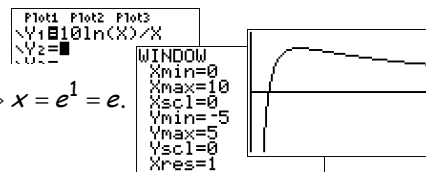
7a  $\frac{x^2+3}{2x} = x-1 (x \neq 0)$   
 $x^2+3 = 2x(x-1)$   
 $x^2+3 = 2x^2-2x$   
 $-x^2+2x+3 = 0$   
 $x^2-2x-3 = 0$   
 $(x-3)(x+1) = 0$   
 $x = 3$  (voldoet)  $\vee x = -1$  (voldoet).

7c  $\frac{\cos(2x)}{4x} = \frac{\cos(2x)}{2x+\pi} (x \neq 0 \text{ en } x \neq -\frac{1}{2}\pi)$   
 $\cos(2x) = 0 \vee 4x = 2x+\pi$   
 $2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = \pi$   
 $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$  (voldoet)  $\vee x = \frac{1}{2}\pi$  (voldoet).

7b  $\frac{4^x-2^x-6}{2^x-4} = 0 (2^x \neq 4 \Rightarrow x \neq 2 \log(4) = 2 \log(2^2) = 2)$   
 $4^x-2^x-6 = 0$   
 $(2^x)^2-1 \cdot 2^x-6 = 0$   
 $(2^x-3)(2^x+2) = 0$   
 $2^x = 3 \vee 2^x = -2$  (kan niet)  
 $x = 2 \log(3)$  (voldoet).

7d  $\frac{\ln^2(x)}{1+\ln(x)} = \frac{2+\ln(x)}{1+\ln(x)} (\ln(x) \neq -1 \Rightarrow x \neq e^{-1} = \frac{1}{e})$   
 $\ln^2(x) = 2+\ln(x)$   
 $\ln^2(x)-1 \cdot \ln(x)-2 = 0$   
 $(\ln(x)-2)(\ln(x)+1) = 0$   
 $\ln(x) = 2 \vee \ln(x) = -1$   
 $x = e^2$  (voldoet)  $\vee x = e^{-1} = \frac{1}{e}$  (voldoet niet).

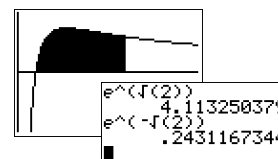
8a  $f(x) = \frac{10\ln(x)}{x} (x > 0) \Rightarrow f'(x) = \frac{x \cdot \frac{10}{x} - 10\ln(x) \cdot 1}{x^2} = \frac{10-10\ln(x)}{x^2}$   
 $f'(x) = 0 \Rightarrow \frac{10-10\ln(x)}{x^2} = 0$  (teller = 0)  $\Rightarrow 10-10\ln(x) = 0 \Rightarrow \ln(x) = 1 \Rightarrow x = e^1 = e$ .  
 $f(e) = \frac{10\ln(e)}{e} = \frac{10}{e}$ . Dus de top is  $(e, \frac{10}{e})$ .



8b  $f'(x) = \frac{10-10\ln(x)}{x^2} (x > 0) \Rightarrow f''(x) = \frac{x^2 \cdot \frac{-10}{x} - (10-10\ln(x)) \cdot 2x}{x^4} = \frac{-10x-20x+20x\ln(x)}{x^4} = \frac{-30x+20x\ln(x)}{x^4} = \frac{-30+20\ln(x)}{x^3}$   
 $f''(x) = 0 \Rightarrow \frac{-30+20\ln(x)}{x^3} = 0$  (teller = 0)  $\Rightarrow 20\ln(x)-30 = 0 \Rightarrow 20\ln(x) = 30 \Rightarrow \ln(x) = 1\frac{1}{2} \Rightarrow x = e^{1\frac{1}{2}} = e\sqrt{e}$ .  
 $f(e\sqrt{e}) = \frac{10\ln(e^{1\frac{1}{2}})}{e\sqrt{e}} = \frac{10 \cdot \frac{1}{2}}{e\sqrt{e}} = \frac{5}{e\sqrt{e}}$ . Dus het buigpunt is  $(e\sqrt{e}, \frac{5}{e\sqrt{e}})$ .

8c Raaklijn door  $O(0, 0) \Rightarrow$  de  $x$ -coördinaat van het raakpunt volgt uit  $f'(x) = \frac{f(x)}{x}$ .  
 $\frac{10-10\ln(x)}{x^2} = \frac{10\ln(x)}{x} \Rightarrow 10-10\ln(x) = 10\ln(x) \Rightarrow -20\ln(x) = -10 \Rightarrow \ln(x) = \frac{1}{2} \Rightarrow x = e^{\frac{1}{2}} = \sqrt{e}$ .  
 $rc_k = f'(\sqrt{e}) = \frac{10-10 \cdot \frac{1}{2}}{e} = \frac{5}{e}$ . Dus  $k: y = \frac{5}{e}x$ .

8d  $F(x) = 5\ln^2(x) \Rightarrow F'(x) = 5 \cdot 2\ln(x) \cdot \frac{1}{x} = \frac{10\ln(x)}{x} = f(x)$ . Dus  $F$  is een primitieve van  $f$ .  
 $O(V) = \int_1^a f(x) dx = [F(x)]_1^a = [5\ln^2(x)]_1^a = 5\ln^2(a) - 5\ln^2(1) = 5\ln^2(a) - 0 = 5\ln^2(a)$ .  
 $O(V) = 10 \Rightarrow 5\ln^2(a) = 10 \Rightarrow \ln^2(a) = 2 \Rightarrow \ln(a) = \sqrt{2} \vee \ln(a) = -\sqrt{2} \Rightarrow a = e^{\sqrt{2}} \vee a = e^{-\sqrt{2}}$  ( $\approx 0,24$  voldoet niet).



9a  $(2x+1)^2 = (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2 = 4x^2 + 4x + 1$ .

9b  $(3x-2)^2 = (3x)^2 + 2 \cdot 3x \cdot (-2) + (-2)^2 = 9x^2 - 12x + 4$ .

9c  $(4x+3)(4x-3) = (4x)^2 - 12x + 12x - 3^2 = 16x^2 - 9$ .

9d  $(x+2)^3 = (x+2) \cdot (x+2)^2 = (x+2) \cdot (x^2+4x+4) = x^3+4x^2+4x+2x^2+8x+8 = x^3+6x^2+12x+8$ .

10a  $(3x\sqrt{2} - \sqrt{5})^2 = 18x^2 - 6x\sqrt{10} + 5$ .

10b  $(2x-1)^3 = (2x-1) \cdot (2x-1)^2 = (2x-1) \cdot (4x^2-4x+1) = 8x^3-8x^2+2x-4x^2+4x-1 = 8x^3-12x^2+6x-1$ .

10c  $\frac{2x^5-32x}{x^2-4} (x^2 \neq 4 \Rightarrow x \neq 2 \wedge x \neq -2) = \frac{2x(x^4-16)}{x^2-4} = \frac{2x(x^2+4)(x^2-4)}{x^2-4} = 2x(x^2+4) (x \neq 2 \wedge x \neq -2)$ .

10d  $\square$   $(4x\sqrt{2} - 3)(4x\sqrt{2} + 3) = 32x^2 - 9.$

10e  $\square$   $(2^x + 1)^3 = (2^x + 1) \cdot (2^x + 1)^2 = (2^x + 1) \cdot (2^{2x} + 2 \cdot 2^x + 1) = 2^{3x} + 2 \cdot 2^{2x} + 2^x + 2^{2x} + 2 \cdot 2^x + 1 = 2^{3x} + 3 \cdot 2^{2x} + 3 \cdot 2^x + 1.$

10f  $\square$   $\frac{x^4 + 4x^2 + 4}{x^4 - 4} (x^4 \neq 4 \Rightarrow x^2 \neq 2 \wedge x^2 \neq -2 \Rightarrow x \neq \sqrt{2} \wedge x \neq -\sqrt{2}) = \frac{(x^2 + 2)^2}{(x^2 + 2)(x^2 - 2)} = \frac{(x^2 + 2)}{(x^2 - 2)} (x \neq \sqrt{2} \wedge x \neq -\sqrt{2}).$

11a  $k: y = mx + n$  door  $(a, a^2)$  en  $(b, b^2)$   $rc_k = m = \frac{b^2 - a^2}{b - a} = \frac{(b + a)(b - a)}{b - a} = b + a = a + b.$

$k: y = (a + b)x + n$  door  $(a, a^2) \Rightarrow a^2 = (a + b) \cdot a + n \Rightarrow a^2 = a^2 + ab + n \Rightarrow -ab = n.$  Dus  $k: y = (a + b)x - ab.$

11b  $O(V) = \int_a^b ((a + b)x - ab - x^2) dx = \left[ (a + b) \cdot \frac{1}{2} x^2 - abx - \frac{1}{3} x^3 \right]_a^b$   
 $= \frac{1}{2} (a + b)b^2 - ab \cdot b - \frac{1}{3} b^3 - \left( \frac{1}{2} (a + b)a^2 - ab \cdot a - \frac{1}{3} a^3 \right) = \frac{1}{2} ab^2 + \frac{1}{2} b^3 - ab^2 - \frac{1}{3} b^3 - \frac{1}{2} a^3 - \frac{1}{2} a^2 b + a^2 b + \frac{1}{3} a^3$   
 $= -\frac{1}{6} a^3 + \frac{1}{2} a^2 b - \frac{1}{2} ab^2 + \frac{1}{6} b^3$  ①  
 $\frac{1}{6} (b - a)^3 = \frac{1}{6} (b - a) \cdot (b - a)^2 = \frac{1}{6} (b - a) \cdot (b^2 - 2ab + a^2) = \frac{1}{6} (b^3 - 2ab^2 + a^2 b - ab^2 + 2a^2 b - a^3)$   
 $= \frac{1}{6} (b^3 - 3ab^2 + 3a^2 b - a^3) = \frac{1}{6} b^3 - \frac{1}{2} ab^2 + \frac{1}{2} a^2 b - \frac{1}{6} a^3$  ②. Uit ① en ②  $\Rightarrow O(V) = \frac{1}{6} (b - a)^3.$

12  $k: y = mx + n$  door  $(-2, 0)$  en  $(a, 4 - a^2) \Rightarrow rc_k = m = \frac{4 - a^2 - 0}{a - (-2)} = \frac{4 - a^2}{a + 2} = \frac{(2 + a)(2 - a)}{a + 2} = 2 - a.$

$k: y = (2 - a)x + n$  door  $(-2, 0) \Rightarrow 0 = (2 - a) \cdot (-2) + n \Rightarrow n = 2(2 - a) = 4 - 2a.$  Dus  $k: y = (2 - a)x + 4 - 2a.$

$O(V) = \int_{-2}^a (4 - x^2 - ((2 - a)x + 4 - 2a)) dx = \int_{-2}^a (4 - x^2 - 2x + ax - 4 + 2a) dx = \int_{-2}^a (-x^2 - 2x + ax + 2a) dx$   
 $= \left[ -\frac{1}{3} x^3 - x^2 + \frac{1}{2} ax^2 + 2ax \right]_{-2}^a = -\frac{1}{3} a^3 - a^2 + \frac{1}{2} a^3 + 2a^2 - \left( -\frac{1}{3} \cdot (-2)^3 - (-2)^2 + \frac{1}{2} a \cdot (-2)^2 + 2a \cdot (-2) \right)$   
 $= -\frac{1}{3} a^3 - a^2 + \frac{1}{2} a^3 + 2a^2 - \left( \frac{8}{3} - 4 + 2a - 4a \right) = \frac{1}{6} a^3 + a^2 - \left( \frac{8}{3} - 4 - 2a \right) = \frac{1}{6} a^3 + a^2 + 2a + \frac{4}{3}$  ①  
 $\frac{1}{6} (a + 2)^3 = \frac{1}{6} (a + 2) \cdot (a + 2)^2 = \frac{1}{6} (a + 2) \cdot (a^2 + 4a + 4) = \frac{1}{6} (a^3 + 4a^2 + 4a + 2a^2 + 8a + 8)$   
 $= \frac{1}{6} (a^3 + 6a^2 + 12a + 8) = \frac{1}{6} a^3 + a^2 + 2a + \frac{4}{3}$  ②. Uit ① en ②  $\Rightarrow O(V) = \frac{1}{6} (a + 2)^3.$

13a  $y = 2x - \frac{1}{x} (x \neq 0) = 2x \cdot \frac{x}{x} - \frac{1}{x} = \frac{2x^2}{x} - \frac{1}{x} = \frac{2x^2 - 1}{x}.$

13b  $y = \frac{x}{x+1} + \frac{x}{x+2} (x \neq -1 \wedge x \neq -2) = \frac{x(x+2)}{(x+1)(x+2)} + \frac{x(x+1)}{(x+1)(x+2)} = \frac{x^2 + 2x}{(x+1)(x+2)} + \frac{x^2 + x}{(x+1)(x+2)} = \frac{2x^2 + 3x}{(x+1)(x+2)}.$

13c  $y = \frac{x}{\left(\frac{x}{2}\right)} (x \neq 0) = x \cdot \frac{2}{x} = \frac{x^2}{2}.$

13d  $y = (x + 1) \cdot \frac{x+2}{x+3} (x \neq -3) = \frac{(x+1)(x+2)}{x+3} = \frac{x^2 + 3x + 2}{x+3}.$

13e  $y = \frac{2}{x} \cdot \frac{x+2}{x+3} (x \neq 0 \wedge x \neq -3) = \frac{2(x+2)}{x(x+3)} = \frac{2x+4}{x(x+3)}.$

13f  $y = \frac{x+1}{\left(\frac{x+2}{x-1}\right)} (x \neq -2 \wedge x \neq 1) = (x+1) \cdot \frac{x-1}{x+2} = \frac{(x+1)(x-1)}{x+2} = \frac{x^2 - 1}{x+2}.$

14  $y = \frac{20}{x-1} \cdot \left(4 - \frac{2}{x-1}\right) (x \neq 1) = \frac{20}{x-1} \cdot \left(\frac{4(x-1)}{x-1} - \frac{2}{x-1}\right) = \frac{20}{x-1} \cdot \frac{4x-4-2}{x-1} = \frac{20(4x-6)}{(x-1)^2} = \frac{80x-120}{(x-1)^2}.$

15a  $\square$   $y = \frac{20}{x} - \frac{5}{2x} (x \neq 0) = \frac{40}{2x} - \frac{5}{2x} = \frac{35}{2x}.$

15b  $\square$   $y = \frac{10}{x-1} - x^2 (x \neq 1) = \frac{10}{x-1} - \frac{x^2(x-1)}{x-1} = \frac{10 - x^2(x-1)}{x-1} = \frac{10 - x^3 + x^2}{x-1} = \frac{-x^3 + x^2 + 10}{x-1}.$

15c  $\square$   $y = \frac{2x^2}{\left(\frac{x+1}{x-1}\right)} (x \neq -1 \wedge x \neq 1) = 2x^2 \cdot \frac{x-1}{x+1} = \frac{2x^2(x-1)}{x+1} = \frac{2x^3 - 2x^2}{x+1}.$

15d  $\square$   $y = \frac{x}{x-1} \cdot \left(x + \frac{1}{x-1}\right) (x \neq 1) = \frac{x}{x-1} \cdot \left(\frac{x(x-1)}{x-1} + \frac{1}{x-1}\right) = \frac{x}{x-1} \cdot \frac{x^2 - x + 1}{x-1} = \frac{x^3 - x^2 + x}{(x-1)^2}.$

15e  $\square$   $y = \frac{5}{x-2} \cdot \frac{6}{x+2} (x \neq 2 \wedge x \neq -2) = \frac{30}{(x-2)(x+2)}.$

15f  $\square$   $y = \frac{\left(\frac{x+1}{2x}\right)}{\left(\frac{x+1}{x-1}\right)} (x \neq 0 \wedge x \neq 1) = \frac{x+1}{2x} \cdot \frac{x-1}{x+1} = \frac{x-1}{2x}.$

$$16a \quad y = \frac{\ln(x)}{x} - \frac{2\ln(x)}{3x} \quad (x > 0) = \frac{3\ln(x)}{3x} - \frac{2\ln(x)}{3x} = \frac{\ln(x)}{3x}.$$

$$16b \quad y = \frac{e^x}{x-1} - 2e^x \quad (x \neq 1) = \frac{e^x}{x-1} - \frac{2e^x(x-1)}{x-1} = \frac{e^x - 2e^x(x-1)}{x-1} = \frac{e^x - 2xe^x + 2e^x}{x-1} = \frac{-2xe^x + 3e^x}{x-1} = \frac{(-2x+3)e^x}{x-1}.$$

$$16c \quad y = \frac{e^x - 2}{\left(\frac{e^x+1}{e^x+2}\right)} = (e^x - 2) \cdot \frac{e^x+2}{e^x+1} = \frac{(e^x-2)(e^x+2)}{e^x+1} = \frac{e^{2x}-4}{e^x+1}.$$

$$16d \quad y = \frac{2^x}{2^x+4} \left(2^x + \frac{1}{2^x}\right) = \frac{2^x \cdot 2^x}{2^x+4} + \frac{2^x \cdot \frac{1}{2^x}}{2^x+4} = \frac{2^{2x}}{2^x+4} + \frac{1}{2^x+4} = \frac{2^{2x}+1}{2^x+4}.$$

$$16e \quad y = \frac{e^x}{e^x-1} \cdot \frac{e^{2x}}{e^x+1} \quad (e^x \neq 1 \Rightarrow x \neq 0) = \frac{e^x \cdot e^{2x}}{(e^x-1)(e^x+1)} = \frac{e^{3x}}{(e^x-1)(e^x+1)}.$$

$$16f \quad y = \frac{\left(\frac{5+\ln(x)}{x}\right)}{x} \quad (x > 0) = \frac{5+\ln(x)}{x} \cdot \frac{1}{x} = \frac{5+\ln(x)}{x^2}.$$

$$17a \quad \frac{4e^x}{e^x+1} - \frac{e^x}{e^x-1} \quad (e^x \neq 1 \Rightarrow x \neq 0) \quad 17b \quad \frac{4e^x}{e^x+1} \cdot \frac{e^x}{e^x-1} = 6 \quad (x \neq 0)$$

$$4e^x(e^x-1) = e^x(e^x+1)$$

$$4(e^x-1) = e^x+1$$

$$4e^x - 4 = e^x + 1$$

$$3e^x = 5$$

$$e^x = \frac{5}{3}$$

$$x = \ln\left(\frac{5}{3}\right).$$

$$\frac{4e^{2x}}{e^{2x}-1} = 6$$

$$4e^{2x} = 6e^{2x} - 6$$

$$-2e^{2x} = -6$$

$$e^{2x} = 3$$

$$2x = \ln(3)$$

$$x = \frac{1}{2}\ln(3).$$

$$17d \quad f'(x) + g'(x) = 0 \quad (e^x \neq 1 \Rightarrow x \neq 0)$$

$$\frac{(e^x+1) \cdot 4e^x - 4e^x \cdot e^x}{(e^x+1)^2} + \frac{(e^x-1) \cdot e^x - e^x \cdot e^x}{(e^x-1)^2} = 0$$

$$\frac{4e^{2x} + 4e^x - 4e^{2x}}{(e^x+1)^2} + \frac{e^{2x} - e^x - e^{2x}}{(e^x-1)^2} = 0$$

$$\frac{4e^x}{(e^x+1)^2} - \frac{e^x}{(e^x-1)^2} = 0$$

$$\frac{4e^x}{(e^x+1)^2} = \frac{e^x}{(e^x-1)^2} \quad (e^x \text{ is alleen positief})$$

$$\frac{4}{(e^x+1)^2} = \frac{1}{(e^x-1)^2}$$

$$4(e^x-1)^2 = (e^x+1)^2$$

$$2(e^x-1) = e^x+1 \vee 2(e^x-1) = -(e^x+1)$$

$$2e^x - 2 = e^x + 1 \vee 2e^x - 2 = -e^x - 1$$

$$e^x = 3 \vee 3e^x = 1$$

$$e^x = 3 \vee e^x = \frac{1}{3}$$

$$x = \ln(3) \vee x = \ln\left(\frac{1}{3}\right).$$

$$17c \quad \frac{4e^x}{e^x+1} - \frac{e^x}{e^x-1} = 3 \quad (x \neq 0)$$

(vermenigvuldigen met  $(e^x+1)(e^x-1)$ )

$$4e^x(e^x-1) - e^x(e^x+1) = 3(e^x+1)(e^x-1)$$

$$4e^{2x} - 4e^x - e^{2x} - e^x = 3(e^{2x}-1)$$

$$3e^{2x} - 5e^x = 3e^{2x} - 3$$

$$-5e^x = -3$$

$$e^x = \frac{3}{5}$$

$$x = \ln\left(\frac{3}{5}\right).$$

$$18 \quad y = \frac{1+\frac{3}{x}}{x+1} \quad (x \neq 0 \wedge x \neq -1) = \frac{1+\frac{3}{x}}{x+1} \cdot \frac{x}{x} = \frac{x+3}{x(x+1)} \Rightarrow \text{I is correct.}$$

II is niet correct, want de factor  $(x+1)$  moet in de noemer staan (zie 18I).

$$y = \frac{1+\frac{3}{x}}{x+1} \quad (x \neq 0 \wedge x \neq -1) = \frac{x+3}{x(x+1)} \quad (\text{zie 18I}) = \frac{x}{x(x+1)} + \frac{3}{x(x+1)} = \frac{1}{x+1} + \frac{3}{x(x+1)} \Rightarrow \text{III is correct.}$$

IV is niet correct, want de factor  $(x+1)$  moet in de noemer van de tweede breuk staan (zie 18III).

$$19a \quad y = \frac{x+\frac{3}{x+1}}{x} \quad (x \neq 0 \wedge x \neq -1) = \frac{x+\frac{3}{x+1}}{x} \cdot \frac{x+1}{x+1} = \frac{x(x+1)+3}{x(x+1)} = \frac{x^2+x+3}{x(x+1)}.$$

$$19b \quad y = \frac{10x}{p+\frac{x^2}{2p}} \quad (p \neq 0) = \frac{10x}{p+\frac{x^2}{2p}} \cdot \frac{2p}{2p} = \frac{20px}{2p^2+x^2}.$$

$$19c \quad y = \frac{10+\frac{x-1}{6-\frac{3}{x-1}}}{x-1} \quad (x \neq 1 \wedge \frac{3}{x-1} \neq 6 \Rightarrow x-1 \neq \frac{1}{2} \Rightarrow x \neq 1\frac{1}{2}) = \frac{10+\frac{x-1}{6-\frac{3}{x-1}}}{x-1} \cdot \frac{x-1}{x-1} = \frac{10(x-1)+5}{6(x-1)-3} = \frac{10x-10+5}{6x-6-3} = \frac{10x-5}{6x-9}.$$

$$19d \quad y = \frac{\frac{3x}{x+4}-5}{\frac{2x}{x+4}-x+5} \quad (x \neq -4) = \frac{\frac{3x}{x+4}-5}{\frac{2x}{x+4}-x+5} \cdot \frac{x+4}{x+4} = \frac{3x-5(x+4)}{2x+(-x+5)(x+4)} = \frac{3x-5x-20}{2x-x^2-4x+5x+20} = \frac{-2x-20}{-x^2+3x+20} = \frac{2x+20}{x^2-3x-20}.$$

$$20a \quad N = \frac{600a}{3b-\frac{a^2}{4b}} \quad (b \neq 0 \wedge 3b \neq \frac{a^2}{4b} \Rightarrow 12b^2 \neq a^2) = \frac{600a}{3b-\frac{a^2}{4b}} \cdot \frac{4b}{4b} = \frac{2400ab}{12b^2-a^2}.$$

$$20b \quad A = 25x + 20 \cdot \frac{\frac{50}{x^2+1}}{x} \quad (x \neq 0) = 25x + 20 \cdot \frac{\frac{50}{x^2+1}}{x} \cdot \frac{x^2+1}{x^2+1} = 25x + 20 \cdot \frac{50}{x(x^2+1)} = 25x + \frac{1000}{x(x^2+1)}.$$

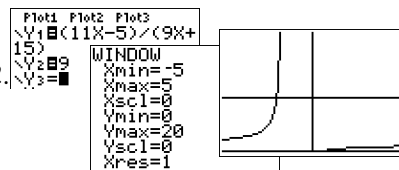
$$20c \quad K = \left(50 + \frac{150}{\frac{p}{q}+5}\right) \cdot p \quad (q \neq 0 \wedge \frac{p}{q}+5 \neq 0) = 50p \cdot \frac{p+5q}{p+5q} + \frac{150p}{\frac{p}{q}+5} \cdot \frac{q}{q} = \frac{50p^2+250pq}{p+5q} + \frac{150pq}{p+5q} = \frac{50p^2+400pq}{p+5q}.$$

21a  $p = \frac{3x}{x+5}$  ( $x \neq -5$ ) invullen in  $N = \frac{4p-1}{2p+3}$  ( $2p+3 \neq 0$ ) geeft  $N = \frac{4 \cdot \frac{3x}{x+5} - 1}{2 \cdot \frac{3x}{x+5} + 3} \cdot \frac{x+5}{x+5} = \frac{12x - (x+5)}{6x + 3(x+5)} = \frac{11x-5}{9x+15}$  ( $x \neq -\frac{15}{9} = -1\frac{2}{3}$ ).

21b  $N > 9 \Rightarrow \frac{11x-5}{9x+15} > 9$  ( $x \neq -\frac{15}{9}$ ).

$\frac{11x-5}{9x+15} = 9 \Rightarrow 11x-5 = 9(9x+15) \Rightarrow 11x-5 = 81x+135 \Rightarrow -70x = 140 \Rightarrow x = -2$ .

$N > 9$  (zie een plot)  $\Rightarrow -2 < x < -1\frac{2}{3}$ .



22a  $N = \frac{x^2+5x-6}{x}$  ( $x \neq 0$ )  $= \frac{x^2}{x} + \frac{5x}{x} - \frac{6}{x} = x + 5 - \frac{6}{x}$ .

22b  $A = \frac{5x^2+1000}{x}$  ( $x \neq 0$ )  $= \frac{5x^2}{x} + \frac{1000}{x} = 5x + \frac{1000}{x}$ .

22c  $K = \frac{6t^2+12t+1500}{3t}$  ( $t \neq 0$ )  $= \frac{6t^2}{3t} + \frac{12t}{3t} + \frac{1500}{3t} = 2t + 4 + \frac{500}{t}$ .

22d  $F = \frac{5a^2+8a}{2a^2}$  ( $a \neq 0$ )  $= \frac{5a^2}{2a^2} + \frac{8a}{2a^2} = 2\frac{1}{2} + \frac{4}{a}$ .

22e  $N = \frac{6p^2-3p-1}{2p}$  ( $p \neq 0$ )  $= \frac{6p^2}{2p} - \frac{3p}{2p} - \frac{1}{2p} = 3p - 1\frac{1}{2} - \frac{1}{2p}$ .

23  $y = \frac{2}{x} \Rightarrow xy = 2 \Rightarrow x = \frac{2}{y}$ .

24a  $A = \frac{B}{B+2}$   
 $A(B+2) = B$   
 $AB + 2A = B$   
 $AB - B = -2A$   
 $B(A-1) = -2A$   
 $B = -\frac{2A}{A-1}$ .

24b  $P = \frac{Q-5}{Q}$   
 $PQ = Q-5$   
 $PQ - Q = -5$   
 $Q(P-1) = -5$   
 $Q = -\frac{5}{P-1}$ .

24c  $R = \frac{F-2}{F-1}$   
 $R(F-1) = F-2$   
 $RF - R = F-2$   
 $RF - F = R-2$   
 $F(R-1) = R-2$   
 $F = \frac{R-2}{R-1}$ .

24d  $L = 320 - \frac{18}{q-1}$   
 $\frac{18}{q-1} = \frac{320-L}{1}$   
 $\frac{18}{320-L} = \frac{q-1}{1}$   
 $q-1 = \frac{18}{320-L}$   
 $q = 1 + \frac{18}{320-L}$ .

25a  $\frac{1}{a} = 2 + \frac{1}{b}$   
 $\frac{1}{a} = \frac{2b+1}{b}$   
 $\frac{1}{a} = \frac{2b+1}{b}$   
 (links en rechts het omgekeerde nemen)  
 $\frac{a}{1} = a = \frac{b}{2b+1}$ .

25b  $\frac{1}{a} = 2 + \frac{1}{b}$   
 $\frac{1}{a} - 2 = \frac{1}{b}$   
 $\frac{1-2a}{a} = \frac{1}{b}$   
 $\frac{1-2a}{a} = \frac{1}{b}$   
 $\frac{a}{1-2a} = \frac{b}{1} = b$ .

26a  $\frac{1}{p} = 5 - \frac{2}{q}$   
 $\frac{1}{p} = \frac{5q-2}{q}$   
 $\frac{1}{p} = \frac{5q-2}{q}$   
 $p = \frac{q}{5q-2}$ .

$\frac{1}{p} = 5 - \frac{2}{q}$   
 $\frac{2}{q} = \frac{5p-1}{p}$  (links en rechts :2)  
 $\frac{2}{q} = \frac{5p-1}{p}$   
 $\frac{1}{q} = \frac{5p-1}{2p}$   
 $q = \frac{2p}{5p-1}$ .

26b  $\frac{1}{m} = \frac{1}{2} - \frac{3}{n}$   
 $\frac{1}{m} = \frac{n-6}{2n}$   
 $m = \frac{2n}{n-6}$ .

$\frac{1}{m} = \frac{1}{2} - \frac{3}{n}$   
 $\frac{m}{3} = \frac{1}{m} - \frac{1}{n}$   
 $\frac{3}{n} = \frac{m-2}{2m}$  (:3)  
 $\frac{3}{n} = \frac{m-2}{2m}$   
 $\frac{1}{n} = \frac{m-2}{6m}$   
 $n = \frac{6m}{m-2}$ .

27a  $\frac{t-2}{t-3} \cdot p = \frac{t}{t-1}$   
 $p = \frac{t}{t-1} \cdot \frac{t-3}{t-2}$   
 $p = \frac{t(t-3)}{(t-1)(t-2)}$ .

27b  $\frac{3x}{x+y} = 5 - y$   
 $3x = (5-y) \cdot (x+y)$   
 $3x = 5x + 5y - xy - y^2$   
 $xy - 2x = 5y - y^2$   
 $x(y-2) = 5y - y^2$   
 $x = \frac{-y^2+5y}{y-2}$ .

27c  $\frac{1500-N}{N} = 50 \cdot 0,95^t$   
 $1500 - N = 50 \cdot 0,95^t \cdot N$   
 $1500 = 50 \cdot 0,95^t \cdot N + N$   
 $1500 = N(50 \cdot 0,95^t + 1)$   
 $N = \frac{1500}{50 \cdot 0,95^t + 1}$ .

27d  $K = 90 - \frac{2N}{N+0,2}$   
 $\frac{2N}{N+0,2} = 90 - K$   
 $2N = (90 - K)(N + 0,2)$   
 $2N = 90N + 18 - KN - 0,2K$   
 $2N + KN - 90N = 18 - 0,2K$   
 $N(K - 88) = 18 - 0,2K$   
 $N = \frac{18 - 0,2K}{K - 88}$ .

28a  $F = \frac{1}{K} + \frac{1}{2K} = \frac{2}{2K} + \frac{1}{2K} = \frac{3}{2K} \Rightarrow \frac{F}{3} = \frac{1}{2K} \Rightarrow 2K = \frac{3}{F} \Rightarrow K = \frac{3}{2F}$ .

28b  $\frac{1}{T} = 10 - \frac{2}{S} = \frac{10S-2}{S} \Rightarrow T = \frac{S}{10S-2}$ .

28c  $\frac{1}{N} + 3 = \frac{2R+2}{5R+2} \Rightarrow \frac{1}{N} = \frac{2R+2}{5R+2} - 3 = \frac{2R+2-3(5R+2)}{5R+2} = \frac{2R+2-15R-6}{5R+2} = \frac{-13R-4}{5R+2} \Rightarrow N = \frac{5R+2}{-13R-4}$ .

$$29a \quad \left. \begin{array}{l} K = \frac{500}{A} + 40A + \frac{60}{B} + 25B \\ AB = 30 \Rightarrow B = \frac{30}{A} \end{array} \right\} \Rightarrow K = \frac{500}{A} + 40A + 60 \cdot \frac{A}{30} + 25 \cdot \frac{30}{A} = \frac{500}{A} + 40A + 2A + \frac{750}{A} = \frac{1250}{A} + 42A.$$

$$29b \quad \left. \begin{array}{l} F = \frac{80}{A-1} + 10A + \frac{40}{AB} + 5A^3B \\ A^2B = 20 \Rightarrow B = \frac{20}{A^2} \end{array} \right\} \Rightarrow F = \frac{80}{A-1} + 10A + \frac{40}{A} \cdot \frac{A^2}{20} + 5A^3 \cdot \frac{20}{A^2} = \frac{80}{A-1} + 10A + 2A + 100A = \frac{80}{A-1} + 112A.$$

$$29c \quad \left. \begin{array}{l} N = 2PQ + P\left(PQ - \frac{2}{P}\right) \\ P^2Q = 10 \Rightarrow PQ = \frac{10}{P} \end{array} \right\} \Rightarrow N = 2 \cdot \frac{10}{P} + P\left(\frac{10}{P} - \frac{2}{P}\right) = \frac{20}{P} + P \cdot \frac{8}{P} = \frac{20}{P} + 8 \Rightarrow \frac{20}{P} = N - 8 \Rightarrow \frac{P}{20} = \frac{1}{N-8} \Rightarrow P = \frac{20}{N-8}.$$

$$30a \quad y = \frac{1}{\sqrt{x}} + \sqrt{x} = \frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{1} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}} = \frac{1+x}{\sqrt{x}} = \frac{x+1}{\sqrt{x}}.$$

$$30b \quad K = 2\sqrt{p+2} + \sqrt{\frac{1}{9}p + \frac{2}{9}} = 2\sqrt{p+2} + \sqrt{\frac{1}{9}(p+2)} = 2\sqrt{p+2} + \frac{1}{3}\sqrt{p+2} = 2\frac{1}{3}\sqrt{p+2}.$$

$$31a \quad y = \sqrt{25x} - \sqrt{x} = 5\sqrt{x} - \sqrt{x} = 4\sqrt{x}.$$

$$31b \quad y = \sqrt{54x} - \sqrt{24x} = \sqrt{9 \cdot 6x} - \sqrt{4 \cdot 6x} = 3\sqrt{6x} - 2\sqrt{6x} = \sqrt{6x}.$$

$$31c \quad N = \sqrt{8a} + \sqrt{\frac{1}{2}a} = \sqrt{4 \cdot 2a} + \sqrt{\frac{1}{2}a \cdot \frac{2}{2}} = 2\sqrt{2a} + \sqrt{\frac{1}{4} \cdot 2a} = 2\sqrt{2a} + \frac{1}{2}\sqrt{2a} = 2\frac{1}{2}\sqrt{2a}.$$

$$31d \quad N = \sqrt{20a} + \sqrt{\frac{9a}{5}} = \sqrt{4 \cdot 5a} + \sqrt{\frac{9a}{5} \cdot \frac{5}{5}} = 2\sqrt{5a} + \sqrt{\frac{9}{5^2} \cdot 5a} = 2\sqrt{5a} + \frac{3}{5}\sqrt{5a} = 2\frac{3}{5}\sqrt{5a}.$$

$$32a \quad y = (x+2)\sqrt{x^2+4} - 2\sqrt{x^2+4} = (x+2-2)\sqrt{x^2+4} = x\sqrt{x^2+4}.$$

$$32b \quad y = \frac{3}{\sqrt{x}} + 2\sqrt{x} = \frac{3}{\sqrt{x}} + \frac{2\sqrt{x}}{1} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3}{\sqrt{x}} + \frac{2x}{\sqrt{x}} = \frac{2x+3}{\sqrt{x}}.$$

$$32c \quad N = \frac{2-t^2}{t} \cdot \sqrt{t} + t \cdot \sqrt{t} = \left(\frac{2-t^2}{t} + t\right) \cdot \sqrt{t} = \left(\frac{2-t^2+t^2}{t}\right) \cdot \sqrt{t} = \frac{2}{t} \cdot \sqrt{t}.$$

$$32d \quad N = \frac{t+1}{\sqrt{2t+1}} - \sqrt{2t+1} = \frac{t+1}{\sqrt{2t+1}} - \frac{2t+1}{\sqrt{2t+1}} = \frac{-t}{\sqrt{2t+1}}.$$

$$33a \quad A = \sqrt{12b} - 6\sqrt{\frac{3}{16}b} = \sqrt{4 \cdot 3b} - 6\sqrt{\frac{1}{16} \cdot 3b} = 2\sqrt{3b} - 6 \cdot \frac{1}{4}\sqrt{3b} = \frac{1}{2}\sqrt{3b}.$$

$$33b \quad B = \frac{5a}{2\sqrt{a}} - \sqrt{2\frac{1}{4}a} = \frac{5a\sqrt{a}}{2a} - \sqrt{\frac{9}{4}a} = \frac{5}{2}\sqrt{a} - \frac{3}{2}\sqrt{a} = \sqrt{a}.$$

$$33c \quad y = \frac{x^2+4}{\sqrt{x-2}} - x\sqrt{x-2} = \frac{x^2+4}{\sqrt{x-2}} - \frac{x(x-2)}{\sqrt{x-2}} = \frac{x^2+4-x^2+2x}{\sqrt{x-2}} = \frac{2x+4}{\sqrt{x-2}}.$$

$$33d \quad N = \frac{3t^2}{(t-1)\sqrt{t-1}} - 3\sqrt{t-1} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3(t-1)^2}{(t-1)\sqrt{t-1}} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3(t^2-2t+1)}{(t-1)\sqrt{t-1}} = \frac{3t^2}{(t-1)\sqrt{t-1}} - \frac{3t^2-6t+3}{(t-1)\sqrt{t-1}} = \frac{6t-3}{(t-1)\sqrt{t-1}}.$$

$$34a \quad f(x) = x^2 \cdot \sqrt{2x+5} \Rightarrow f'(x) = 2x \cdot \sqrt{2x+5} + x^2 \cdot \frac{1}{2 \cdot \sqrt{2x+5}} \cdot 2 = \frac{2x(2x+5)}{\sqrt{2x+5}} + \frac{x^2}{\sqrt{2x+5}} = \frac{5x^2+10x}{\sqrt{2x+5}} \Rightarrow a = 5 \text{ en } b = 10.$$

$$34b \quad f'(x) = 0 \Rightarrow \frac{5x^2+10x}{\sqrt{2x+5}} = 0 \text{ (teller = 0)} \Rightarrow 5x(x+2) = 0 \Rightarrow x = 0 \vee x = -2.$$

$$x_A = -2 \Rightarrow f(-2) = (-2)^2 \cdot \sqrt{2 \cdot (-2) + 5} = 4 \cdot \sqrt{1} = 4. \quad \text{Dus } A(-2, 4).$$

$$34c \quad F(x) = (px^2 + qx + r)(2x+5)^{\frac{1}{2}} \Rightarrow F'(x) = (2px+q) \cdot (2x+5)^{\frac{1}{2}} + (px^2 + qx + r) \cdot \frac{1}{2}(2x+5)^{-\frac{1}{2}} \cdot 2 \\ = (2px+q) \cdot (2x+5) \cdot \sqrt{2x+5} + 3(px^2 + qx + r) \cdot \sqrt{2x+5} \\ = (4px^2 + 10px + 2qx + 5q + 3px^2 + 3qx + 3r) \cdot \sqrt{2x+5} \\ = (7px^2 + (10p+5q)x + 5q + 3r) \cdot \sqrt{2x+5}.$$

$$F'(x) = f(x) \Rightarrow 7p = 1 \wedge 10p + 5q = 0 \wedge 5q + 3r = 0$$

$$p = \frac{1}{7} \wedge \frac{10}{7} + 5q = 0 \wedge 5q + 3r = 0$$

$$p = \frac{1}{7} \wedge 5q = -\frac{10}{7} \wedge -\frac{10}{7} + 3r = 0$$

$$p = \frac{1}{7} \wedge q = -\frac{2}{7} \wedge 3r = \frac{10}{7}$$

$$p = \frac{1}{7} \wedge q = -\frac{2}{7} \wedge r = \frac{10}{21}.$$

$$34d \quad O(V) = \int_{-2\frac{1}{2}}^0 f(x) dx = \left[ \left(\frac{1}{7}x^2 - \frac{2}{7}x + \frac{10}{21}\right)(2x+5)\sqrt{2x+5} \right]_{-2\frac{1}{2}}^0 = \frac{10}{21} \cdot 5 \cdot \sqrt{5} - \left(\frac{1}{7} \cdot (-2\frac{1}{2})^2 - \frac{2}{7} \cdot (-2\frac{1}{2}) + \frac{10}{21}\right) \cdot 0 = \frac{50}{21} \sqrt{5}.$$

35a  $y = 2\sqrt{x} \ (x \geq 0) \Rightarrow 2\sqrt{x} = y \ (\text{kwadrateren}) \Rightarrow 4x = y^2 \ (y \geq 0) \Rightarrow x = \frac{1}{4}y^2 \ (y \geq 0).$

35b  $y = \sqrt{x-2} \ (x-2 \geq 0 \Rightarrow x \geq 2) \Rightarrow \sqrt{x-2} = y \ (\text{kwadrateren}) \Rightarrow x-2 = y^2 \ (y \geq 0) \Rightarrow x = y^2 + 2 \ (y \geq 0).$

35c  $y = 2\sqrt{x-2} \ (x \geq 2) \Rightarrow 2\sqrt{x-2} = y \ (\text{kwadrateren}) \Rightarrow 4(x-2) = y^2 \ (y \geq 0) \Rightarrow x-2 = \frac{1}{4}y^2 \Rightarrow x = \frac{1}{4}y^2 + 2 \ (y \geq 0).$

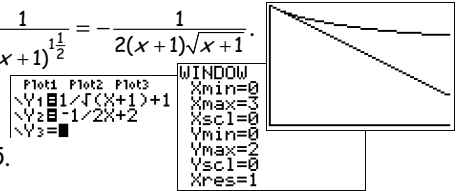
36a  $\square \ R = 2\sqrt{4A-1}$   
 $2\sqrt{4A-1} = R \ (\text{kwadrateren})$   
 $4(4A-1) = R^2$   
 $4A-1 = \frac{1}{4}R^2 \Rightarrow 4A = \frac{1}{4}R^2 + 1 \Rightarrow A = \frac{1}{16}R^2 + \frac{1}{4}.$

36c  $\square \ x\sqrt{y} - 2x = 4$   
 $x\sqrt{y} = 2x + 4$   
 $\sqrt{y} = \frac{2x+4}{x} \ (\text{kwadrateren})$   
 $y = \frac{(2x+4)^2}{x^2}.$

36b  $\square \ A = 6 - \frac{2}{\sqrt{B}}$   
 $\frac{2}{\sqrt{B}} = 6 - A$   
 $\frac{2}{6-A} = \sqrt{B} \ (\text{kwadrateren})$   
 $B = \frac{4}{(6-A)^2}.$

36d  $\square \ \frac{p\sqrt{q}}{q} - \sqrt{p} = 4$   
 $\frac{p\sqrt{q}}{q} = 4 + \sqrt{p}$   
 $\frac{1}{\sqrt{q}} = \frac{4 + \sqrt{p}}{p} \Rightarrow \sqrt{q} = \frac{p}{4 + \sqrt{p}} \ (\text{kwadrateren})$   
 $q = \frac{p^2}{(4 + \sqrt{p})^2}.$

37a  $f(x) = \frac{1}{\sqrt{x+1}} + 1 = \frac{1}{(x+1)^{\frac{1}{2}}} + 1 = (x+1)^{-\frac{1}{2}} + 1 \Rightarrow f'(x) = -\frac{1}{2}(x+1)^{-\frac{3}{2}} \cdot 1 = -\frac{1}{2(x+1)^{\frac{3}{2}}} = -\frac{1}{2(x+1)\sqrt{x+1}}.$   
 $f(0) = \frac{1}{\sqrt{1}} + 1 = 1 + 1 = 2$  en  $f'(0) = -\frac{1}{2 \cdot 1 \cdot \sqrt{1}} = -\frac{1}{2} \Rightarrow k: y = -\frac{1}{2}x + 2.$   
 $O(V) = \int_0^3 \left( \frac{1}{\sqrt{x+1}} + 1 \right) dx = [2\sqrt{x+1} + x]_0^3 = 2\sqrt{4} + 3 - (2\sqrt{1} + 0) = 4 + 3 - 2 = 5.$



De oppervlakte van het deel van  $V$  onder de lijn  $k$  is  $\int_0^3 \left(-\frac{1}{2}x + 2\right) dx = \left[-\frac{1}{4}x^2 + 2x\right]_0^3 = -\frac{9}{4} + 6 - (0 + 0) = \frac{15}{4}.$

De verhouding van de oppervlakten van beide vlakdelen is  $\frac{15}{4} : \frac{5}{4} = 15 : 5 = 3 : 1.$

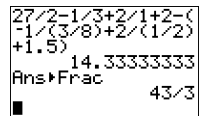
37b  $x$  vrijmaken in  $y = \frac{1}{\sqrt{x+1}} + 1 \Rightarrow y - 1 = \frac{1}{\sqrt{x+1}} \Rightarrow \sqrt{x+1} = \frac{1}{y-1} \Rightarrow x+1 = \frac{1}{(y-1)^2} \Rightarrow x = \frac{1}{(y-1)^2} - 1.$   
 $f(0) = \frac{1}{\sqrt{1}} + 1 = 1 + 1 = 2$  en  $f(3) = \frac{1}{\sqrt{4}} + 1 = \frac{1}{2} + 1 = 1\frac{1}{2}.$

$$I(V) = I(\text{cilinder}) + \int_{1\frac{1}{2}}^2 \pi x^2 dy = \pi \cdot 3^2 \cdot \frac{3}{2} + \int_{1\frac{1}{2}}^2 \pi \left( \frac{1}{(y-1)^2} - 1 \right)^2 dy = \frac{27}{2}\pi + \int_{1\frac{1}{2}}^2 \pi \left( \frac{1}{(y-1)^4} - \frac{2}{(y-1)^2} + 1 \right) dy$$

$$= 13\frac{1}{2}\pi + \int_{1\frac{1}{2}}^2 \pi \left( (y-1)^{-4} - 2(y-1)^{-2} + 1 \right) dy = 13\frac{1}{2}\pi + \left[ \pi \left( -\frac{1}{3}(y-1)^{-3} + 2(y-1)^{-1} + y \right) \right]_{1\frac{1}{2}}^2$$

$$= 13\frac{1}{2}\pi + \left[ \pi \left( -\frac{1}{3(y-1)^3} + \frac{2}{y-1} + y \right) \right]_{1\frac{1}{2}}^2 = 13\frac{1}{2}\pi + \pi \left( -\frac{1}{3} + \frac{2}{1} + 2 \right) - \pi \left( -\frac{1}{\frac{3}{8}} + \frac{2}{\frac{1}{2}} + 1\frac{1}{2} \right)$$

$$= 13\frac{1}{2}\pi + 3\frac{2}{3}\pi - \pi \left( -\frac{8}{3} + 4 + 1\frac{1}{2} \right) = 13\frac{1}{2}\pi + 3\frac{2}{3}\pi - 2\frac{5}{6}\pi = 14\frac{1}{3}\pi.$$



38a  $\frac{1}{x^3} = x^{-3}.$

38c  $x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}}.$

38e  $\frac{x^6}{x^2} = x^4.$

38b  $\sqrt[4]{x^3} = x^{\frac{3}{4}}.$

38d  $x^3 \cdot x^2 = x^5.$

38f  $\frac{x}{\sqrt{x}} = x : \sqrt{x} = x^1 : x^{\frac{1}{2}} = x^{\frac{1}{2}}.$

39a  $x^5 = 18$   
 $x = \sqrt[5]{18}.$

39b  $\sqrt[3]{x} = 4$   
 $x = 4^3$   
 $x = 64.$

40a  $\square \ y = \frac{6}{x} \cdot x^{1.4} = 6 \cdot \frac{x^{1.4}}{x} = 6x^{0.4}.$

40c  $\square \ y = (2x^{0.4})^3 \cdot 5 \cdot x^{0.5} = 8x^{1.2} \cdot 5x^{0.5} = 40x^{1.7}.$

40b  $\square \ y = \frac{15}{x^2 \cdot \sqrt{x}} \cdot \sqrt[4]{x} = 15 \cdot \frac{x^{\frac{1}{4}}}{x^{2\frac{1}{2}}} = 15x^{-2\frac{1}{4}}.$

40d  $\square \ y = (4\sqrt{x})^3 \cdot \left(\frac{1}{2}x\right)^4 = \left(4x^{\frac{1}{2}}\right)^3 \cdot \frac{1}{16}x^4 = 64x^{1\frac{1}{2}} \cdot \frac{1}{16}x^4 = 4x^{5\frac{1}{2}}.$

$$41a \quad y = \frac{1}{3} \cdot \sqrt[3]{x} - 7$$

$$y + 7 = \frac{1}{3} \cdot \sqrt[3]{x}$$

$$3y + 21 = \sqrt[3]{x}$$

$$x = (3y + 21)^3$$

$$41c \quad T = \frac{1}{16} \cdot S^4$$

$$16T = S^4$$

$$S = (16T)^{\frac{1}{4}}$$

$$S = 16^{\frac{1}{4}} \cdot T^{\frac{1}{4}}$$

$$S = (2^4)^{\frac{1}{4}} \cdot T^{\frac{1}{4}}$$

$$S = 2T^{\frac{1}{4}}$$

$$41d \quad A = 15 \cdot (4B)^{-1,6}$$

$$\frac{1}{15} A = (4B)^{-1,6}$$

$$4B = \left(\frac{1}{15} A\right)^{-\frac{1}{1,6}}$$

$$B = \frac{1}{4} \left(\frac{1}{15} A\right)^{-\frac{1}{1,6}}$$

$$B = \frac{1}{4} \cdot \left(\frac{1}{15}\right)^{-\frac{1}{1,6}} \cdot A^{-\frac{1}{1,6}}$$

$$B \approx 1,358 \cdot A^{-0,625}$$

$$41b \quad P = \frac{1}{2} \cdot \sqrt[4]{2q-1} + 3$$

$$P - 3 = \frac{1}{2} \cdot \sqrt[4]{2q-1}$$

$$2P - 6 = \sqrt[4]{2q-1}$$

$$2q - 1 = (2P - 6)^4$$

$$2q = (2P - 6)^4 + 1$$

$$q = \frac{1}{2} (2P - 6)^4 + \frac{1}{2}$$

$$42a \quad P = 20 \cdot (3x^{0,6})^2 \cdot (2y^3)^{0,8} = 20 \cdot 3^2 \cdot x^{1,2} \cdot 2^{0,8} \cdot y^{2,4} \approx 313,40 x^{1,2} y^{2,4}$$

$$42b \quad s \text{ elimineren (uitstoten)} \Rightarrow L = 0,6 \cdot (5t)^{0,35} \cdot \left(6 \cdot \frac{1}{3} t^3\right)^{0,18} = 0,6 \cdot 5^{0,35} \cdot t^{0,35} \cdot 2^{0,18} \cdot t^{0,54} \approx 1,19 t^{0,89}$$

$$42c \quad t \text{ elimineren} \Rightarrow N = 0,12 \cdot \left(3 \cdot \frac{1}{2} k^{-0,4}\right)^{1,6} \cdot (5k)^{2,3} = 0,12 \cdot 1,5^{1,6} \cdot k^{-0,64} \cdot 5^{2,3} \cdot k^{2,3} \approx 9,30 k^{1,66}$$

$$42d \quad a \text{ elimineren} \Rightarrow y = \frac{1536}{(4 \cdot \sqrt{x^2 + 20})^5} = 1536 \cdot \left(4 \cdot (x^2 + 20)^{0,5}\right)^{-5} = 1536 \cdot 4^{-5} \cdot (x^2 + 20)^{-2,5} = 1,5 \cdot (x^2 + 20)^{-2,5}$$

$$43a \quad V = 6 \cdot (5R)^{1,8}$$

$$\frac{1}{6} V = (5R)^{1,8}$$

$$\left(\frac{1}{6} V\right)^{\frac{1}{1,8}} = 5R$$

$$R = \frac{1}{5} \cdot \left(\frac{1}{6} V\right)^{\frac{1}{1,8}}$$

$$R = \frac{1}{5} \cdot \left(\frac{1}{6}\right)^{\frac{1}{1,8}} \cdot V^{\frac{1}{1,8}}$$

$$R \approx 0,07 \cdot V^{0,56}$$

$$43c \quad O = 16a^2$$

$$\frac{1}{16} O = a^2$$

$$a = \left(\frac{1}{16} O\right)^{\frac{1}{2}} = \frac{1}{4} O^{\frac{1}{2}}$$

$$K = 8 \cdot \left(\frac{1}{4} O^{\frac{1}{2}}\right)^3$$

$$K = 8 \cdot \left(\frac{1}{4}\right)^3 \cdot O^{\frac{3}{2}}$$

$$K = \frac{1}{8} O^{\frac{3}{2}}$$

$$K = 8a^3$$

$$\frac{1}{8} K = a^3$$

$$a = \left(\frac{1}{8} K\right)^{\frac{1}{3}} = \frac{1}{2} K^{\frac{1}{3}}$$

$$O = 16 \cdot \left(\frac{1}{2} K^{\frac{1}{3}}\right)^2$$

$$O = 16 \cdot \left(\frac{1}{2}\right)^2 \cdot K^{\frac{2}{3}}$$

$$O = 4 \cdot K^{\frac{2}{3}}$$

$$43b \quad S = \frac{2}{5} \cdot \sqrt[3]{4t} + 2$$

$$S - 2 = \frac{2}{5} \cdot \sqrt[3]{4t}$$

$$2,5 \cdot (S - 2) = \sqrt[3]{4t}$$

$$(2,5 \cdot (S - 2))^3 = 4t$$

$$t = \frac{1}{4} \cdot (2,5S - 5)^3$$

$$43d \quad L = 4 \cdot (r - 3)^3$$

$$\frac{1}{4} L = (r - 3)^3$$

$$r - 3 = \left(\frac{1}{4} L\right)^{\frac{1}{3}}$$

$$F = 2 \cdot \left(\left(\frac{1}{4} L\right)^{\frac{1}{3}}\right)^4$$

$$F = 2 \cdot \left(\frac{1}{4} L\right)^{\frac{4}{3}}$$

$$F = 2 \cdot (r - 3)^4$$

$$\frac{1}{2} F = (r - 3)^4$$

$$r - 3 = \left(\frac{1}{2} F\right)^{\frac{1}{4}}$$

$$L = 4 \cdot \left(\left(\frac{1}{2} F\right)^{\frac{1}{4}}\right)^3$$

$$L = 4 \cdot \left(\frac{1}{2} F\right)^{\frac{3}{4}}$$

$$44 \quad y = 2^x \cdot 2^{3x+1} = 2^x \cdot 2^{3x} \cdot 2^1 = 2^{4x} \cdot 2 = 2 \cdot (2^4)^x = 2 \cdot 16^x$$

$$45a \quad 10^{\log(2)} = 10^{10 \log(2)} = 2 \quad (10^{\dots} \text{ en } 10 \log \dots \text{ heffen elkaar op}) \quad e^{\ln(7)} = e^{e \log(7)} = 7 \quad (e^{\dots} \text{ en } \ln \dots \text{ heffen elkaars op})$$

$$45b \quad 3 = 10^{\log(3)} \quad (10^{\dots} \text{ en } 10 \log \dots \text{ zijn elkaars inverse}) \quad 5 = e^{\ln(5)} \quad (e^{\dots} \text{ en } \ln \dots \text{ zijn elkaars inverse})$$

$$46a \quad N = 25 \cdot 1,3^{4t-2} = 25 \cdot 1,3^{4t} \cdot 1,3^{-2} = 25 \cdot 1,3^{-2} \cdot (1,3^4)^t \approx 14,79 \cdot 2,86^t$$

$$46b \quad N = 180 \cdot 0,8^{5-t} = 180 \cdot 0,8^5 \cdot 0,8^{-t} = 180 \cdot 0,8^5 \cdot (0,8^{-1})^t \approx 58,98 \cdot 1,25^t$$

$$47a \quad N = 3^t \cdot 3^{t+3} = 3^{2t+3} = 3^{2t} \cdot 3^3 = 3^3 \cdot (3^2)^t = 27 \cdot 9^t$$

$$47b \quad N = \left(\frac{1}{2}\right)^{t-3} \cdot \left(\frac{1}{2}\right)^{-3t+2} = \left(\frac{1}{2}\right)^{-2t-1} = (2^{-1})^{-2t-1} = 2^{2t+1} = 2^{2t} \cdot 2^1 = 2 \cdot (2^2)^t = 2 \cdot 4^t$$



47c  $N = 3^{2t-1} \cdot 9^{t+1} = 3^{2t-1} \cdot (3^2)^{t+1} = 3^{2t-1} \cdot 3^{2t+2} = 3^{4t+1} = 3^{4t} \cdot 3^1 = (3^4)^t \cdot 3 = 3 \cdot 81^t.$

47d  $N = \left(\frac{1}{4}\right)^{t+1} \cdot 2^{3t+4} = (2^{-2})^{t+1} \cdot 2^{3t+4} = 2^{-2t-2} \cdot 2^{3t+4} = 2^{t+2} = 2^t \cdot 2^2 = 4 \cdot 2^t.$

48a  $y = 15 \cdot 5^{x-1} = 15 \cdot 5^x \cdot 5^{-1} = 15 \cdot \frac{1}{5} \cdot (10^{\log(5)})^x = 3 \cdot 10^x \cdot \log(5) \approx 3 \cdot 10^{0,70x} \cdot \log(5)$

48b  $y = 15 \cdot 5^{x-1} = 15 \cdot 5^x \cdot 5^{-1} = 15 \cdot \frac{1}{5} \cdot (e^{\ln(5)})^x = 3 \cdot e^{x \cdot \ln(5)} \approx 3 \cdot e^{1,61x}$

48c  $T = 37,2 \cdot 1,7^{3t-2} = 37,2 \cdot 1,7^{3t} \cdot 1,7^{-2} = 37,2 \cdot 1,7^{-2} \cdot (10^{\log(1,7)})^{3t} = 37,2 \cdot 1,7^{-2} \cdot 10^{3t \cdot \log(1,7)} \approx 12,87 \cdot 10^{0,69t}.$

48d  $T = 37,2 \cdot 1,7^{3t-2} = 10^{\log(37,2)} \cdot (10^{\log(1,7)})^{3t-2} = 10^{\log(37,2)} \cdot 10^{(3t-2)\log(1,7)} = 10^{\log(37,2)+3t\log(1,7)-2\log(1,7)} \approx 10^{0,69t+1,11}.$

49a  $N = 18 - 5(6 - 1,5^{4t}) = 18 - 30 + 5 \cdot 1,5^{4t} = -12 + 5 \cdot (1,5^4)^t \approx -12 + 5 \cdot 5,06^t.$

49b  $N = \frac{8^{2t+1}}{4^{t-1}} = \frac{(2^3)^{2t+1}}{(2^2)^{t-1}} = \frac{2^{6t+3}}{2^{2t-2}} = 2^{6t+3-2t+2} = 2^{4t+5} = 2^{4t} \cdot 2^5 = 32 \cdot (2^4)^t = 32 \cdot 16^t.$

49c  $K = 150 \cdot 1,12^{6q+3} = e^{\ln(150)} \cdot (e^{\ln(1,12)})^{6q+3} = e^{\ln(150)} \cdot e^{(6q+3)\ln(1,12)} = e^{\ln(150)+6q\ln(1,12)+3\ln(1,12)} \approx e^{0,68q+5,35}$

50a  $T = 27 \cdot 0,4^t \cdot (3 - 0,4^{2t}) = 27 \cdot 0,4^t \cdot 3 - 27 \cdot 0,4^{3t} = 81 \cdot 0,4^t - 27 \cdot (0,4^3)^t = 81 \cdot 0,4^t - 27 \cdot 0,064^t.$

50b  $\rho = 3^{2t+1} \cdot 2^{3t+1} = 3^{2t} \cdot 3^1 \cdot 2^{3t} \cdot 2^1 = 6 \cdot (e^{\ln(3)})^{2t} \cdot (e^{\ln(2)})^{3t} = 6 \cdot e^{2t\ln(3)} \cdot e^{3t\ln(2)} = 6 \cdot e^t \cdot (2\ln(3)+3\ln(2)) \approx 6e^{4,28t}.$

51  $y = e^{x-1} \Rightarrow \ln(y) = \ln(e^{x-1})$  ( $\ln \dots$  en  $e^{\dots}$  zijn elkaars inverse)  $= x - 1 \Rightarrow \ln(y) + 1 = x \Rightarrow x = 1 + \ln(y).$

52a  $y = 20 \cdot 3^{x-4}$

$\frac{1}{20}y = 3^{x-4}$

${}^3\log(0,05y) = x - 4$

$x = 4 + {}^3\log(0,05y)$

$x = 4 + {}^3\log(0,05) + {}^3\log(y)$

$x = 4 + \frac{\log(0,05)}{\log(3)} + \frac{\log(y)}{\log(3)}$

$x \approx 1,27 + 2,10\log(y).$

$$\begin{matrix} 1/20 \\ 4+1\log(0,05)/\log(3) \\ 1/\log(3) \\ 1,273166972 \\ 2,095903274 \end{matrix}$$

52b  $y = 0,65 \cdot 1,16^{x-1}$

$\frac{1}{0,65}y = 1,16^{x-1}$

${}^{1,16}\log\left(\frac{1}{0,65}y\right) = x - 1$

$x = 1 + {}^{1,16}\log\left(\frac{1}{0,65}y\right)$

$x = 1 + {}^{1,16}\log\left(\frac{1}{0,65}\right) + {}^{1,16}\log(y)$

$x = 1 + \frac{\ln\left(\frac{1}{0,65}\right)}{\ln(1,16)} + \frac{\ln(y)}{\ln(1,16)}$

$x \approx 3,90 + 6,74\ln(y).$

$$\begin{matrix} 1+\ln(1/0,65)/\ln(1,16) \\ 1/\ln(1,16) \\ 3,902458572 \\ 6,737636205 \end{matrix}$$

52c  $N = 250 \cdot 10^{2t-3}$

$\frac{1}{250}N = 10^{2t-3}$

$\log\left(\frac{1}{250}N\right) = 2t - 3$

$2t = 3 + \log\left(\frac{1}{250}N\right)$

$t = 1\frac{1}{2} + \frac{1}{2}\log\left(\frac{1}{250}N\right).$

52d  $\rho = 120 \cdot e^{3-q}$

$\frac{1}{120}\rho = e^{3-q}$

$\ln\left(\frac{1}{120}\rho\right) = 3 - q$

$q = 3 - \ln\left(\frac{1}{120}\rho\right).$

53a  $f(x) = g(x)$

$e^{2x-1} = e^{2-3x}$

$2x - 1 = 2 - 3x$

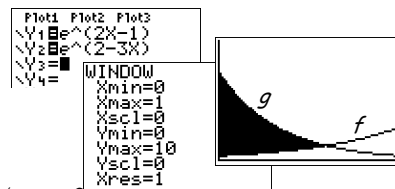
$5x = 3$

$x = \frac{3}{5}.$

$$0 = \int_0^{\frac{3}{5}} (g(x) - f(x)) dx = \int_0^{\frac{3}{5}} (e^{2-3x} - e^{2x-1}) dx$$

$$= \left[ -\frac{1}{3} \cdot e^{2-3x} - \frac{1}{2} \cdot e^{2x-1} \right]_0^{\frac{3}{5}} = -\frac{1}{3} \cdot e^{\frac{1}{5}} - \frac{1}{2} \cdot e^{\frac{1}{5}} - \left( -\frac{1}{3} \cdot e^2 - \frac{1}{2} \cdot e^{-1} \right)$$

$$= -\frac{5}{6} \cdot e^{\frac{1}{5}} + \frac{1}{3} \cdot e^2 + \frac{1}{2} \cdot e^{-1} = \frac{1}{3}e^2 - \frac{5}{6}\sqrt[5]{e} + \frac{1}{2e}.$$



53b  $h(x) = f(x) \cdot g(x) = e^{2x-1} \cdot e^{2-3x} = e^{1-x}.$

$y = e^{1-x} \Rightarrow \ln(y) = 1 - x \Rightarrow x = 1 - \ln(y).$

53c  $y\ln^2(y) - 4y\ln(y) + 5y$  heeft als afgeleide  $1 \cdot \ln^2(y) + y \cdot 2\ln(y) \cdot \frac{1}{y} - 4 \cdot \ln(y) - 4y \cdot \frac{1}{y} + 5$

$$= \ln^2(y) + 2\ln(y) - 4\ln(y) - 4 + 5 = 1 - 2\ln(y) + \ln^2(y) = (1 - \ln(y))^2.$$

Dus  $y\ln^2(y) - 4y\ln(y) + 5y$  is een primitieve van  $(1 - \ln(y))^2$ .

53d  $h(0) = e^{1-0} = e^1 = e.$

$$I(L) = \int_{y=1}^{y=e} (\pi \cdot x^2) dy = \int_1^e (\pi \cdot (1 - \ln(y))^2) dy = \left[ \pi \cdot (y \ln^2(y) - 4y \ln(y) + 5y) \right]_1^e$$

$$= \pi \cdot (e \ln^2(e) - 4e \ln(e) + 5e) - (\pi \cdot (1 \ln^2(1) - 4 \ln(1) + 5)) = \pi \cdot (e \cdot 1^2 - 4e \cdot 1 + 5e) - (\pi \cdot (0 - 0 + 5)) = 2\pi e - 5\pi.$$



54  $y = \frac{1}{2} \cdot 3 \log(x) - 2 \Rightarrow 2y = 3 \log(x) - 4 \Rightarrow 2y + 4 = 3 \log(x) \Rightarrow x = 3^{2y+4} = 3^{2y} \cdot 3^4 = 3^4 \cdot (3^2)^y = 81 \cdot 9^y.$

55a  $\log(N) = 2,15 + 0,07t \Rightarrow N = 10^{2,15+0,07t} = 10^{2,15} \cdot 10^{0,07t} = 10^{2,15} \cdot (10^{0,07})^t \approx 141 \cdot 1,175^t.$

55b  $\log(P) = 2,85 + 0,75 \log(q) \Rightarrow P = 10^{2,85+0,75 \log(q)} = 10^{2,85} \cdot 10^{0,75 \log(q)} = 10^{2,85} \cdot (10^{\log(q)})^{0,75} \approx 708 \cdot q^{0,75}.$

55c  $y = \frac{1}{4} \cdot \ln(5x+2) + 3 \Rightarrow 4y = \ln(5x+2) + 12 \Rightarrow 4y - 12 = \ln(5x+2) \Rightarrow 5x+2 = e^{4y-12} \Rightarrow 5x = e^{4y-12} - 2 \Rightarrow x = \frac{1}{5} e^{4y-12} - \frac{2}{5}.$

55d  $\ln(2A+3) = 5 + \ln(B) \Rightarrow \ln(2A+3) - \ln(B) = 5 \Rightarrow \ln\left(\frac{2A+3}{B}\right) = 5 \Rightarrow \frac{2A+3}{B} = e^5 \Rightarrow 2A+3 = e^5 B \Rightarrow 2A = e^5 B - 3 \Rightarrow A = \frac{1}{2} e^5 B - 1\frac{1}{2}.$

56a  $F = 2 \log(N) - 0,4 \Rightarrow F + 0,4 = 2 \log(N) \Rightarrow \log(N) = 0,5F + 0,2 \Rightarrow N = 10^{0,5F+0,2}.$

56b  $D = \log(4Q+1) + 1 \Rightarrow \log(4Q+1) = D - 1 \Rightarrow 4Q+1 = 10^{D-1} \Rightarrow 4Q = 10^{D-1} - 1 \Rightarrow Q = \frac{1}{4} \cdot 10^{D-1} - \frac{1}{4}.$

56c  $K \cdot \sqrt{2} = \ln(2R-3) - \sqrt{6} \Rightarrow \ln(2R-3) = K \cdot \sqrt{2} + \sqrt{6} \Rightarrow 2R-3 = e^{K\sqrt{2}+\sqrt{6}} \Rightarrow 2R = e^{K\sqrt{2}+\sqrt{6}} + 3 \Rightarrow$   
 $R = \frac{1}{2} e^{K\sqrt{2}+\sqrt{6}} + 1\frac{1}{2} = \frac{1}{2} \cdot (e^{\sqrt{2}})^K \cdot e^{\sqrt{6}} + 1\frac{1}{2} \approx 5,8 \cdot 4,1^K + 1\frac{1}{2}.$

56d  ${}^2 \log(3x-1) = -4 + {}^2 \log(2y+1) \Rightarrow 4 = {}^2 \log(2y+1) - {}^2 \log(3x-1) \Rightarrow 4 = {}^2 \log\left(\frac{2y+1}{3x-1}\right) \Rightarrow \frac{2y+1}{3x-1} = 2^4 = 16 \Rightarrow$   
 $2y+1 = 48x-16 \Rightarrow 2y = 48x-17 \Rightarrow y = 24x-8\frac{1}{2}.$

57  $\log(a+b) = \log(a) + \log(b) \Rightarrow \log(a+b) = \log(a \cdot b) \Rightarrow a+b = ab \Rightarrow a-ab = -b \Rightarrow a(1-b) = -b \Rightarrow a = -\frac{b}{1-b} = \frac{b}{b-1}.$

58a  $D = 50 \text{ (cm)} \Rightarrow \log(N) = 5,3 - 1,7 \log(50) \Rightarrow N = 10^{5,3-1,7 \log(50)} \approx 258 \text{ (bomen per ha).}$

58b  $N = \frac{2000}{8} = 250 \text{ (bomen/ha)} \Rightarrow \log(250) = 5,3 - 1,7 \log(D) \text{ (intersect of)} \Rightarrow$

$$1,7 \log(D) = 5,3 - \log(250) \Rightarrow \log(D) = \frac{5,3 - \log(250)}{1,7} \Rightarrow D = 10^{\text{Ans}} \approx 51 \text{ (cm).}$$

58c  $\log(N) = 5,3 - 1,7 \log(D) \Rightarrow 1,7 \log(D) = 5,3 - \log(N) \Rightarrow \log(D) = \frac{5,3 - \log(N)}{1,7} \Rightarrow$

$$D = 10^{\frac{5,3 - \frac{\log(N)}{1,7}}{1,7}} = 10^{\frac{5,3}{1,7} - \frac{1}{1,7} \log(N)} = 10^{\frac{5,3}{1,7}} \cdot (10^{\log(N)})^{-\frac{1}{1,7}} = 10^{\frac{5,3}{1,7}} \cdot N^{-\frac{1}{1,7}} \approx 1310 \cdot N^{-0,59}.$$

59a  $G = 185 \text{ (cm)} \Rightarrow s = 290 \log(185+100) - 550 \approx 162 \text{ (cm).}$

59b  $s = 210 \text{ (cm)} \Rightarrow 210 = 290 \log(G+100) - 550 \Rightarrow 760 = 290 \log(G+100) \Rightarrow$

$$\frac{760}{290} = \log(G+100) \Rightarrow G+100 = 10^{\frac{760}{290}} \Rightarrow G = 10^{\frac{760}{290}} - 100 \approx 318 \text{ (cm).}$$

59c  $s = 290 \log(G+100) - 550 \Rightarrow s + 550 = 290 \log(G+100) \Rightarrow \frac{s+550}{290} = \log(G+100) \Rightarrow$

$$G+100 = 10^{\frac{s+550}{290}} = 10^{\frac{1}{290} s} \cdot 10^{\frac{550}{290}} \Rightarrow G \approx 78,8 \cdot 10^{0,00345s} - 100.$$

59d  $s = 180 \text{ (cm)} \Rightarrow G \approx 78,8 \cdot 10^{0,00345 \cdot 180} - 100 \approx 229 \text{ en}$

$s = 220 \text{ (cm)} \Rightarrow G \approx 78,8 \cdot 10^{0,00345 \cdot 220} - 100 \approx 352.$

Dus de spanwijdten  $G$  liggen tussen 229 en 352 cm.

60a  $(-1, 6)$  invullen in  $y = x^2 + bx + c$  geeft  $6 = (-1)^2 + b \cdot (-1) + c \Rightarrow 6 = 1 - b + c \Rightarrow 5 = -b + c \Rightarrow -b + c = 5$  ①.

$(4, 11)$  invullen in  $y = x^2 + bx + c$  geeft  $11 = 4^2 + b \cdot 4 + c \Rightarrow 11 = 16 + 4b + c \Rightarrow -5 = 4b + c \Rightarrow 4b + c = -5$  ②.

60b 
$$\begin{cases} -b + c = 5 & \text{①} \\ 4b + c = -5 & \text{②} \end{cases}$$

$-5b = 10 \Rightarrow b = -\frac{10}{5} = -2$  in ①  $\Rightarrow 2 + c = 5 \Rightarrow c = 3.$



61a (1, 4  $\frac{1}{2}$ ) invullen geeft  $\frac{9}{2} = \frac{1+a}{1+b} \Rightarrow 2+2a=9+9b \Rightarrow 2a-9b=7$  ①.

(2, 2  $\frac{2}{5}$ ) invullen geeft  $\frac{12}{5} = \frac{4+a}{4+b} \Rightarrow 20+5a=48+12b \Rightarrow 5a-12b=28$  ②.

$$\begin{cases} 2a-9b=7 & \text{①} \\ 5a-12b=28 & \text{②} \end{cases} \Rightarrow \begin{cases} 10a-45b=35 & \text{③} \\ 10a-24b=56 & \text{④} \end{cases}$$

$-21b=-21 \Rightarrow b=1$  in ①  $\Rightarrow 2a-9=7 \Rightarrow 2a=16 \Rightarrow a=8$ .

61b (-5, 15) invullen geeft  $\frac{15}{1} = \frac{25+a}{\sqrt{-5+b}} \Rightarrow 25+a=15\sqrt{-5+b}$  ①.

(0, 1  $\frac{2}{3}$ ) invullen geeft  $\frac{5}{3} = \frac{a}{\sqrt{b}} \Rightarrow 3a=5\sqrt{b} \Rightarrow a=\frac{5}{3}\sqrt{b}$  ② (kwadrateren)  $\Rightarrow a^2 = \frac{25}{9}b \Rightarrow b = \frac{9}{25}a^2$  ③.

③ in ①  $\Rightarrow 25+a=15\sqrt{-5+\frac{9}{25}a^2}$  (nog eens kwadrateren)

$25^2 + 2 \cdot 25 \cdot a + a^2 = 15^2 \cdot (-5 + \frac{9}{25}a^2)$

$625 + 50a + a^2 = 225 \cdot (-5 + \frac{9}{25}a^2)$

$625 + 50a + a^2 = -1125 + 81a^2$

$-80a^2 + 50a + 1750 = 0$  (hiernaast verder)

$8a^2 - 5a - 175 = 0$

$D = (-5)^2 - 4 \cdot 8 \cdot -175 = 5625 \Rightarrow \sqrt{D} = 75$

$a = \frac{5+75}{2 \cdot 8} = \frac{80}{2 \cdot 8} = 5 \vee a = \frac{5-75}{2 \cdot 8} = \frac{-70}{2 \cdot 8} = -\frac{35}{8}$

voldoet niet aan ②

$a=5$  in ③  $\Rightarrow b = \frac{9}{25} \cdot 25 = 9$ .

$5^2 - 4 \cdot 8 \cdot -175$	5625
$\sqrt{\text{Ans}}$	75

62a  $\begin{cases} x^2 + 2y^2 = 18 & \text{①} \\ x^2 + y = 17 & \text{②} \end{cases}$

$2y^2 - y = 1 \Rightarrow 2y^2 - y - 1 = 0$  met  $D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9 \Rightarrow \sqrt{D} = 3 \Rightarrow y = \frac{1+3}{2 \cdot 2} = \frac{4}{4} = 1 \vee y = \frac{1-3}{2 \cdot 2} = \frac{-2}{4} = -\frac{1}{2}$

$y=1$  in ② geeft  $x^2 + 1 = 17 \Rightarrow x^2 = 16 \Rightarrow x = 4 \vee x = -4$

$y=-\frac{1}{2}$  in ② geeft  $x^2 - \frac{1}{2} = 17 \Rightarrow x^2 = 17\frac{1}{2} \Rightarrow x = \sqrt{17\frac{1}{2}} = \sqrt{\frac{35}{2}} = \sqrt{\frac{70}{4}} = \frac{1}{2}\sqrt{70} \vee x = -\frac{1}{2}\sqrt{70}$ .

62b  $\begin{cases} x^2 + 2y^2 = 19 & \text{①} \\ xy = 3 & \text{②} \end{cases} \Rightarrow \begin{cases} x^2 + 2y^2 = 19 & \text{①} \\ y = \frac{3}{x} & \text{③} \end{cases}$

$x^2 = 1 \vee x^2 = 18$

$x=1 \vee x=-1 \vee x=\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2} \vee x = -3\sqrt{2}$

$x=1$  in ③ geeft  $y = \frac{3}{1} = 3$

$x=-1$  in ③ geeft  $y = \frac{3}{-1} = -3$

$x=3\sqrt{2}$  in ③ geeft  $y = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2}$

$x=-3\sqrt{2}$  in ③ geeft  $y = \frac{3}{-3\sqrt{2}} = -\frac{1}{2}\sqrt{2}$ .

③ in ①  $\Rightarrow x^2 + 2(\frac{3}{x})^2 = 19$

$x^2 + 2 \cdot \frac{9}{x^2} = 19$  (vermenigvuldigen met  $x^2$ )

$x^4 - 19x^2 + 18 = 0$

$(x^2 - 1)(x^2 - 18) = 0$  (hiernaast verder)

62c  $\begin{cases} a+b=8 & \text{①} \\ 2^a + 2^{b-1} = 24 & \text{②} \end{cases} \Rightarrow \begin{cases} b=8-a & \text{③} \\ 2^a + 2^{b-1} = 24 & \text{④} \end{cases}$

③ in ④  $\Rightarrow 2^a + 2^{8-a-1} = 24$

$2^a + 2^{7-a} = 24$

$2^a + \frac{2^7}{2^a} = 24$  (vermenigvuldigen met  $2^a$ )

$(2^a)^2 - 24 \cdot 2^a + 2^7 = 0$  (hiernaast verder)

$(2^a)^2 - 24 \cdot 2^a + 128 = 0$

$(2^a - 8)(2^a - 16) = 0$

$2^a = 8 = 2^3 \vee 2^a = 16 = 2^4$

$a=3 \vee a=4$

$a=3$  in ③ geeft  $b=8-3=5$

$a=4$  in ③ geeft  $b=8-4=4$ .

62d  $\begin{cases} a+2b=14 & \text{①} \\ 2\log(a) + 2\log(b-1) = 4 & \text{②} \end{cases} \Rightarrow \begin{cases} a=14-2b & \text{③} \\ 2\log(a \cdot (b-1)) = 4 & \text{④} \end{cases} \Rightarrow \begin{cases} a=14-2b & \text{⑤} \\ a \cdot (b-1) = 2^4 = 16 & \text{⑥} \end{cases}$

③ in ⑥  $\Rightarrow (14-2b) \cdot (b-1) = 16$

$(b-3)(b-5) = 0$

$14b - 14 - 2b^2 + 2b - 16 = 0$

$b=3 \vee b=5$

$-2b^2 + 16b - 30 = 0$

$b=3$  in ⑥ geeft  $a=14-6=8$

$b^2 - 8b + 15 = 0$  (hiernaast verder)

$b=5$  in ⑥ geeft  $a=14-10=4$ .

63a  $f_{0,0}(x) = \frac{3^{x+0}+1}{3^{x+0}-1} = \frac{3^x+1}{3^x-1}$ . Nu is  $f_{0,0}(a) = \frac{3^a+1}{3^a-1}$  en  $f_{0,0}(-a) = \frac{3^{-a}+1}{3^{-a}-1} = \frac{3^{-a}+1}{\frac{1}{3^a}-1} = \frac{3^0+3^a}{3^0-3^a} = \frac{1+3^a}{1-3^a} = -\frac{3^a+1}{3^a-1}$ .  
Voor elke  $a$  is  $f_{0,0}(a) = -f_{0,0}(-a) \Rightarrow$  de grafiek van  $f_{0,0}$  is puntsymmetrisch in de oorsprong.

63b  $f_{-1,-1}(x) = \frac{3^{x-1}+1}{3^{x-1}-1} = \frac{3^{x-1}+1}{3^{x-1}-1} \cdot \frac{3}{3} = \frac{3^x+3}{3^x-3}$ .

63c (3,  $\frac{1}{4}$ ) invullen in  $f_{p,q}(x) = \frac{3^{x+p}+1}{3^{x+p}-1}$  geeft  $\frac{1}{4} = \frac{3^{3+p}+1}{3^{3+p}-1} \Rightarrow 3^{3+p} - 1 = 4 \cdot 3^{3+p} + 4 \Rightarrow 3^{3+p} = 4 \cdot 3^{3+p} + 5$  ①.

(5,  $\frac{1}{8}$ ) invullen geeft  $\frac{1}{8} = \frac{3^{5+p}+1}{3^{5+p}-1} \Rightarrow 3^{5+p} - 1 = 8 \cdot 3^{5+p} + 8 \Rightarrow 3^{5+p} = 8 \cdot 3^{5+p} + 9$  (delen door  $3^2$ )  $\Rightarrow 3^{3+p} = 8 \cdot 3^{3+p} + 1$  ②.

① in ②  $\Rightarrow 4 \cdot 3^{3+p} + 5 = 8 \cdot 3^{3+p} + 1 \Rightarrow -4 \cdot 3^{3+p} = -4 \Rightarrow 3^{3+p} = 1 = 3^0 \Rightarrow 3+p=0 \Rightarrow p=-3$ .

$p=-3$  in ①  $\Rightarrow 3^{3+q} = 4 \cdot 3^0 + 5 = 4 \cdot 1 + 5 = 9 = 3^2 \Rightarrow 3+q=2 \Rightarrow q=-1$ .

64a  $f(x) = \frac{4}{x+1} + \frac{5}{x+2} = 4 \cdot \frac{1}{x+1} + 5 \cdot \frac{1}{x+2} \Rightarrow F(x) = 4 \cdot \ln|x+1| + 5 \cdot \ln|x+2|.$

64b  $f(x) = \frac{4}{x+1} + \frac{5}{x+2} = \frac{4 \cdot x+2}{x+2} + \frac{5 \cdot x+1}{x+1} = \frac{4(x+2)+5(x+1)}{(x+1)(x+2)} = \frac{4x+8+5x+5}{(x+1)(x+2)} = \frac{9x+13}{(x+1)(x+2)}.$

64c  $g(x) = \frac{9x+13}{x^2+3x+2} = \frac{9x+13}{(x+1)(x+2)} = f(x) \Rightarrow G(x) = F(x) = 4 \cdot \ln|x+1| + 5 \cdot \ln|x+2|.$

65a  $f(x) = \frac{5x-14}{x^2-6x+8} = \frac{3}{x-4} + \frac{2}{x-2}$  (zie de berekening hieronder)  $\Rightarrow F(x) = 3\ln|x-4| + 2\ln|x-2| + c.$

$$f(x) = \frac{5x-14}{x^2-6x+8} = \frac{5x-14}{(x-4)(x-2)} = \frac{a}{x-4} + \frac{b}{x-2} = \frac{a(x-2)}{(x-4)(x-2)} + \frac{b(x-4)}{(x-4)(x-2)} = \frac{ax-2a+bx-4b}{(x-4)(x-2)} = \frac{(a+b)x-2a-4b}{(x-4)(x-2)}$$

$$\begin{cases} a + b = 5 & \textcircled{1} \\ -2a - 4b = -14 & \textcircled{2} \end{cases} \Rightarrow \begin{cases} 2a + 2b = 10 & \textcircled{3} \\ -2a - 4b = -14 & \textcircled{2} \end{cases} \\ -2b = -4 \Rightarrow b = 2 \text{ in } \textcircled{1} \Rightarrow a + 2 = 5 \Rightarrow a = 3.$$

65b  $g(x) = \frac{x+11}{x^2+4x+3} = -\frac{4}{x+3} + \frac{5}{x+1}$  (zie de berekening hieronder)  $\Rightarrow G(x) = -4\ln|x+3| + 5\ln|x+1| + c.$

$$g(x) = \frac{x+11}{x^2+4x+3} = \frac{x+11}{(x+3)(x+1)} = \frac{a}{x+3} + \frac{b}{x+1} = \frac{a(x+1)}{(x+3)(x+1)} + \frac{b(x+3)}{(x+3)(x+1)} = \frac{ax+a+bx+3b}{(x+3)(x+1)} = \frac{(a+b)x+a+3b}{(x+3)(x+1)}$$

$$\begin{cases} a + b = 1 & \textcircled{1} \\ a + 3b = 11 & \textcircled{2} \end{cases} \\ -2b = -10 \Rightarrow b = 5 \text{ in } \textcircled{1} \Rightarrow a + 5 = 1 \Rightarrow a = -4.$$

65c  $h(x) = \frac{3x+24}{x^2+7x+10} = -\frac{3}{x+5} + \frac{6}{x+2}$  (zie de berekening hieronder)  $\Rightarrow H(x) = -3\ln|x+5| + 6\ln|x+2| + c.$

$$h(x) = \frac{3x+24}{x^2+7x+10} = \frac{3x+24}{(x+5)(x+2)} = \frac{a}{x+5} + \frac{b}{x+2} = \frac{a(x+2)}{(x+5)(x+2)} + \frac{b(x+5)}{(x+5)(x+2)} = \frac{ax+2a+bx+5b}{(x+5)(x+2)} = \frac{(a+b)x+2a+5b}{(x+5)(x+2)}$$

$$\begin{cases} a + b = 3 & \textcircled{1} \\ 2a + 5b = 24 & \textcircled{2} \end{cases} \Rightarrow \begin{cases} 2a + 2b = 6 & \textcircled{3} \\ 2a + 5b = 24 & \textcircled{2} \end{cases} \\ -3b = -18 \Rightarrow b = 6 \text{ in } \textcircled{1} \Rightarrow a + 6 = 3 \Rightarrow a = -3.$$

65d  $j(x) = \frac{x}{x^2+7x+12} = \frac{4}{x+4} - \frac{3}{x+3}$  (zie de berekening hieronder)  $\Rightarrow J(x) = 4\ln|x+4| - 3\ln|x+3| + c.$

$$j(x) = \frac{x}{x^2+7x+12} = \frac{x}{(x+4)(x+3)} = \frac{a}{x+4} + \frac{b}{x+3} = \frac{a(x+3)}{(x+4)(x+3)} + \frac{b(x+4)}{(x+4)(x+3)} = \frac{ax+3a+bx+4b}{(x+4)(x+3)} = \frac{(a+b)x+3a+4b}{(x+4)(x+3)}$$

$$\begin{cases} a + b = 1 & \textcircled{1} \\ 3a + 4b = 0 & \textcircled{2} \end{cases} \Rightarrow \begin{cases} 3a + 3b = 3 & \textcircled{3} \\ 3a + 4b = 0 & \textcircled{2} \end{cases} \\ -b = 3 \Rightarrow b = -3 \text{ in } \textcircled{1} \Rightarrow a - 3 = 1 \Rightarrow a = 4.$$

66a  $f(x) = \frac{8x+4}{x^2-4} = \frac{3}{x^2-4}$  ( $x^2 \neq 4 \Rightarrow x \neq \pm 2$ )

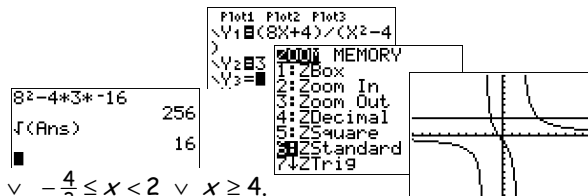
$$3x^2 - 12 = 8x + 4$$

$$3x^2 - 8x - 16 = 0$$

$$D = (-8)^2 - 4 \cdot 3 \cdot (-16) = 256 \Rightarrow \sqrt{D} = 16$$

$$x = \frac{8 \pm 16}{2 \cdot 3} = \frac{24}{6} = 4 \vee x = \frac{8 - 16}{2 \cdot 3} = \frac{-8}{6} = -\frac{4}{3}$$

$$f(x) \leq 3 \text{ (zie hierboven en een plot)} \Rightarrow x < -2 \vee -\frac{4}{3} \leq x < 2 \vee x \geq 4.$$



66b  $f(x) = \frac{8x+4}{x^2-4} \Rightarrow f'(x) = \frac{(x^2-4) \cdot 8 - (8x+4) \cdot 2x}{(x^2-4)^2} = \frac{8x^2-32-16x^2-8x}{(x^2-4)^2} = \frac{-8x^2-8x-32}{(x^2-4)^2}.$

$$f(0) = \frac{0+4}{0-4} = -1 \text{ en } f'(0) = \frac{0-0-32}{(0-4)^2} = \frac{-32}{16} = -2.$$

$$k: y = -2x + b \text{ door } (0, -1) \Rightarrow k: y = -2x - 1.$$

66c  $f(x) = \frac{8x+4}{x^2-4} = \frac{4}{3}$

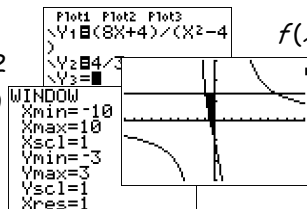
$$4x^2 - 16 = 24x + 12$$

$$4x^2 - 24x - 28 = 0$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$x = 7 \vee x = -1.$$



$$f(x) = \frac{8x+4}{x^2-4} = \frac{8x+4}{(x-2)(x+2)} = \frac{a}{x-2} + \frac{b}{x+2} = \frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x-2)(x+2)} \\ = \frac{ax+2a+bx-2b}{(x-2)(x+2)} = \frac{(a+b)x+2a-2b}{(x-2)(x+2)}$$

$$\begin{cases} a + b = 8 & \textcircled{1} \\ 2a - 2b = 4 & \textcircled{2} \end{cases} \Rightarrow \begin{cases} 2a + 2b = 16 & \textcircled{3} \\ 2a - 2b = 4 & \textcircled{2} \end{cases} \\ 4a = 20 \Rightarrow a = 5 \text{ in } \textcircled{1} \Rightarrow 5 + b = 8 \Rightarrow b = 3.$$

$$O(V) = \int_{-1}^0 \left( \frac{1}{3} - f(x) \right) dx = \int_{-1}^0 \left( \frac{1}{3} - \left( \frac{5}{x-2} + \frac{3}{x+2} \right) \right) dx = \int_{-1}^0 \left( \frac{1}{3} - \frac{5}{x-2} - \frac{3}{x+2} \right) dx = \left[ \frac{1}{3}x - 5\ln|x-2| - 3\ln|x+2| \right]_{-1}^0 \\ = 0 - 5\ln(2) - 3\ln(2) - \left( -\frac{1}{3} - 5\ln(3) - 3\ln(1) \right) = -8\ln(2) + 1\frac{1}{3} + 5\ln(3).$$

67a  $e^{x-p} = 6$   $\textcircled{1}$  invullen in  $-2p \cdot e^{x-p} = 24$   $\textcircled{2}$  geeft  $-2p \cdot 6 = 24.$

67b  $-2p \cdot 6 = 24 \Rightarrow -12p = 24 \Rightarrow p = -2$  in  $\textcircled{1} \Rightarrow e^{x+2} = 6 \Rightarrow x+2 = \ln(6) \Rightarrow x = -2 + \ln(6).$



68a  $p\sqrt{x^2+4} = 4$  ② invullen in  $px\sqrt{x^2+4} = x^2+3$  ① geeft  $4x = x^2+3$   
 $x^2 - 4x + 3 = 0$   $x = 3$  in ②  $\Rightarrow p\sqrt{3^2+4} = 4 \Rightarrow p = \frac{4}{\sqrt{13}} = \frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4}{13}\sqrt{13}$  en  
 $(x-3)(x-1) = 0$   $x = 1$  in ②  $\Rightarrow p\sqrt{1^2+4} = 4 \Rightarrow p = \frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4}{5}\sqrt{5}$   
 $x = 3 \vee x = 1$  (hiernaast verder)

68b  $pe^{x^2-1} = x^2+1$  ① invullen in  $pxe^{x^2-1} = x^3+x^2-6$  ② geeft  $x(x^2+1) = x^3+x^2-6$   
 $x^3+x = x^3+x^2-6$   $x = 3$  in ①  $\Rightarrow pe^{3^2-1} = 3^2+1 \Rightarrow pe^8 = 10 \Rightarrow \frac{10}{e^8}$  en  
 $x^2-x-6 = 0$   $x = -2$  in ①  $\Rightarrow pe^{4-1} = 4+1 \Rightarrow pe^3 = 5 \Rightarrow \frac{5}{e^3}$   
 $(x-3)(x+2) = 0$   
 $x = 3 \vee x = -2$  (hiernaast verder)

68c  $2 + a\ln(x+1) = x \Rightarrow a\ln(x+1) = x-2$  ① invullen in  $2ax\ln(x+1) = 48$  (2) geeft  $2x \cdot (x-2) = 48$   
 $2x^2 - 4x = 48$   $x = 6$  in ①  $\Rightarrow a\ln(6+1) = 6-2 \Rightarrow a = \frac{4}{\ln(7)}$  en  
 $x^2 - 2x - 24 = 0$   $x = -4$  in ①  $\Rightarrow a\ln(-4+1) = \dots$  heeft geen oplossing.  
 $(x-6)(x+4) = 0$   
 $x = 6 \vee x = -4$  (hiernaast verder)

68d  $x + \frac{x+p}{2x+3} = 2 \Rightarrow \frac{x+p}{2x+3} = 2-x$  ① invullen in  $\sqrt{\frac{x+p}{2x+3}} = x$  ② geeft  $\sqrt{2-x} = x$  (\* kwadrateren)  
 $2-x = x^2$   
 $x^2+x-2 = 0$   
 $(x+2)(x-1) = 0$   
 $x = -2$  (voldoet niet aan \*)  $\vee x = 1$  in ①  $\Rightarrow \frac{1+p}{2+3} = 2-1 \Rightarrow 1+p = 5 \cdot 1 \Rightarrow p = 5-1 = 4$ .

69  $f_p(x) = e^{x^2+p} + x \Rightarrow f_p'(x) = e^{x^2+p} \cdot 2x + 1$ .  
 $x$ -as raken  $\Rightarrow f_p(x) = 0$  (op de  $x$ -as komen)  $\wedge f_p'(x) = 0$  (horizontale raaklijn)  
 $e^{x^2+p} + x = 0 \Rightarrow e^{x^2+p} = -x$  ① invullen in  $e^{x^2+p} \cdot 2x + 1 = 0$  ② geeft  $-x \cdot 2x + 1 = 0$   
 $-2x^2 + 1 = 0$   
 $2x^2 = 1$   
 $x^2 = \frac{1}{2}$   
 $x = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2} \vee x = -\sqrt{\frac{1}{2}} = -\frac{1}{2}\sqrt{2}$  (hierboven verder) 
 $x = \frac{1}{2}\sqrt{2} = \sqrt{\frac{1}{2}}$  in ①  $\Rightarrow e^{\frac{1}{2}+p} = -\frac{1}{2}\sqrt{2}$  heeft geen oplossing en  
 $x = -\frac{1}{2}\sqrt{2} = -\sqrt{\frac{1}{2}}$  in ①  $\Rightarrow e^{\frac{1}{2}+p} = \frac{1}{2}\sqrt{2} \Rightarrow \frac{1}{2} + p = \ln(\frac{1}{2}\sqrt{2}) \Rightarrow p = -\frac{1}{2} + \ln(\frac{1}{2}\sqrt{2})$ .
  Dus  $x = -\frac{1}{2}\sqrt{2}$  en het raakpunt is  $(-\frac{1}{2}\sqrt{2}, 0)$ .

70  $f_p(x) = \sqrt{x^2+p} \Rightarrow f_p'(x) = \frac{1}{2\sqrt{x^2+p}} \cdot 2x = \frac{x}{\sqrt{x^2+p}}$ .  
 $f_p(x) = 5x+5$  ( $y = 5x+5$  snijden)  $\wedge f_p'(x) \cdot 5 = -1$  (loodrecht op  $y = 5x+5$ )  
 $\sqrt{x^2+p} = 5x+5$  ① invullen in  $\frac{x}{\sqrt{x^2+p}} \cdot 5 = -1$  ② geeft  $\frac{x}{5x+5} \cdot 5 = -1$   
 $\frac{x}{x+1} = \frac{-1}{1}$   
 $x = -x-1$   
 $2x = -1$   
 $x = -\frac{1}{2}$  in ①  $\Rightarrow \sqrt{\frac{1}{4}+p} = 5 \cdot -\frac{1}{2} + 5 \Rightarrow \sqrt{\frac{1}{4}+p} = 2\frac{1}{2}$  (\* kwadrateren)  $\Rightarrow \frac{1}{4} + p = \frac{25}{4} \Rightarrow p = \frac{24}{4} = 6$  (voldoet aan \*).

71  $f_p(x) = \ln(p-x^2) \Rightarrow f_p'(x) = \frac{1}{p-x^2} \cdot -2x = -\frac{2x}{p-x^2}$  en  $g_q(x) = -x^2+q \Rightarrow g_q'(x) = -2x$ .  
 $f_p(x) = g_q(x) \wedge f_p'(x) = g_q'(x)$   
 $\ln(p-x^2) = -x^2+q$  ①  $\wedge -\frac{2x}{p-x^2} = -2x$  ②  
 $\ln(p-x^2) = -x^2+q \wedge (x=0 \vee p-x^2=1)$   
 $(\ln(p-x^2) = -x^2+q \wedge x=0) \vee (\ln(p-x^2) = -x^2+q \wedge p-x^2=1)$   
 $(\ln(p) = q) \vee (\ln(1) = -x^2+q)$   
 $(p = e^q) \vee (x^2 = q \text{ in } ①) \Rightarrow p = e^q \vee \ln(p-q) = 0 \Rightarrow p = e^q \vee p-q = e^0 = 1 \Rightarrow p = e^q \vee p = 1+q$ .

**Diagnostische toets**

D1a  $\square$   $(x^2 - 4)^2 = (-x + 2)^2$   
 $x^2 - 4 = -x + 2 \vee x^2 - 4 = x - 2$   
 $x^2 + x - 6 = 0 \vee x^2 - x - 2 = 0$   
 $(x + 3)(x - 2) = 0 \vee (x - 2)(x + 1) = 0$   
 $x = -3 \vee x = 2 \vee x = 2 \vee x = -1$   
 $x = -3 \vee x = 2 \vee x = -1.$

D1b  $\square$   $e^x \cdot \sin(x) = 5 \sin(x)$   
 $\sin(x) = 0 \vee e^x = 5$   
 $x = k \cdot \pi \vee x = \ln(5).$

D1c  $\square$   $\frac{\ln(x)}{\ln(x)-1} = \frac{\ln(x)+2}{\ln(x)-4}$   
(stel tijdelijk  $\ln(x)=t$ )  
 $\frac{t}{t-1} = \frac{t+2}{t-4}$   
 $t(t-4) = (t-1)(t+2)$   
 $t^2 - 4t = t^2 + t - 2$   
 $-5t = -2$   
 $t = \ln(x) = \frac{2}{5} \Rightarrow x = e^{\frac{2}{5}} = \sqrt[5]{e^2}.$

D2a  $\square$   $((x - \sqrt{2})(x + \sqrt{2}))^2 = (x^2 - 2)^2 = x^4 - 4x^2 + 4.$

D2b  $\square$   $(2x + 3)^3 = (2x + 3)(2x + 3)^2 = (2x + 3)(4x^2 + 12x + 9) = 8x^3 + 24x^2 + 18x + 12x^2 + 36x + 27 = 8x^3 + 36x^2 + 54x + 27.$

D2c  $\square$   $\frac{x^4 - 16}{x^2 - 4} (x^2 \neq 4 \Rightarrow x \neq 2 \wedge x \neq -2) = \frac{(x^2 + 4)(x^2 - 4)}{x^2 - 4} = x^2 + 4 (x \neq 2 \wedge x \neq -2).$

D3a  $\square$   $y = \frac{6}{x+1} \cdot \frac{2}{x-1} (x \neq -1 \wedge x \neq 1) = \frac{12}{(x+1)(x-1)}.$

D3b  $\square$   $y = \frac{\frac{x-1}{x+1}}{2x^2} (x \neq 0 \wedge x \neq -1) = \frac{x-1}{x+1} \cdot \frac{1}{2x^2} = \frac{x-1}{2x^2(x+1)}.$

D3c  $\square$   $y = \frac{x-1}{\frac{x+1}{2x^2}} (x \neq 0 \wedge x \neq -1) = (x-1) \cdot \frac{2x^2}{x+1} = \frac{2x^2(x-1)}{x+1}.$

D3d  $\square$   $y = \frac{2e^x}{e^x-1} + \frac{e^{-x}}{e^x+1} (e^x \neq 1 \Rightarrow x \neq 0) = \frac{2e^x(e^x+1) + e^{-x}(e^x-1)}{(e^x-1)(e^x+1)} = \frac{2e^{2x} + 2e^x + 1 - e^{-x}}{(e^x-1)(e^x+1)} \cdot \frac{e^x}{e^x} = \frac{2e^{3x} + 2e^{2x} + e^x - 1}{e^x(e^x-1)(e^x+1)}.$

D3e  $\square$   $y = \frac{e^{-x}}{x+1} \cdot 3e^{-x} (x \neq -1) = \frac{3e^{-2x}}{x+1} \cdot \frac{e^{2x}}{e^{2x}} = \frac{3}{e^{2x}(x+1)}.$

D3f  $\square$   $y = \frac{x + \frac{x}{x-1}}{x+1} (x \neq 1 \wedge x \neq -1) = \frac{x(x-1) + x}{(x+1)(x-1)} = \frac{x^2 - x + x}{2x^2} = \frac{x^2}{(x+1)(x-1)}.$

D4a  $\square$   $\frac{1}{b} + \frac{1}{v} - \frac{1}{f} = 0 (b \neq 0 \wedge v \neq 0 \wedge f \neq 0) \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{b} \Rightarrow \frac{1}{v} = \frac{b-f}{bf} - \frac{f}{bf} = \frac{b-f-f}{bf} \Rightarrow v = \frac{bf}{b-f} (b \neq 0 \wedge v \neq 0 \wedge f \neq 0 \wedge b \neq f).$

D4b  $\square$   $\frac{3}{p} + \frac{4}{q} = 6 (p \neq 0 \wedge q \neq 0) \Rightarrow \frac{3}{p} = 6 - \frac{4}{q} \Rightarrow \frac{3}{p} = \frac{6q-4}{q} - \frac{4}{q} = \frac{6q-4-4}{q} \Rightarrow \frac{p}{3} = \frac{q}{6q-4} \Rightarrow p = \frac{3q}{6q-4} (p \neq 0 \wedge q \neq 0 \wedge q \neq \frac{2}{3}).$

D5a  $\square$   $y = \frac{2}{\sqrt{x}} + \frac{4}{3}\sqrt{x} = \frac{2}{\sqrt{x}} \cdot \frac{3}{3} + \frac{4\sqrt{x}}{3} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{6}{3\sqrt{x}} + \frac{4x}{3\sqrt{x}} = \frac{6+4x}{3\sqrt{x}} (x > 0).$

D5b  $\square$   $A = \sqrt{8p} - \sqrt{\frac{32}{p}} = \sqrt{4 \cdot 2p} - \sqrt{\frac{16 \cdot 2}{p} \cdot \frac{p}{p}} = 2 \cdot \sqrt{2p} - \frac{4}{p} \cdot \sqrt{2p} = \left(2 - \frac{4}{p}\right) \cdot \sqrt{2p} (p > 0).$

D5c  $\square$   $y = \frac{3}{\sqrt{x^2+1}} - \sqrt{x^2+1} = \frac{3}{\sqrt{x^2+1}} - \frac{x^2+1}{\sqrt{x^2+1}} = \frac{3-x^2-1}{\sqrt{x^2+1}} = \frac{2-x^2}{\sqrt{x^2+1}}.$

D6a  $\square$   $x\sqrt{2y-1} - \frac{2}{\sqrt{x}} = 0 (x > 0 \wedge y \geq \frac{1}{2})$

$x\sqrt{2y-1} = \frac{2}{\sqrt{x}}$   
 $\sqrt{2y-1} = \frac{2}{x\sqrt{x}} \text{ (kwadrateren)}$   
 $2y-1 = \frac{4}{x^3}$   
 $2y = \frac{4}{x^3} + 1 \Rightarrow y = \frac{2}{x^3} + \frac{1}{2}.$

D6b  $\square$   $A = 2\sqrt{p-2A} (p \geq 2A)$

$A^2 = 4(p-2A)$   
 $p-2A = \frac{A^2}{4}$   
 $p = \frac{A^2}{4} + 2A.$

D7a  $\square$   $y = \left(\frac{2}{x \cdot \sqrt[5]{x^2}}\right)^2 \cdot x\sqrt{x} = \left(\frac{2}{x^{1\frac{1}{5}}}\right)^2 \cdot x^{1\frac{1}{2}} = \frac{4}{x^{2\frac{2}{5}}} \cdot x^{1\frac{1}{2}} = 4 \cdot x^{-2\frac{4}{5}} \cdot x^{1\frac{1}{2}} = 4x^{-1\frac{3}{10}}.$

D7b  $\square$   $y = (2x^{0.7})^3 \cdot (81x^3)^{0.5} = 2^3 \cdot x^{2.1} \cdot 81^{\frac{1}{2}} \cdot x^{1.5} = 8 \cdot x^{2.1} \cdot 9 \cdot x^{1.5} = 72x^{3.6}.$

D8a  $\square$   $p = 7 \cdot (5A)^{-1.3} \Rightarrow p = \frac{7}{(5A)^{1.3}} \Rightarrow (5A)^{1.3} = \frac{7}{p} \Rightarrow 5A = \left(\frac{7}{p}\right)^{\frac{1}{1.3}} = \frac{7^{\frac{1}{1.3}}}{p^{\frac{1}{1.3}}} \Rightarrow A = \frac{1}{5} \cdot 7^{\frac{1}{1.3}} \cdot p^{-\frac{1}{1.3}} \approx 0,89p^{-0,77}.$

$1\sqrt{5*7^{(1/1.3)}}$
$-1/1.3$
$8935223313$
$.7692307692$

D8b  $\square$   $x = 2t \cdot \sqrt[3]{t} = 2t \cdot t^{\frac{1}{3}} = 2t^{\frac{4}{3}}$  invullen in  $y = 3x^2 \cdot \sqrt{x} = 3x^{2\frac{1}{2}}$  geeft  $y = 3\left(2t^{\frac{4}{3}}\right)^{\frac{5}{2}} = 3 \cdot 2^{2\frac{1}{2}} \cdot t^{\frac{10}{3}} = 3 \cdot 4\sqrt{2} \cdot t^{\frac{10}{3}} = 12\sqrt{2} \cdot t^{\frac{10}{3}}.$

D9a  $\square$   $N = 2^{t-1} \cdot \left(\frac{1}{4}\right)^{3t+2} = 2^{t-1} \cdot (2^{-2})^{3t+2} = 2^{t-1} \cdot 2^{-6t-4} = 2^{-5t-5} = 2^{-5t} \cdot 2^{-5} = (2^{-5})^t \cdot \frac{1}{32} = \frac{1}{32} \cdot \left(\frac{1}{32}\right)^t$   $\square$   $2^{-5}$  32

D9b  $\square$   $N = 2 \cdot 3^{2t-1} = 2 \cdot 3^{2t} \cdot 3^{-1} = 2 \cdot (3^2)^t \cdot \frac{1}{3} = \frac{2}{3} \cdot 9^t$

D10a  $\square$   $y = 1200 \cdot 2^{x-1} = 1200 \cdot 2^x \cdot 2^{-1} = 1200 \cdot (e^{\ln(2)})^x \cdot \frac{1}{2} = 600 \cdot e^{x \ln(2)} \approx 600 \cdot e^{0,69x}$   $\square$   $\ln(2)$  .6931471806  $2^6$  64  
Ans: 1100  
-log(2)  
-1.3010299957

D10b  $\square$   $y = \frac{1}{1100} \cdot \left(\frac{1}{2}\right)^{x-6} = \frac{1}{1100} \cdot (2^{-1})^{x-6} = \frac{1}{1100} \cdot 2^{-x} \cdot 2^6 = \frac{1}{1100} \cdot (10^{\log(2)})^{-x} \cdot 64 = \frac{64}{1100} \cdot 10^{-x \log(2)} \approx 0,06 \cdot 10^{-0,30x}$

D10c  $\square$   $y = 10 \cdot 2,21^{2x+3} = e^{\ln(10)} \cdot (e^{\ln(2,21)})^{2x+3} = e^{\ln(10)+2x \ln(2,21)+3 \ln(2,21)} \approx e^{1,59x+4,68}$   $\square$   $\ln(10)+3 \ln(2,21)$  4,68156264  
 $2 \ln(2,21)$  1,585985031

D11a  $\square$   $N = 10 \cdot 2,21^{2x+3}$   
 $\frac{1}{10} N = 2,21^{2x+3}$   
 $\log\left(\frac{1}{10} N\right) = \log(2,21^{2x+3})$   
 $\log\left(\frac{1}{10}\right) + \log(N) = (2x+3) \log(2,21)$   
 $\frac{\log(10^{-1}) + \log(N)}{\log(2,21)} = 2x+3$   
 $2x = \frac{-1 + \log(N)}{\log(2,21)} - 3$   
 $x = \frac{-1}{2 \log(2,21)} + \frac{\log(N)}{2 \log(2,21)} - \frac{3}{2}$   
 $x \approx -2,95 + 1,45 \log(N)$   $\square$   $\frac{-1 + \log(2,21)}{2 \log(2,21)} - \frac{3}{2}$  -2,951832803  
 $\frac{\log(2,21)}{2 \log(2,21)}$  1,451832803

D11b  $\square$   $y = \frac{1}{3} \cdot 2^{5x-6}$   
 $3y = 2^{5x-6}$   
 $\ln(3y) = \ln(2^{5x-6})$   
 $\ln(3) + \ln(y) = (5x-6) \ln(2)$   
 $\frac{\ln(3) + \ln(y)}{\ln(2)} = 5x-6$   
 $5x = \frac{\ln(3) + \ln(y)}{\ln(2)} + 6$   
 $x = \frac{\ln(3) + \ln(y)}{5 \ln(2)} + \frac{6}{5}$   
 $x \approx 1,52 + 0,29 \ln(y)$   $\square$   $\frac{\ln(3) + \ln(2)}{5 \ln(2)} + \frac{6}{5}$  1,5169925  
 $\frac{\ln(2)}{5 \ln(2)}$  .2885390082

D12a  $\square$   $\log(N) = 2,1 + 0,1t \Rightarrow N = 10^{2,1+0,1t} = 10^{2,1} \cdot (10^{0,1})^t \approx 126 \cdot 1,259^t$   $\square$   $10^{2,1}$  125,8925412  
 $10^{0,1}$  1,258925412  $10^{1,7}$  50,11872336

D12b  $\square$   $\log(p) = 1,7 - 0,02 \log(q) \Rightarrow \log(p) = \log(10^{1,7}) + \log(q^{-0,02}) = \log(10^{1,7} \cdot q^{-0,02}) \Rightarrow p = 10^{1,7} \cdot q^{-0,02} \approx 50,12 \cdot q^{-0,02}$

D12c  $\square$   $\ln(y+2) = 2 + \ln(x-4) \Rightarrow \ln(y+2) = \ln(e^2) + \ln(x-4) = \ln(e^2(x-4)) \Rightarrow y+2 = e^2 x - 4e^2 \Rightarrow$   
 $y = e^2 x - 4e^2 - 2 \approx 7,39x - 31,56$   $\square$   $e^2$  7,389056099  
 $-4e^2 - 2$  -31,5562244

D12d  $\square$   $K = 3 \ln(2F-1) + 4 \Rightarrow K-4 = 3 \ln(2F-1) \Rightarrow \frac{K-4}{3} = \ln(2F-1) \Rightarrow 2F-1 = e^{\frac{1}{3}K - \frac{4}{3}} \Rightarrow 2F = 1 + e^{\frac{1}{3}K - \frac{4}{3}} \Rightarrow F = \frac{1}{2} + \frac{1}{2} e^{\frac{1}{3}K - \frac{4}{3}}$

D13a  $\square$   $\begin{cases} 2x^2 + 6y^2 = 20 & \textcircled{1} \\ 2x^2 + 6y = 8 & \textcircled{2} \end{cases}$   $y^2 - y - 2 = 0$   
 $(y-2)(y+1) = 0$   
 $y = 2$  in  $\textcircled{2} \Rightarrow 2x^2 + 12 = 8 \Rightarrow 2x^2 = -4 \Rightarrow x^2 = -2$  geen oplossing  
 $y = -1$  in  $\textcircled{2} \Rightarrow 2x^2 - 6 = 8 \Rightarrow 2x^2 = 14 \Rightarrow x^2 = 7 \Rightarrow x = 7 \vee x = -7$   
 $6y^2 - 6y = 12$   
 $y^2 - y = 2$  (hiernaast verder)

D13b  $\square$   $x + y = 4 \Rightarrow x = 4 - y$   $\textcircled{1}$  invullen in  $2^x + 2^{-y} = 17$   $\textcircled{2}$  geeft  $2^{4-y} + 2^{-y} = 17$   
 $2^{4-y} + 2^{-y} = 17$   
 $2^4 \cdot 2^{-y} + 2^{-y} = 17$   
 $16 \cdot 2^{-y} + 1 \cdot 2^{-y} = 17$   
 $17 \cdot 2^{-y} = 17$  (hiernaast verder)  
 $2^{-y} = 1 = 2^0$   
 $-y = 0$   
 $y = 0$   
 $y = 0$  in  $\textcircled{1} \Rightarrow x + 0 = 4 \Rightarrow x = 4$

D13c  $\square$   $2a - 7 = 2\sqrt{b-2}$  (kwadrateren)  $\Rightarrow 4a^2 - 28a + 49 = 4(b-2) \Rightarrow 4a^2 - 28a + 49 = 4b - 8$   $\textcircled{1}$   
 $a = 2\sqrt{b}$  (kwadrateren)  $\Rightarrow a^2 = 4b \Rightarrow 4b = a^2$   $\textcircled{2}$   
 $\textcircled{2}$  in  $\textcircled{1} \Rightarrow 4a^2 - 28a + 49 = a^2 - 8$   
 $3a^2 - 28a + 57 = 0$   $\square$   $(-28)^2 - 4 \cdot 3 \cdot 57$  100  
 $D = (-28)^2 - 4 \cdot 3 \cdot 57 = 100 \Rightarrow \sqrt{D} = 10$  (hiernaast verder)  
 $a = \frac{28+10}{6} = \frac{38}{6} = 6\frac{1}{3} \vee a = \frac{28-10}{6} = \frac{18}{6} = 3$   $\square$   $(6+1/3)^2 \cdot \frac{1}{36}$  361/9  
 $a = 6\frac{1}{3}$  in  $\textcircled{2} \Rightarrow 4b = \frac{361}{9} \Rightarrow b = \frac{361}{36} = 10\frac{1}{36}$  (voldoet)  
 $a = 3$  in  $\textcircled{2} \Rightarrow 4b = 9 \Rightarrow b = \frac{9}{4} = 2\frac{1}{4}$  (voldoet niet).

D13d  $\square$   $p \ln(x) = 2x$   $\textcircled{2}$  invullen in  $p x^2 \ln(x) = x^3 + 8$   $\textcircled{1}$  geeft  $2x \cdot x^2 = x^3 + 8 \Rightarrow 2x^3 = x^3 + 8 \Rightarrow x^3 = 8 \Rightarrow x = 2$   
 $x = 2$  in  $\textcircled{2} \Rightarrow p \ln(2) = 4 \Rightarrow p = \frac{4}{\ln(2)}$

$$D14a \quad f(x) = \frac{x+2}{x^2-3x+2} = \frac{-3}{x-1} + \frac{4}{x-2} \text{ (zie de berekening hieronder)} \Rightarrow F(x) = -3\ln|x-1| + 4\ln|x-2| + c.$$

$$f(x) = \frac{x+2}{x^2-3x+2} = \frac{x+2}{(x-1)(x-2)} = \frac{a}{x-1} + \frac{b}{x-2} = \frac{a(x-2)}{(x-1)(x-2)} + \frac{b(x-1)}{(x-1)(x-2)} = \frac{ax-2a+bx-b}{(x-1)(x-2)} = \frac{(a+b)x-2a-b}{(x-1)(x-2)}$$

$$\begin{cases} a+b=1 & \textcircled{1} \\ -2a-b=2 & \textcircled{2} \end{cases}$$

$$-a = 3 \Rightarrow a = -3 \text{ in } \textcircled{1} \Rightarrow -3+b=1 \Rightarrow b=4.$$

$$D14b \quad g(x) = \frac{6x-4}{x^2-4} = \frac{4}{x+2} + \frac{2}{x-2} \text{ (zie de berekening hieronder)} \Rightarrow G(x) = 4\ln|x+2| + 2\ln|x-2| + c.$$

$$g(x) = \frac{6x-4}{x^2-4} = \frac{6x-4}{(x+2)(x-2)} = \frac{a}{x+2} + \frac{b}{x-2} = \frac{a(x-2)}{(x+2)(x-2)} + \frac{b(x+2)}{(x+2)(x-2)} = \frac{ax-2a+bx+2b}{(x+2)(x-2)} = \frac{(a+b)x-2a+2b}{(x+2)(x-2)}$$

$$\begin{cases} a+b=6 & \textcircled{1} \\ -2a+2b=-4 & \textcircled{2} \end{cases} \Rightarrow \begin{cases} 2a+2b=12 & \textcircled{3} \\ -2a+2b=-4 & \textcircled{2} \end{cases}$$

$$4b=8 \Rightarrow b=2 \text{ in } \textcircled{1} \Rightarrow a+2=6 \Rightarrow a=4.$$

$$D15 \quad f_a(x) = ax - x^2 \Rightarrow f'_a(x) = a - 2x \text{ en } g(x) = \frac{1}{2}x^2 \Rightarrow g'(x) = x.$$

$$f'_a(x) = g(x) \wedge f'_a(x) \cdot g'(x) = -1$$

$$ax - x^2 = \frac{1}{2}x^2 \wedge (a-2x) \cdot x = -1$$

$$ax = 1\frac{1}{2}x^2 \Rightarrow a = 1\frac{1}{2}x \textcircled{1} \wedge ax - 2x^2 = -1 \textcircled{2}$$

$$\textcircled{1} \text{ in } \textcircled{2} \Rightarrow 1\frac{1}{2}x^2 - 2x^2 = -1 \Rightarrow -\frac{1}{2}x^2 = -1 \Rightarrow x^2 = 2 \Rightarrow x = \sqrt{2} \vee x = -\sqrt{2}.$$

$$x = \sqrt{2} \text{ in } \textcircled{1} \Rightarrow a = 1\frac{1}{2}\sqrt{2} \text{ en } x = -\sqrt{2} \text{ in } \textcircled{1} \Rightarrow a = -1\frac{1}{2}\sqrt{2}.$$

### Gemengde opgaven 14. Algebraïsche vaardigheden

$$G27a \quad \frac{x^2-5x+4}{x+1} = x+1 \quad (x \neq -1)$$

$$x^2 - 5x + 4 = (x+1)^2$$

$$x^2 - 5x + 4 = (x+1)(x+1)$$

$$x^2 - 5x + 4 = x^2 + 2x + 1$$

$$-7x = -3$$

$$x = \frac{-3}{-7} = \frac{3}{7}.$$

$$G27b \quad (\sin(x)-3)^4 = (2\sin(x)-4)^4$$

$$\sin(x)-3 = 2\sin(x)-4 \vee \sin(x)-3 = -(2\sin(x)-4)$$

$$-\sin(x) = -1 \vee \sin(x)-3 = -2\sin(x)+4$$

$$\sin(x) = 1 \vee 3\sin(x) = 7$$

$$x = \frac{1}{2}\pi + k \cdot 2\pi \vee \sin(x) = \frac{7}{3} > 1 \text{ (geen oplossing).}$$

$$G27c \quad \frac{e^x}{e^x+2} = \frac{e^x}{3e^x-1} \quad (e^x \neq \frac{1}{3})$$

$$e^x(3e^x-1) = 2e^x(e^x+2)$$

$$e^x = 0 \text{ (kan niet)} \vee 3e^x-1 = 2(e^x+2)$$

$$3e^x-1 = 2e^x+4$$

$$e^x = 5$$

$$x = \ln(5).$$

$$G27d \quad (2\ln(x)-1)(3\ln^2(x)-10) = 4\ln(x)-2$$

$$(2\ln(x)-1)(3\ln^2(x)-10) = 2(2\ln(x)-1)$$

$$2\ln(x)-1 = 0 \vee 3\ln^2(x)-10 = 2$$

$$2\ln(x) = 1 \vee 3\ln^2(x) = 12$$

$$\ln(x) = \frac{1}{2} \vee \ln^2(x) = 4$$

$$x = e^{\frac{1}{2}} \vee \ln(x) = 2 \vee \ln(x) = -2$$

$$x = \sqrt{e} \vee x = e^2 \vee x = e^{-2}$$

$$x = \sqrt{e} \vee x = e^2 \vee x = \frac{1}{e^2}.$$



$$G28a \quad y = \frac{10^x}{10^x-1} \left( 10^x - \frac{1}{10^x} \right) = \frac{10^x \cdot 10^x}{10^x-1} - \frac{10^x}{(10^x-1)10^x} = \frac{10^{2x}}{10^x-1} - \frac{1}{10^x-1} = \frac{10^{2x}-1}{10^x-1} = \frac{(10^x+1)(10^x-1)}{10^x-1} = 10^x + 1.$$

$$G28b \quad y = \frac{10 - \frac{2x}{x+3}}{5 + \frac{2}{x+3}} = \frac{10 - \frac{2x}{x+3}}{\frac{5(x+3) + 2}{x+3}} \cdot \frac{x+3}{x+3} = \frac{10(x+3) - 2x}{5(x+3) + 2} = \frac{10x+30-2x}{5x+15+2} = \frac{8x+30}{5x+17}.$$

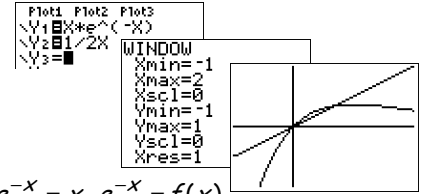
$$G28c \quad p = \frac{\frac{2}{a} - \frac{3}{a+1}}{1-a} = \frac{\frac{2}{a} - \frac{3}{a+1}}{1-a} \cdot \frac{a(a+1)}{a(a+1)} = \frac{2(a+1) - 3a}{a(a+1)(1-a)} = \frac{2a+2-3a}{a(a+1)(1-a)} = \frac{2-a}{a(a+1)(1-a)}.$$

$$G28d \quad p = \frac{\frac{5}{a} + \frac{6}{b}}{2b - \frac{3}{a}} = \frac{\frac{5}{a} + \frac{6}{b}}{2b - \frac{3}{a}} \cdot \frac{ab}{ab} = \frac{5b+6a}{2ab^2-3b}.$$

$$G29a \quad f(x) = x \cdot e^{-x} \Rightarrow f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1) = 1 \cdot e^{-x} - x \cdot e^{-x} = (1-x)e^{-x}.$$

$$f'(x) = 0 \Rightarrow (1-x)e^{-x} = 0 \Rightarrow 1-x = 0 \Rightarrow -x = -1 \Rightarrow x = 1.$$

maximum (zie een plot)  $f(1) = 1 \cdot e^{-1} = e^{-1} = \frac{1}{e}.$



$$G29b \quad F(x) = (-x-1) \cdot e^{-x} \Rightarrow F'(x) = -1 \cdot e^{-x} + (-x-1) \cdot e^{-x} \cdot (-1) = -e^{-x} + x \cdot e^{-x} + e^{-x} = x \cdot e^{-x} = f(x).$$

$$G29c \quad f(x) = ax$$

$$x \cdot e^{-x} = ax$$

$$x=0 \vee e^{-x} = a$$

$$x=0 \vee -x = \ln(a)$$

$$x=0 \vee x = -\ln(a).$$

$$O(V) = \int_0^{-\ln(a)} (f(x) - ax) dx = \left[ (-x-1)e^{-x} - \frac{1}{2}ax^2 \right]_0^{-\ln(a)}$$

$$= (\ln(a)-1)e^{\ln(a)} - \frac{1}{2}a(-\ln(a))^2 - (-1e^0 - 0)$$

$$= (\ln(a)-1) \cdot a - \frac{1}{2}a \cdot \ln^2(a) + 1$$

$$= a \ln(a) - a - \frac{1}{2}a \cdot \ln^2(a) + 1.$$

$$G30a \quad \frac{a+b}{b+2} = \frac{3}{a}$$

$$a(a+b) = 3(b+2)$$

$$a^2 + ab = 3b + 6$$

$$ab - 3b = -a^2 + 6$$

$$b(a-3) = -a^2 + 6$$

$$b = \frac{-a^2 + 6}{a-3}.$$

$$G30b \quad \frac{3x+2}{x-1} = \frac{6y+1}{y+3}$$

$$(3x+2)(y+3) = (x-1)(6y+1)$$

$$3xy + 9x + 2y + 6 = 6xy + x - 6y - 1$$

$$-3xy + 8y = -8x - 7$$

$$3xy - 8y = 8x + 7 \quad \textcircled{1}$$

$$y(3x-8) = 8x+7$$

$$y = \frac{8x+7}{3x-8}$$

Uit  $\textcircled{1}$  volgt ook  $3xy - 8x = 8y + 7$

$$x(3y-8) = 8y+7$$

$$x = \frac{8y+7}{3y-8}.$$

$$G31a \quad f(x) = (2x+1) \cdot \sqrt{x^2+1} \Rightarrow f'(x) = 2 \cdot \sqrt{x^2+1} + (2x+1) \cdot \frac{1}{2 \cdot \sqrt{x^2+1}} \cdot 2x = 2\sqrt{x^2+1} + \frac{x(2x+1)}{\sqrt{x^2+1}}$$

$$= 2\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} + \frac{x(2x+1)}{\sqrt{x^2+1}} = \frac{2(x^2+1)}{\sqrt{x^2+1}} + \frac{x(2x+1)}{\sqrt{x^2+1}} = \frac{2x^2+2+2x^2+x}{\sqrt{x^2+1}} = \frac{4x^2+x+2}{\sqrt{x^2+1}}.$$

$$G31b \quad G(x) = (px+q) \cdot \sqrt{x^2+1} \Rightarrow G'(x) = p \cdot \sqrt{x^2+1} + (px+q) \cdot \frac{1}{2 \cdot \sqrt{x^2+1}} \cdot 2x = p\sqrt{x^2+1} + \frac{x(px+q)}{\sqrt{x^2+1}}$$

$$= p\sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} + \frac{x(px+q)}{\sqrt{x^2+1}} = \frac{p(x^2+1)}{\sqrt{x^2+1}} + \frac{x(px+q)}{\sqrt{x^2+1}} = \frac{px^2+p+px^2+qx}{\sqrt{x^2+1}} = \frac{2px^2+qx+p}{\sqrt{x^2+1}}.$$

$G'(x) = g(x)$  geeft  $2p=2 \wedge q=1 \wedge p=1.$   
 $p=1 \wedge q=1.$

$$G31c \quad \int_0^{\sqrt{3}} g(x) dx = \left[ (x+1) \cdot \sqrt{x^2+1} \right]_0^{\sqrt{3}} = (\sqrt{3}+1) \cdot \sqrt{3+1} - 1 \cdot \sqrt{1} = (\sqrt{3}+1) \cdot 2 - 1 = 2\sqrt{3} + 2 - 1 = 2\sqrt{3} + 1.$$

$$G32a \quad N = 2^{3t} \cdot 3^{2t-1}$$

$$= 2^{3t} \cdot 3^{2t} \cdot 3^{-1}$$

$$= (2^3)^t \cdot (3^2)^t \cdot \frac{1}{3}$$

$$= \frac{1}{3} \cdot 8^t \cdot 9^t$$

$$= \frac{1}{3} \cdot (8 \cdot 9)^t$$

$$= \frac{1}{3} \cdot 72^t.$$

$$G32b \quad N = \frac{3^{2t-1}}{5^{t+1}}$$

$$= 3^{2t-1} \cdot 5^{-t-1}$$

$$= 3^{2t} \cdot 3^{-1} \cdot 5^{-t} \cdot 5^{-1}$$

$$= (3^2)^t \cdot \frac{1}{3} \cdot (5^{-1})^t \cdot \frac{1}{5}$$

$$= \frac{1}{15} \cdot 9^t \cdot \left(\frac{1}{5}\right)^t$$

$$= \frac{1}{15} \cdot \left(9 \cdot \frac{1}{5}\right)^t$$

$$= \frac{1}{15} \cdot \left(\frac{9}{5}\right)^t.$$

$$G32c \quad N = 15 \cdot 3^{0,6t+1} = 15 \cdot 3^{0,6t} \cdot 3^1$$

$$45 \cdot 3^{0,6t} = N$$

$$3^{0,6t} = \frac{1}{45} N$$

$$\ln(3^{0,6t}) = \ln\left(\frac{1}{45} N\right)$$

$$0,6t \cdot \ln(3) = \ln\left(\frac{1}{45}\right) + \ln(N)$$

$$t = \frac{\ln\left(\frac{1}{45}\right) + \ln(N)}{0,6 \ln(3)} = \frac{\ln(N)}{0,6 \ln(3)} - \frac{\ln(45)}{0,6 \ln(3)}$$

$$t \approx -5,77 + 1,52 \ln(N).$$

$\frac{\ln(1/45)}{0,6 \ln(3)}$	$\frac{\ln(N)}{0,6 \ln(3)}$
-5,774955868	$1 \times 0,61 \ln(3)$
	1,517065378

G33  $\square$  Stel  $x_D = p (> 0)$  dan is  $x_B = 3p$  (omdat  $CD : DB = 1 : 2$ ).  
 $y_D = y_B$  geeft dan  $f_a(p) = f_a(3p)$ .

$$\frac{p}{2} + \frac{a}{p} = \frac{3p}{2} + \frac{a}{3p} \Rightarrow \frac{a}{p} - \frac{a}{3p} = \frac{3p}{2} - \frac{p}{2} \Rightarrow \frac{2a}{3p} = \frac{2p}{2} \Rightarrow \frac{2a}{3p} = \frac{p}{1} \Rightarrow 3p^2 = 2a \Rightarrow p^2 = \frac{2}{3}a$$

$$O(ABCD) = OA \cdot AB = 3p \cdot f_a(3p) = 3p \cdot f_a(p) = 3p \cdot \left(\frac{p}{2} + \frac{a}{p}\right) = \frac{3}{2}p^2 + 3a \Rightarrow O(ABCD) = \frac{3}{2} \cdot \frac{2}{3}a + 3a = 4a.$$

G34a  $\square$   $L = 80$  geeft  $T = -2,57 \ln\left(\frac{87-80}{63}\right) = -2,57 \ln\left(\frac{7}{63}\right) = -2,57 \ln\left(\frac{1}{9}\right) \approx 5,65$  (jaar).  
De leeftijd is ongeveer 5 jaar en 8 maanden.

```
-2,57ln((87-80)/63)
Ans=-5,646867164
Ans*12
7,762405965
```

```
10^-2,57
3,891050584
e^(Ans)
0,204238778
Ans*63
1,2867043
```

G34b  $\square$   $T = 10$  geeft  $10 = -2,57 \ln\left(\frac{87-L}{63}\right)$  (algebraïsch of intersect)  $\Rightarrow L \approx 85,7$  (feet).  
Dus de lengte is ongeveer  $\text{Ans} \cdot 0,314 \approx 26,9$  meter.

```
Ans-87
-85,7132957
Ans*-1
85,7132957
Ans*0,314
26,91397485
```

G34c  $\square$   $T = -2,57 \ln\left(\frac{87-L}{63}\right)$   
 $\frac{1}{-2,57} T = \ln\left(\frac{87-L}{63}\right)$   
 $\frac{87-L}{63} = e^{-\frac{1}{2,57} T}$   
 $87-L = 63e^{-\frac{1}{2,57} T}$   
 $-L = -87 + 63e^{-\frac{1}{2,57} T}$   
 $L \approx 87 - 63e^{-0,39T}$

```
-1/2,57
-0,3891050584
```

G35  $\square$  Loodrecht snijden geeft:  $f(x) = g_p(x) \wedge f'(x) \cdot g_p'(x) = -1$ .

$$f(x) = x^2 - 8 \Rightarrow f'(x) = 2x \text{ en } g_p(x) = \sqrt{2x+p} \Rightarrow g_p'(x) = \frac{1}{2 \cdot \sqrt{2x+p}} \cdot 2 = \frac{1}{\sqrt{2x+p}}$$

$$x^2 - 8 = \sqrt{2x+p} \wedge 2x \cdot \frac{1}{\sqrt{2x+p}} = -1$$

$$\sqrt{2x+p} = x^2 - 8 \text{ invullen in } 2x \cdot \frac{1}{\sqrt{2x+p}} = -1 \text{ geeft } 2x \cdot \frac{1}{x^2 - 8} = -1 \Rightarrow$$

$$2x = -(x^2 - 8) \Rightarrow x^2 + 2x - 8 = 0 \Rightarrow (x+4)(x-2) = 0 \Rightarrow x = -4 \vee x = 2.$$

$$x = -4 \text{ en } \sqrt{2x+p} = x^2 - 8 \text{ geeft } \sqrt{-8+p} = 8 \Rightarrow -8+p = 64 \Rightarrow p = 72.$$

$$(x = 2 \text{ en } \sqrt{2x+p} = x^2 - 8 \text{ geeft } \sqrt{4+p} = -4 \text{ heeft geen oplossing})$$

G36a  $\square$   $b = 0,018$  en  $S = 58 \Rightarrow 58 = \frac{a+0,018}{a \cdot 0,018} \Rightarrow 58 \cdot 0,018a = a + 0,018 \Rightarrow (1,044 - 1)a = 0,018 \Rightarrow a \approx 0,41$ .

$b = 0,018$  en  $S = 63 \Rightarrow 63 = \frac{a+0,018}{a \cdot 0,018} \Rightarrow 63 \cdot 0,018a = a + 0,018 \Rightarrow (1,044 - 1)a = 0,018 \Rightarrow a \approx 0,13$ .

Het rechthoek kan tussen 0,13 meter en 0,41 meter scherp zien.

G36b  $\square$   $S = \frac{a+b}{a \cdot b} \Rightarrow abS = a + b \Rightarrow abS - a = b \Rightarrow a(bS - 1) = b \Rightarrow a = \frac{b}{bS - 1}$ .

G36c  $\square$   $b = 0,017$  en  $a = 0,15 \Rightarrow S = \frac{0,15+0,017}{0,15 \cdot 0,017} \approx 65,49$ .

$$b = 0,017 \Rightarrow S = \frac{a+0,017}{a \cdot 0,017} = \frac{a}{a \cdot 0,017} + \frac{0,017}{a \cdot 0,017} = \frac{1}{0,017} + \frac{1}{a}$$

Voor grote waarden van  $a$  nadert  $S = \frac{1}{0,017} + \frac{1}{a}$  naar  $S = \frac{1}{0,017} \approx 58,82$ .

Dus  $S$  kan de waarden 59 tot en met 65 aannemen.

```
(0,15+0,017)/(0,15*0,017)
65,49019608
1/0,017
58,82352941
```

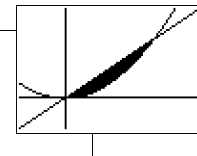
```
58*0,018
1,044
Ans-1
0,044
0,018/Ans
0,4090909091
63*0,018
1,134
Ans-1
0,134
0,018/Ans
0,1343283582
```

G37a  $\square$   $\frac{1}{2}x^2 = x \Rightarrow x = 0 \vee \frac{1}{2}x = 1 \Rightarrow x = 0 \vee x = 2$ .

$$I = I_{\text{kegel}} - \int_0^2 \pi \left(\frac{1}{2}x^2\right)^2 dx = \frac{1}{3}\pi \cdot 2^2 \cdot 2 - \int_0^2 \frac{1}{4}\pi x^4 dx = \frac{8}{3}\pi - \left[\frac{1}{20}\pi x^5\right]_0^2$$

$$= \frac{8}{3}\pi - \left(\frac{1}{20}\pi \cdot 2^5 - 0\right) = \frac{8}{3}\pi - \frac{32}{20}\pi = 1\frac{1}{15}\pi$$

```
Plot1 Plot2 Plot3
V1=1/2x^2
V2=x
V3=
-2
8/3-32/20>Frac
16/15
```



G37b  $\square$  Omdat  $OP_n Q_n R_n$  een vierkant is, geldt:  $\frac{1}{n}x^2 = x \Rightarrow x = 0 \vee \frac{1}{n}x = 1 \Rightarrow x = 0$  (in de oorsprong)  $\vee x = n$  (in  $P_n$ ).

De richtingscoëfficiënt van de raaklijn in  $P_n$  is  $\left[\frac{dy}{dx}\right]_{x=n} = \left[\frac{2}{n} \cdot x\right]_{x=n} = \frac{2}{n} \cdot n = 2$  en dit is onafhankelijk van  $n$ .

G38a  $\square$   $100 = 200 - 180 \cdot e^{-0,29t} \Rightarrow 180 \cdot e^{-0,29t} = 100 \Rightarrow e^{-0,29t} = \frac{100}{180} \Rightarrow -0,29t = \ln\left(\frac{100}{180}\right) \Rightarrow t \approx 2,027$ .

Het opwarmen wordt gestopt na 2 uur en 2 minuten, dus om 17:02.

G38b  $\square$   $S(t) = 200 - 180 \cdot e^{-0,29t} \Rightarrow S'(t) = -180 \cdot e^{-0,29t} \cdot -0,29 = 52,2 \cdot e^{-0,29t}$ .

Om 16:00 is  $t = 1$  en  $S'(1) = 52,2 \cdot e^{-0,29} \approx 39,06$  ( $^{\circ}\text{C}/\text{uur}$ ).

Dit is een snelheid van ongeveer 0,7 graden Celcius per minuut.

```
-180*-0,29
52,2
Ans*e^(-0,29)
39,05935823
Ans/60
0,6509893038
```

```
100/180
0,5555555556
ln(Ans)
-0,5877866649
Ans/-0,29
2,026850569
<Ans-2>*60
```

G38c  $\square$   $S = 200 - 180 \cdot e^{-0,29t}$

$$180 \cdot e^{-0,29t} = 200 - S$$

$$e^{-0,29t} = \frac{200-S}{180}$$

$$-0,29t = \ln\left(\frac{200-S}{180}\right)$$

$$t = -\frac{1}{0,29} \cdot \ln\left(\frac{200-S}{180}\right) \approx -3,45 \cdot \ln\left(\frac{200-S}{180}\right)$$

```
-1/0,29
-3,448275862
```