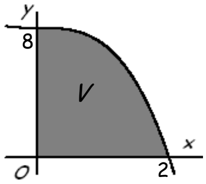
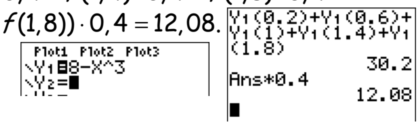
1 De tweede benadering is de beste.

2 Omdat de grafiek van f dalend is, is het minimum van f op elk interval de functiewaarde in de rechtergrens van het interval. En zo is het maximum van f op elk interval de functiewaarde in de linkergrens van het interval.

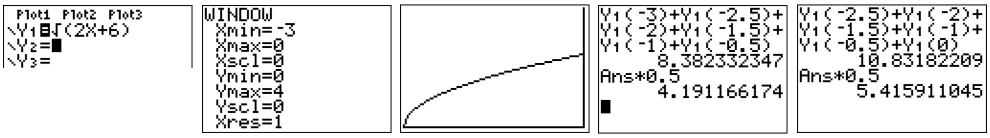
3 $y = 0$ (de x-as)
$f(x) = 0$
$8 - x^3 = 0$
$x^3 = 8$
$x = 2.$



De middens van de intervallen zijn 0,2; 0,6; 1; 1,4 en 1,8.
$O(V) \approx f(0{,}2)\cdot 0{,}4 + f(0{,}6)\cdot 0{,}4 + f(1)\cdot 0{,}4 + f(1{,}4)\cdot 0{,}4 + f(1{,}8)\cdot 0{,}4$
$= (f(0{,}2) + f(0{,}6) + f(1) + f(1{,}4) + f(1{,}8))\cdot 0{,}4 = 12{,}08.$

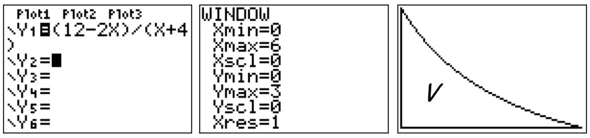


4 $f(x) = 0$
$\sqrt{2x+6} = 0$
$2x + 6 = 0$
$2x = -6$
$x = -3.$

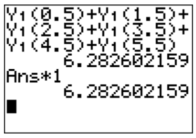


ondersom $= (f(-3) + f(-2{,}5) + f(-2) + f(-1{,}5) + f(-1) + f(-0{,}5)) \times 0{,}5 \approx 4{,}19.$
bovensom $= (f(-2{,}5) + f(-2) + f(-1{,}5) + f(-1) + f(-0{,}5) + f(0)) \times 0{,}5 \approx 5{,}42.$ Dus $4{,}19 \le O(V) \le 5{,}42.$

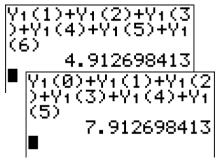
5a $y = 0$ (de x-as)
$f(x) = 0$
$\frac{12-2x}{x+4} = 0$ (teller = 0)
$12 = 2x$
$x = 6.$



De middens van de intervallen zijn 0,5; 1,5; 2,5; 3,5; 4,5 en 5,5.
$O(V) \approx (f(0{,}5) + f(1{,}5) + f(2{,}5) + f(3{,}5) + f(4{,}5) + f(5{,}5))\cdot 1 \approx 6{,}28.$



5b ondersom $= (f(1) + f(2) + f(3) + f(4) + f(5) + f(6))\cdot 1 \approx 4{,}91.$
bovensom $= (f(0) + f(1) + f(2) + f(3) + f(4) + f(5))\cdot 1 \approx 7{,}91.$ Dus $4{,}91 \le O(V) \le 7{,}91.$

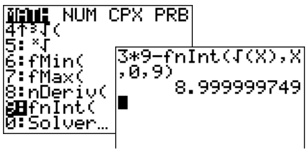


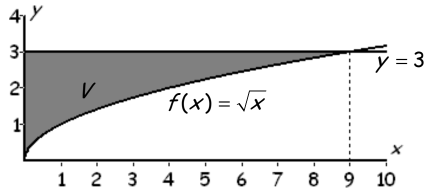
6a De oppervlakte van de blauwe rechthoek rechts wordt gehalveerd.

6b De oppervlakte van de blauwe rechthoek rechts is het verschil van de ondersom en de bovensom. De breedte van deze rechhoek is Δx. Als $\Delta x \to 0$ dan nadert de oppervlakte van deze blauwe rechthoek naar 0 en nadert het verschil van de ondersom en de bovensom dus ook naar 0.

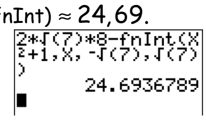
*** **Neem GR - practicum 10 door.** (zie aan het eind van deze uitwerkingen)

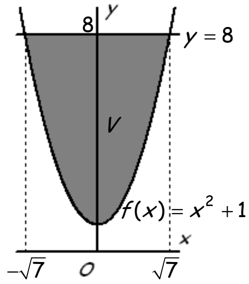
7 $f(x) = 3 \Rightarrow \sqrt{x} = 3$ (kwadrateren) $\Rightarrow x = 9.$
$O(V) = 3\cdot 9 - \int_0^9 \sqrt{x}\,dx$ (fnInt) $\approx 9{,}00.$



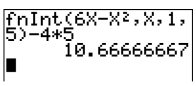


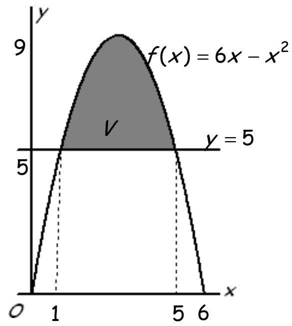
8 $f(x) = 8 \Rightarrow x^2 + 1 = 8$
$x^2 = 7$
$x = \pm\sqrt{7}.$
$O(V) = 2\cdot\sqrt{7}\cdot 8 - \int_{-\sqrt{7}}^{\sqrt{7}} (x^2+1)\,dx$ (fnInt) $\approx 24{,}69.$





9 $f(x) = 5 \Rightarrow 6x - x^2 = 5$
$-x^2 + 6x - 5 = 0$
$x^2 - 6x + 5 = 0$
$(x-1)\cdot(x-5) = 0$
$x = 1 \vee x = 5.$
$O(V) = \int_1^5 (6x - x^2)\,dx$ (fnInt) $- 4\cdot 5 \approx 10{,}67.$





10a $f(x)=1 \Rightarrow x^3-5x^2+6x+1=1$

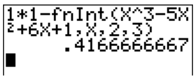
$x^3-5x^2+6x=0$

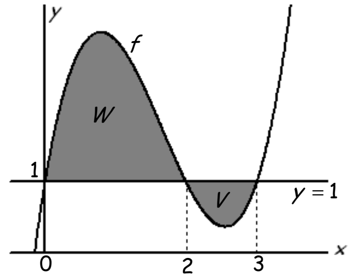
$x\cdot(x^2-5x+6)=0$

$x\cdot(x-2)\cdot(x-3)=0$

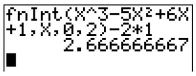
$x=0 \vee x=2 \vee x=3.$

$O(V)=1\cdot 1-\int_2^3 (x^3-5x^2+6x+1)\,dx$ (fnInt) $\approx 0{,}42.$





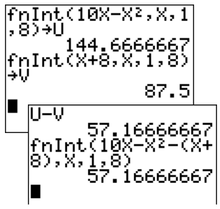
10b $O(W)=\int_0^2 (x^3-5x^2+6x+1)\,dx$ (fnInt) $-2\cdot 1\approx 2{,}67.$



11a $O(U)=\int_1^8 f(x)\,dx=\int_1^8 (10x-x^2)\,dx$ (fnInt) $\approx 144{,}67$ en $O(V)=\int_1^8 g(x)\,dx=\int_1^8 (x+8)\,dx$ (fnInt) $=87{,}5.$

11b $O(W)=O(U)-O(V)=\int_1^8 f(x)\,dx-\int_1^8 g(x)\,dx$ (zie 11a) $\approx 57{,}17.$

11c $\int_1^8 f(x)\,dx-\int_1^8 g(x)\,dx=\int_1^8 (f(x)-g(x))\,dx.$ (zie figuur 10.5)

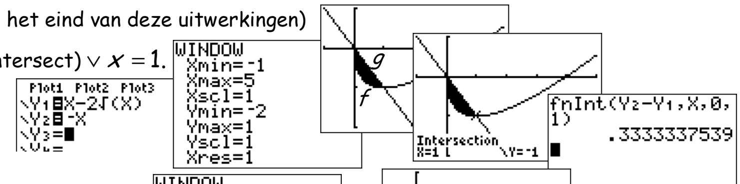


✻✻✻ **Neem GR - practicum 11 door.** (zie aan het eind van deze uitwerkingen)

12a $f(x)=g(x)$ (intersect) $\Rightarrow x=0$ (lukt niet met intersect) $\vee\ x=1.$

Op $[0,1]$ (zie plot) is $g(x)\geq f(x)$, dus

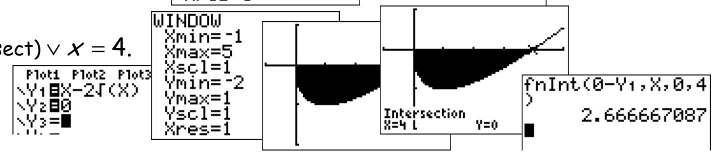
$O(V)=\int_0^1 (g(x)-f(x))\,dx$ (fnInt) $\approx 0{,}33.$



12b $f(x)=0$ (intersect) $\Rightarrow x=0$ (lukt niet met intersect) $\vee\ x=4.$

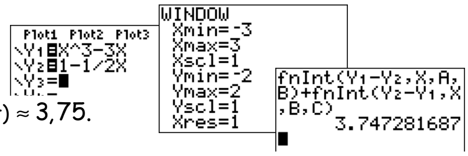
Op $[0,4]$ (zie plot) is $f(x)\leq 0$, dus

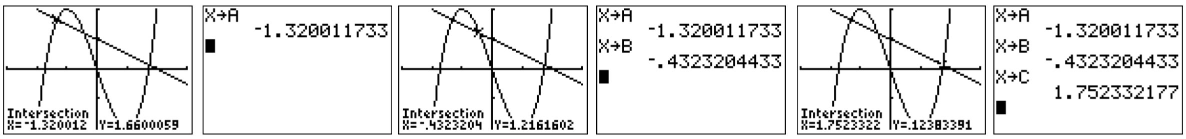
$O(V)=\int_0^4 (0-f(x))\,dx$ (fnInt) $\approx 2{,}67.$



13 $f(x)=g(x)$ (intersect) $\Rightarrow x\approx -1{,}32 \vee x\approx -0{,}43 \vee x\approx 1{,}75.$

$O(V)=\int_{-1{,}32}^{-0{,}43} (f(x)-g(x))\,dx+\int_{-0{,}43}^{1{,}75} (g(x)-f(x))\,dx$ (fnInt) $\approx 3{,}75.$



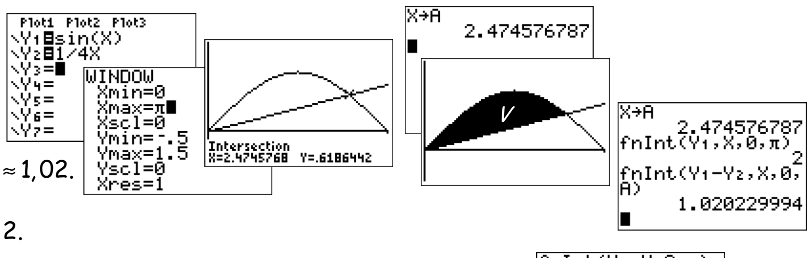


14 $f(x)=g(x)$ (intersect) $\Rightarrow x\approx 2{,}47.$

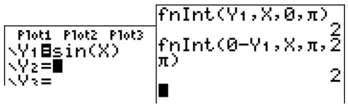
$\int_0^{\pi} f(x)\,dx$ (fnInt) $=2$ en

$O(V)=\int_0^{2{,}47} (f(x)-g(x))\,dx$ (fnInt) $\approx 1{,}02.$

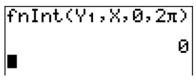
Het is dus niet waar want $1{,}02\neq \frac{1}{2}\cdot 2.$



15a $O(V)=\int_0^{\pi} \sin(x)\,dx$ (fnInt) $=2$ en $O(W)=\int_{\pi}^{2\pi} (0-\sin(x))\,dx$ (fnInt) $=2.$

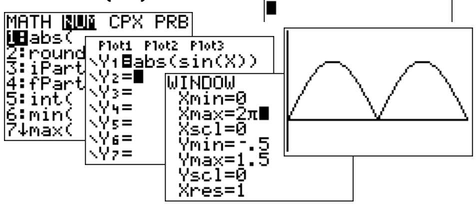


15b $\int_0^{2\pi} \sin(x)\,dx$ (fnInt) $=0.$ $\int_0^{2\pi} \sin(x)\,dx=\int_0^{\pi} \sin(x)\,dx+\int_{\pi}^{2\pi} \sin(x)\,dx=2+(-2)=0.$



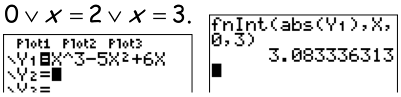
De integraal is negatief als de grafiek onder de x-as ligt.

15c $\int_0^{2\pi} |\sin(x)|\,dx$ (fnInt) $=4.$ Dus $O(V)+O(W)=\int_0^{2\pi} |\sin(x)|\,dx.$



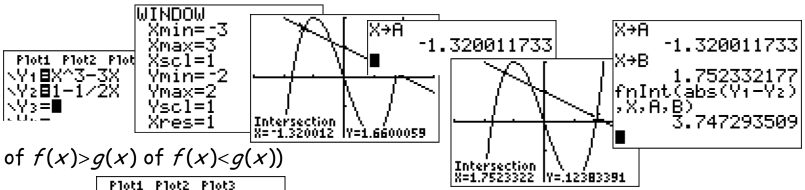
16 $f(x)=0$ (intersect of) $\Rightarrow x\cdot(x^2-5x+6)=0 \Rightarrow x\cdot(x-2)\cdot(x-3)=0 \Rightarrow x=0 \vee x=2 \vee x=3$.

$O(V)+O(W)=\int_0^3 |f(x)|\,dx$ (fnInt) $\approx 3{,}08$.



17 $O=\int_{-1{,}32}^{1{,}75} |f(x)-g(x)|\,dx$ (fnInt) $\approx 3{,}75$.

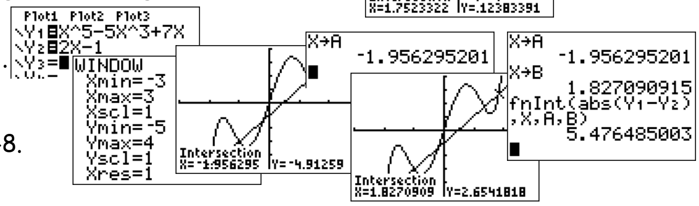
(vanwege de absolute waarde maakt het niet uit of $f(x)>g(x)$ of $f(x)<g(x)$)



18 $f(x)=g(x)$ (intersect) $\Rightarrow x\approx -1{,}96 \vee ... \vee x\approx 1{,}83$.

(alleen de buitenste snijpunten zijn van belang)

$O(V)+O(W)=\int_{-1{,}96}^{1{,}83} |f(x)-g(x)|\,dx$ (fnInt) $\approx 5{,}48$.

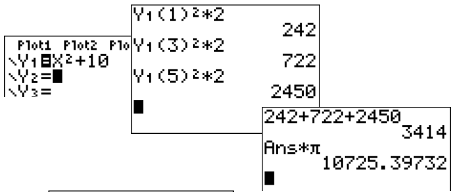


19a Op [0,2]: $I_{\text{cilinder}}=\pi\cdot r^2\cdot h=\pi\cdot f(1)^2\cdot 2=\pi\cdot 11^2\cdot 2=242\pi$.

Op [2,4]: $I_{\text{cilinder}}=\pi\cdot r^2\cdot h=\pi\cdot f(3)^2\cdot 2=\pi\cdot 19^2\cdot 2=722\pi$.

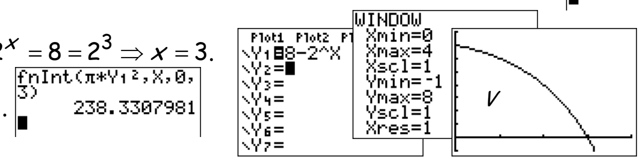
Op [4,6]: $I_{\text{cilinder}}=\pi\cdot r^2\cdot h=\pi\cdot f(5)^2\cdot 2=\pi\cdot 35^2\cdot 2=2\,450\pi$.

19b $I_{3\text{ cilinders}}=242\pi+722\pi+2\,450\pi=3\,414\pi\approx 10\,725$.



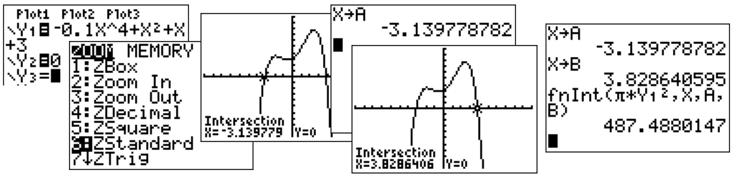
20 $f(x)=0$ (intersect of) $\Rightarrow 8-2^x=0 \Rightarrow 2^x=8=2^3 \Rightarrow x=3$.

$I(L)=\int_0^3 \pi\cdot(f(x))^2\,dx$ (fnInt) $\approx 238{,}33$.



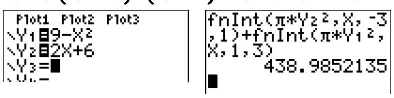
21 $f(x)=0$ (intersect) $\Rightarrow x=-3{,}14... \vee x=3{,}83...$

$I(L)=\int_{-3{,}14...}^{3{,}83...} \pi\cdot(f(x))^2\,dx$ (fnInt) $\approx 487{,}49$.



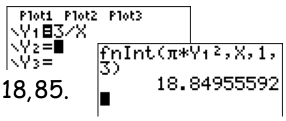
22 $f(x)=2x+6$ (intersect of) $\Rightarrow 9-x^2=2x+6 \Rightarrow x^2+2x-3=0 \Rightarrow (x+3)\cdot(x-1)=0 \Rightarrow x=-3 \vee x=1$.

$I(L)=\int_{-3}^{1} \pi\cdot(2x+6)^2\,dx+\int_1^3 \pi\cdot(f(x))^2\,dx$ (fnInt) $\approx 438{,}99$.



23a $g(x)=f(x)-2=2+\frac{3}{x}-2=\frac{3}{x}$.

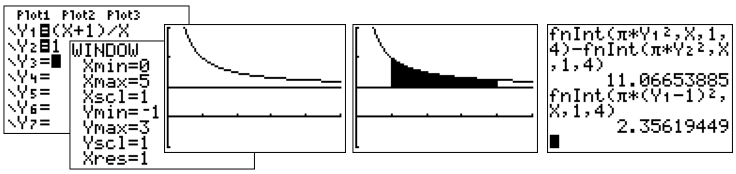
23b $I(M)=\int_1^3 \pi\cdot(g(x))^2\,dx$ (fnInt) $\approx 18{,}85$.



Wentelen van V om de lijn $y=2$ levert een lichaam met dezelfde inhoud op als wentelen van W om de x-as, omdat V en de lijn $y=2$ beide 2 omlaag zijn verschoven.

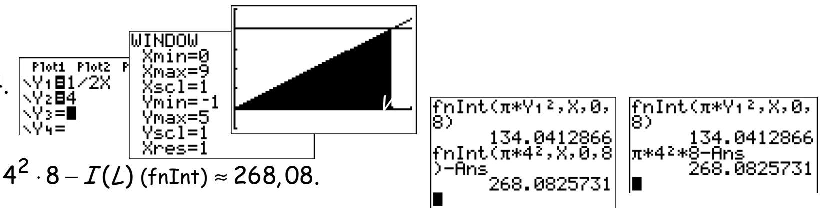
24a $I=\int_1^4 \pi\cdot(f(x))^2\,dx-\int_1^4 \pi\cdot 1^2\,dx$ (fnInt) $\approx 11{,}07$.

24b $I=\int_1^4 \pi\cdot(f(x)-1)^2\,dx$ (fnInt) $\approx 2{,}36$.



25a $I(L)=\int_0^8 \pi\cdot(\frac{1}{2}x)^2\,dx$ (fnInt) $\approx 134{,}04$.

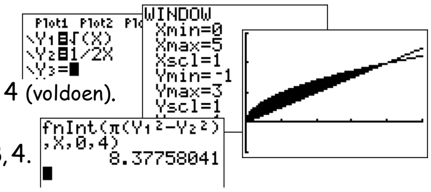
25b $I(M)=\int_0^8 \pi\cdot 4^2\,dx-I(L)$ (fnInt) $=\pi\cdot 4^2\cdot 8-I(L)$ (fnInt) $\approx 268{,}08$.



26 $I(L)=\pi\cdot 2^2\cdot 5-\pi\cdot 1^2\cdot 5=20\pi-5\pi=15\pi$.

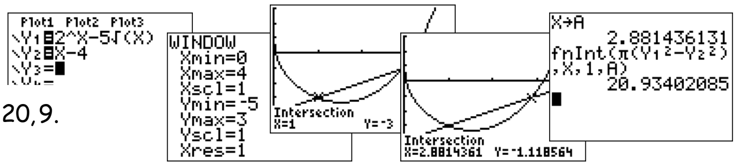
27 $f(x)=g(x)$ (intersect of) $\Rightarrow \sqrt{x}=\frac{1}{2}x \Rightarrow x=\frac{1}{4}x^2 \Rightarrow x^2=4x \Rightarrow x=0 \vee x=4$ (voldoen).

$I(L)=\int_0^4 \pi\cdot(f(x))^2\,\mathrm{d}x-\int_0^4 \pi\cdot(g(x))^2\,\mathrm{d}x=\int_0^4 \pi\cdot(f(x)^2-g(x)^2)\,\mathrm{d}x$ (fnInt) $\approx 8,4$.



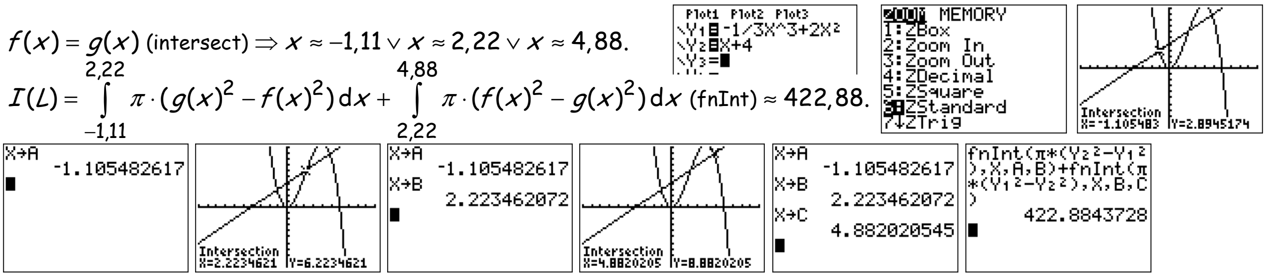
28 $f(x)=g(x)$ (intersect) $\Rightarrow x=1 \vee x\approx 2,88$.

$I(L)=\int_1^{2,88} \pi\cdot(f(x)^2-g(x)^2)\,\mathrm{d}x$ (fnInt) $\approx 20,9$.



29 $f(x)=g(x)$ (intersect) $\Rightarrow x\approx -1,11 \vee x\approx 2,22 \vee x\approx 4,88$.

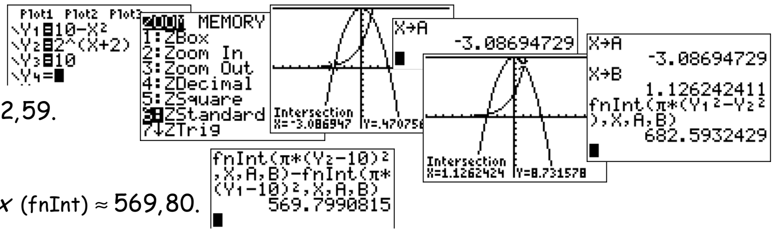
$I(L)=\int_{-1,11}^{2,22} \pi\cdot(g(x)^2-f(x)^2)\,\mathrm{d}x+\int_{2,22}^{4,88} \pi\cdot(f(x)^2-g(x)^2)\,\mathrm{d}x$ (fnInt) $\approx 422,88$.



30a $f(x)=g(x)$ (intersect) $\Rightarrow x\approx -3,09 \vee x\approx 1,13$.

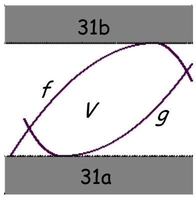
$I(L)=\int_{-3,09}^{1,13} \pi\cdot(f(x)^2-g(x)^2)\,\mathrm{d}x$ (fnInt) $\approx 682,59$.

30b $I(M)=\int_{-3,09}^{1,13} \pi\cdot((g(x)-10)^2-(f(x)-10)^2)\,\mathrm{d}x$ (fnInt) $\approx 569,80$.



31a Er moet gelden $f(x)\ge g(x)\ge 0$ voor x op $[x_A,x_B]$,
dus het laagste punt van g ligt op of boven de x-as.

31b Er moet gelden $g(x)\le f(x)\le 0$ voor x op $[x_A,x_B]$,
dus het hoogste punt van f ligt op of onder de x-as.

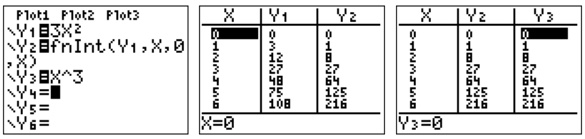


32a $O(p)=O(\text{rechthoek})+O(\text{driehoek})=p\cdot b+\frac{1}{2}p\cdot(ap+b-b)=p\cdot b+\frac{1}{2}p\cdot ap=\frac{1}{2}ap^2+bp$.

32b $O(p)=\frac{1}{2}ap^2+bp \Rightarrow \frac{\mathrm{d}O}{\mathrm{d}p}=O'(p)=ap+b$. Dus $\frac{\mathrm{d}O}{\mathrm{d}p}=O'(p)=f(p)$.

33a

p	1	2	3	4	5
$O(p)$	1	8	27	64	125



33b Voor elke p uit de tabel geldt $O(p)=p^3$.
(vermoedelijk is dat voor andere waarden van p ook waar)

33c $O(p)=p^3 \Rightarrow \frac{\mathrm{d}O}{\mathrm{d}p}=O'(p)=3p^2$. Dus $\frac{\mathrm{d}O}{\mathrm{d}p}=O'(p)=f(p)$.

33d $O(p)=\int_0^p f(x)\,\mathrm{d}x=10 \Rightarrow p^3=10 \Rightarrow p=\sqrt[3]{10}$.

34a $F(x)=(\boxed{x^2+1})^6+1 \Rightarrow F'(x)=6\cdot(x^2+1)^5\cdot 2x=12x(x^2+1)^5$.
Dus $F'(x)=f(x)$ ofwel F is een primitieve van f.

34b $G(x)=(\frac{1}{2}x-\frac{1}{4})\cdot e^{\boxed{2x}}-2 \Rightarrow G'(x)=\frac{1}{2}\cdot e^{2x}+(\frac{1}{2}x-\frac{1}{4})\cdot e^{2x}\cdot 2=(\frac{1}{2}+x-\frac{1}{2})\cdot e^{2x}=x\cdot e^{2x}$.
Dus $G'(x)=g(x)$ ofwel G is een primitieve van g.

34c $H(x)=2\ln(x)+\ln^2(x)+3=2\ln(x)+(\boxed{\ln(x)})^2+3 \Rightarrow H'(x)=2\cdot\frac{1}{x}+2\ln(x)\cdot\frac{1}{x}=\frac{2+2\ln(x)}{x}$.
Dus $H'(x)=h(x)$ ofwel H is een primitieve van h.

34d $J(x)=\frac{e^{\boxed{3x}}-10}{2e^x}-4 \Rightarrow J'(x)=\frac{2e^x\cdot e^{3x}\cdot 3-(e^{3x}-10)\cdot 2e^x}{(2e^x)^2}=\frac{3e^{3x}-e^{3x}+10}{2e^x}=\frac{2e^{3x}+10}{2e^x}=\frac{e^{3x}+5}{e^x}$.
Dus $J'(x)=j(x)$ ofwel J is een primitieve van j.

35a $F(x)=\frac{1}{5}x^5 \Rightarrow F'(x)=f(x)=\frac{1}{5}\cdot 5x^4=x^4$. Dus $F(x)=\frac{1}{5}x^5$ is een primitieve van $f(x)=x^4$.

35b $G(x)=\frac{1}{4}e^{\boxed{4x+1}} \Rightarrow G'(x)=g(x)=\frac{1}{4}e^{4x+1}\cdot 4=e^{4x+1}$. Dus $G(x)=\frac{1}{4}e^{4x+1}$ is een primitieve van $g(x)=e^{4x+1}$.

35c $H(x)=\frac{3^x}{\ln(3)}=\frac{1}{\ln(3)}\cdot 3^x \Rightarrow H'(x)=h(x)=\frac{1}{\ln(3)}\cdot 3^x\cdot\ln(3)=3^x$. Dus $H(x)=\frac{3^x}{\ln(3)}$ is een primitieve van $h(x)=3^x$.

35d $J(x)=x\cdot\ln(x) \Rightarrow J'(x)=1\cdot\ln(x)+x\cdot\frac{1}{x}=\ln(x)+1$. Dus $J(x)=x\ln(x)$ is een primitieve van $j(x)=\ln(x)+1$.

36a $F(x)=\frac{a}{n+1}x^{n+1}+c$ met $n\neq 1 \Rightarrow F'(x)=f(x)=\frac{a}{n+1}\cdot(n+1)\cdot x^n=ax^n$.

36b $F(x)=\frac{g^x}{\ln(g)}+c=\frac{1}{\ln(g)}\cdot g^x+c \Rightarrow F'(x)=f(x)=\frac{1}{\ln(g)}\cdot g^x\cdot\ln(g)=g^x$.

36c $F(x)=e^x+c \Rightarrow F'(x)=f(x)=e^x$.

36d $F(x)=\ln|x|+c \Rightarrow F'(x)=f(x)=\frac{1}{x}$.

36e $F(x)=x\ln(x)-x+c \Rightarrow F'(x)=f(x)=1\cdot\ln(x)+x\cdot\frac{1}{x}-1=\ln(x)+1-1=\ln(x)$.

36f $F(x)=\frac{1}{\ln(g)}\cdot(x\ln(x)-x)+c \Rightarrow F'(x)=f(x)=\frac{1}{\ln(g)}\cdot(1\cdot\ln(x)+x\cdot\frac{1}{x}-1)=\frac{1}{\ln(g)}\cdot\ln(x)=\frac{\ln(x)}{\ln(g)}={}^{g}\log(x)$.

37a $f(x)=ax^{-1}$ geeft $F(x)=\frac{a}{-1+1}x^{-1+1}=\frac{a}{0}x^0$. Dit kan niet kloppen omdat $\frac{a}{0}$ niet bestaat en $x^0=1$ voor $x\neq 0$.

37b $f(x)=x^{-1}=\frac{1}{x} \Rightarrow F(x)=\ln|x|+c$.

37c $[a\cdot F(x)]'=a\cdot F'(x)=a\cdot f(x)$.

38a $f(x)=6x^2 \Rightarrow F(x)=\frac{6}{3}x^3+c=2x^3+c$.

38b $f(x)=2x^3+5x^4 \Rightarrow F(x)=\frac{2}{4}x^4+\frac{5}{5}x^5+c=\frac{1}{2}x^4+x^5+c$.

38c $f(x)=\frac{x^4-2x}{2x^3}=\frac{x^4}{2x^3}-\frac{2x}{2x^3}=\frac{1}{2}x-x^{-2} \Rightarrow F(x)=\frac{1}{2}\cdot\frac{1}{2}\cdot x^2-\frac{1}{-1}x^{-1}+c=\frac{1}{4}x^2+\frac{1}{x}+c$.

38d $f(x)=10^x \Rightarrow F(x)=\frac{10^x}{\ln(10)}+c$.

38e $f(x)=5\cdot 2^x \Rightarrow F(x)=5\cdot\frac{2^x}{\ln(2)}+c=\frac{5\cdot 2^x}{\ln(2)}+c$.

38f $f(x)=\frac{x^3+2}{x^4}=\frac{x^3}{x^4}+\frac{2}{x^4}=\frac{1}{x}+2x^{-4} \Rightarrow F(x)=\ln|x|+\frac{2}{-3}x^{-3}+c=\ln|x|-\frac{2}{3x^3}+c$.

39a $f(x)=x^3-3x \Rightarrow F(x)=\frac{1}{4}x^4-\frac{3}{2}x^2+c$.

39b $f(x)=5e^x \Rightarrow F(x)=5e^x+c$.

39c $f(x)=\frac{x^4-6}{2x^3}=\frac{x^4}{2x^3}-\frac{6}{2x^3}=\frac{1}{2}x-3x^{-3} \Rightarrow F(x)=\frac{1}{2\cdot 2}x^2-\frac{3}{-2}x^{-2}+c=\frac{1}{4}x^2+\frac{3}{2x^2}+c$.

39d $f(x)=3^x+x^3 \Rightarrow F(x)=\frac{3^x}{\ln(3)}+\frac{1}{4}x^4+c$.

39e $f(x)=2\ln(x) \Rightarrow F(x)=2\cdot(x\ln(x)-x)+c=2x\ln(x)-2x+c$.

39f $f(x)=\ln(2x)=\ln(2)+\ln(x) \Rightarrow F(x)=\ln(2)\cdot x+x\ln(x)-x+c=x\ln(2)+x\ln(x)-x+c$.

40a $f(x)=e^{x+1}=e^x\cdot e^1=e\cdot e^x \Rightarrow F(x)=e\cdot e^x+c=e^{x+1}+c$ of $f(x)=e^{\boxed{x+1}} \Rightarrow F(x)=\frac{1}{1}\cdot e^{x+1}+c=e^{x+1}+c$.

40b $f(x)=\frac{8}{x^3}=8x^{-3} \Rightarrow F(x)=\frac{8}{-2}x^{-2}+c=-4x^{-2}+c=-\frac{4}{x^2}+c$.

40c $f(x)=\frac{-x^2+2x+3}{x^4}=\frac{-x^2}{x^4}+\frac{2x}{x^4}+\frac{3}{x^4}=-x^{-2}+2x^{-3}+3x^{-4} \Rightarrow F(x)=-\frac{1}{-1}x^{-1}+\frac{2}{-2}x^{-2}+\frac{3}{-3}x^{-3}+c=\frac{1}{x}-\frac{1}{x^2}-\frac{1}{x^3}+c$.

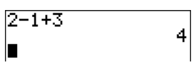
40d $f(x)=\ln(x\sqrt{x})=\ln(x^{1\frac{1}{2}})=1\frac{1}{2}\cdot\ln(x) \Rightarrow F(x)=1\frac{1}{2}\cdot(x\ln(x)-x)+c=1\frac{1}{2}x\ln(x)-1\frac{1}{2}x+c$.

40e $f(x)={}^{2}\log(\frac{1}{x})={}^{2}\log(x^{-1})=-{}^{2}\log(x) \Rightarrow F(x)=-\frac{1}{\ln(2)}\cdot(x\ln(x)-x)+c=\frac{-x\ln(x)+x}{\ln(2)}+c$.

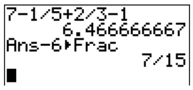
40f $f(x)=5\cdot\log(2x)=5\cdot\log(2)+5\cdot\log(x) \Rightarrow F(x)=5\cdot\log(2)\cdot x+5\cdot\frac{1}{\ln(10)}\cdot(x\ln(x)-x)+c=5x\log(2)+\frac{5x\ln(x)-5x}{\ln(10)}+c$.

41a $f(x)=2x-3 \Rightarrow F(x)=\frac{2}{2}x^2-3x+c=x^2-3x+c$.

41b $F(x)=x^2-3x+c$ door $(1, 2) \Rightarrow 2=1^2-3\cdot 1+c \Rightarrow c=4$. Dus $F(x)=x^2-3x+4$.



41c $F(x) = x^2 - 3x + c$ (waarvan de grafiek een parabool is) raakt de x-as $\Rightarrow$
$F(x) = 0$ (grafiek komt op de x-as) én $F'(x) = f(x) = 0$ (de helling in het punt op de x-as is nul)
$x^2 - 3x + c = 0$ én $2x - 3 = 0$
$x^2 - 3x + c = 0$ én $2x = 3$
$x^2 - 3x + c = 0$ én $x = \frac{3}{2}$
$(\frac{3}{2})^2 - 3 \cdot \frac{3}{2} + c = 0$ én $x = \frac{3}{2}$
$c = 3 \cdot \frac{3}{2} - (\frac{3}{2})^2 = \frac{9}{2} - \frac{9}{4} = \frac{18}{4} - \frac{9}{4} = \frac{9}{4} \Rightarrow F(x) = x^2 - 3x + \frac{9}{4}.$

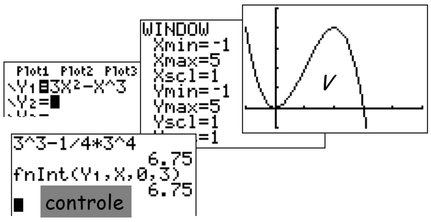


42 $f(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1 \Rightarrow F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c.$
$F(x)$ door $(1, 7) \Rightarrow 7 = \frac{1}{5} \cdot 1^5 - \frac{2}{3} \cdot 1^3 + 1 + c \Rightarrow 7 - \frac{1}{5} + \frac{2}{3} - 1 = c \Rightarrow c = 6\frac{7}{15}$. Dus $F(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + 6\frac{7}{15}$.

43a $f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3 + c.$

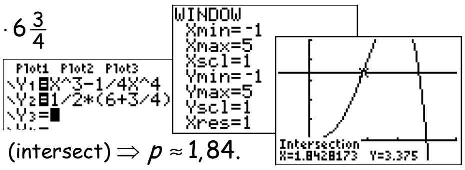
43b $O(x) = F(x) = \frac{1}{3}x^3 + c$ én $O(0) = 0 \Rightarrow \frac{1}{3} \cdot 0^3 + c = 0 \Rightarrow c = 0.$

43c $O(x) = \frac{1}{3}x^3$ én $O(p) = 10 \Rightarrow \frac{1}{3} \cdot p^3 = 10 \Rightarrow p^3 = 30 \Rightarrow p = \sqrt[3]{30}.$

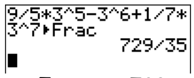


44a $f(x) = 0 \Rightarrow 3x^2 - x^3 = 0 \Rightarrow x^2 \cdot (3 - x) = 0 \Rightarrow x = 0 \vee x = 3.$
$O(V) = \int_0^3 (3x^2 - x^3)\,dx = \left[x^3 - \frac{1}{4}x^4\right]_0^3 = 3^3 - \frac{1}{4} \cdot 3^4 - (0^3 - \frac{1}{4} \cdot 0^4) = 6\frac{3}{4}.$

44b $\int_0^p (3x^2 - x^3)\,dx = \frac{1}{2} \cdot 6\frac{3}{4}$

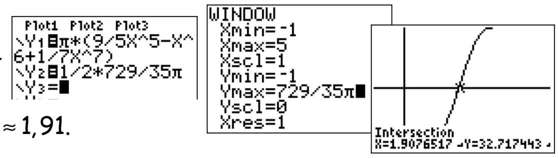


$\left[x^3 - \frac{1}{4}x^4\right]_0^p = 3\frac{3}{8}$
$p^3 - \frac{1}{4} \cdot p^4 - 0 = 3\frac{3}{8}$ (intersect) $\Rightarrow p \approx 1{,}84.$



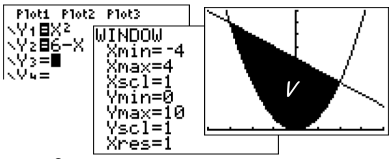
44c $I(L) = \int_0^3 \pi(3x^2 - x^3)^2\,dx = \int_0^3 \pi(9x^4 - 6x^5 + x^6)\,dx = \left[\pi(\frac{9}{5}x^5 - x^6 + \frac{1}{7}x^7)\right]_0^3 = \pi(\frac{9}{5} \cdot 3^5 - 3^6 + \frac{1}{7} \cdot 3^7) - 0 = \frac{729}{35}\pi.$

44d $\int_0^q \pi \cdot f(x)^2\,dx = \left[\pi(\frac{9}{5}x^5 - x^6 + \frac{1}{7}x^7)\right]_0^q = \frac{1}{2} \cdot \frac{729}{35}\pi$

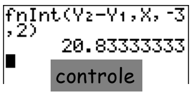


$\pi(\frac{9}{5} \cdot q^5 - q^6 + \frac{1}{7} \cdot q^7) - 0 = \frac{1}{2} \cdot \frac{729}{35}\pi$ (intersect) $\Rightarrow q \approx 1{,}91.$

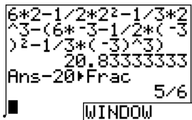
45a $x^2 = 6 - x$ (intersect of)



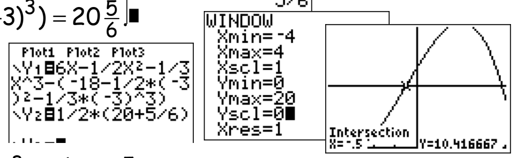
$x^2 + x - 6 = 0$
$(x + 3)(x - 2) = 0$
$x = -3 \vee x = 2.$



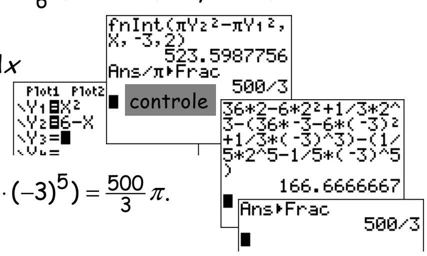
$O(V) = \int_{-3}^{2} (g(x) - f(x))\,dx = \int_{-3}^{2} (6 - x - x^2)\,dx$ (niet met fnInt)



$= \left[6x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-3}^{2} = 6 \cdot 2 - \frac{1}{2} \cdot 2^2 - \frac{1}{3} \cdot 2^3 - (6 \cdot (-3) - \frac{1}{2} \cdot (-3)^2 - \frac{1}{3} \cdot (-3)^3) = 20\frac{5}{6}$



45b $\int_{-3}^{p} (6 - x - x^2)\,dx = \frac{1}{2} \cdot 20\frac{5}{6}$
$\left[6x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-3}^{p} = 6 \cdot p - \frac{1}{2} \cdot p^2 - \frac{1}{3} \cdot p^3 - (6 \cdot (-3) - \frac{1}{2} \cdot (-3)^2 - \frac{1}{3} \cdot (-3)^3) = \frac{1}{2} \cdot 20\frac{5}{6}$ (intersect) $\Rightarrow p = -0{,}5.$



45c $I(L) = \int_{-3}^{2} \pi \cdot (6 - x)^2\,dx - \int_{-3}^{2} \pi \cdot (x^2)^2\,dx = \int_{-3}^{2} \pi \cdot (36 - 12x + x^2)\,dx - \int_{-3}^{2} \pi \cdot x^4\,dx$
$= \left[\pi \cdot (36x - 6x^2 + \frac{1}{3}x^3)\right]_{-3}^{2} - \left[\pi \cdot \frac{1}{5}x^5\right]_{-3}^{2}$
$= \pi \cdot (36 \cdot 2 - 6 \cdot 2^2 + \frac{1}{3} \cdot 2^3) - \pi \cdot (36 \cdot (-3) - 6 \cdot (-3)^2 + \frac{1}{3} \cdot (-3)^3) - (\pi \cdot \frac{1}{5} \cdot 2^5 - \pi \cdot \frac{1}{5} \cdot (-3)^5) = \frac{500}{3}\pi.$

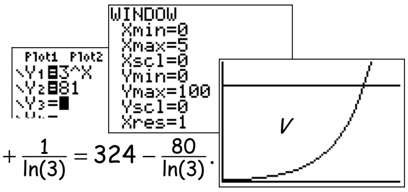
46a $f(x) = -1\frac{1}{2} \Rightarrow \frac{x^2+x+1}{x} = -1\frac{1}{2} \Rightarrow x^2 + x + 1 = -1\frac{1}{2}x \Rightarrow x^2 + 2\frac{1}{2}x + 1 = 0 \Rightarrow (x+2)(x+\frac{1}{2}) = 0 \Rightarrow x = -2 \vee x = -\frac{1}{2}.$

$$O(V) = \int_{-2}^{-\frac{1}{2}} \left(\frac{x^2+x+1}{x} - (-1\tfrac{1}{2})\right) dx = \int_{-2}^{-\frac{1}{2}} (x + 1 + \tfrac{1}{x} + 1\tfrac{1}{2})\, dx = \int_{-2}^{-\frac{1}{2}} (x + \tfrac{1}{x} + 2\tfrac{1}{2})\, dx = \left[\tfrac{1}{2}x^2 + \ln|x| + 2\tfrac{1}{2}x\right]_{-2}^{-\frac{1}{2}}$$

$$= \tfrac{1}{2}\cdot(-\tfrac{1}{2})^2 + \ln(\tfrac{1}{2}) + 2\tfrac{1}{2}\cdot -\tfrac{1}{2} - (\tfrac{1}{2}\cdot(-2)^2 + \ln(2) + 2\tfrac{1}{2}\cdot -2) = \tfrac{1}{8} + \ln(\tfrac{1}{2}) - 1\tfrac{1}{4} - (2 + \ln(2) - 5) = 1\tfrac{7}{8} + \ln(\tfrac{1}{2}) - \ln(2).$$

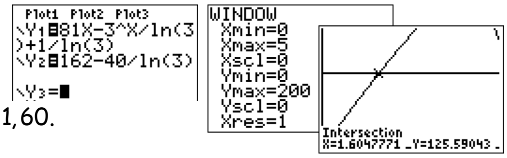
46b $$O(W) = \int_1^p \left(\frac{x^2+x+1}{x} - (x+1)\right) dx = \int_1^p (x + 1 + \tfrac{1}{x} - x - 1)\, dx = \int_1^p \tfrac{1}{x}\, dx = \left[\ln|x|\right]_1^p = \ln(p) - \ln(1) = \ln(p).$$

$O(W) = 2 \Rightarrow \ln(p) = 2 \Rightarrow p = e^2.$



47a $f(x) = 81 \Rightarrow 3^x = 81 = 3^4 \Rightarrow x = 4.$

$$O(V) = \int_0^4 (81 - 3^x)\, dx = \left[81x - \frac{3^x}{\ln(3)}\right]_0^4 = 81\cdot 4 - \frac{3^4}{\ln(3)} - (81\cdot 0 - \frac{3^0}{\ln(3)}) = 324 - \frac{81}{\ln(3)} + \frac{1}{\ln(3)} = 324 - \frac{80}{\ln(3)}.$$



47b $$\int_0^a (81 - 3^x)\, dx = \tfrac{1}{2}\cdot\left(324 - \frac{80}{\ln(3)}\right)$$

$$\left[81x - \frac{3^x}{\ln(3)}\right]_0^a = 81a - \frac{3^a}{\ln(3)} + \frac{1}{\ln(3)} = 162 - \frac{40}{\ln(3)} \text{ (intersect)} \Rightarrow a \approx 1{,}60.$$

48 $f(x) = 8 \Rightarrow \frac{8}{x^2} = \frac{8}{1} \Rightarrow 8x^2 = 8 \Rightarrow x^2 = 1 \Rightarrow x = -1$ (zoeken we niet) $\vee\ x = 1.$

$$O(V) = 8\cdot 1 + \int_1^8 \frac{8}{x^2}\, dx = 8\cdot 1 + \int_1^8 8x^{-2}\, dx = 8 + \left[-8x^{-1}\right]_1^8 = 8 + \left[-\frac{8}{x}\right]_1^8 = 8 + (-\tfrac{8}{8} - -\tfrac{8}{1}) = 8 + (-1 + 8) = 8 + 7 = 15.$$

$$O(V_1) = \tfrac{2}{3}\cdot 15 = 10 \Rightarrow 8 + \int_1^a \frac{8}{x^2}\, dx = 8 + \left[-\frac{8}{x}\right]_1^a = 8 + (-\tfrac{8}{a} - -\tfrac{8}{1}) = 8 - \tfrac{8}{a} + 8 = 16 - \tfrac{8}{a} = 10 \Rightarrow 6 = \tfrac{8}{a} \Rightarrow 6a = 8 \Rightarrow a = \tfrac{8}{6} = \tfrac{4}{3}.$$

49 $$I(L_1 + L_2) = \int_2^8 \pi(\sqrt{x-2})^2\, dx = \int_2^8 \pi(x-2)\, dx = \left[\pi(\tfrac{1}{2}x^2 - 2x)\right]_2^8 = \pi(32-16) - \pi(2-4) = 16\pi - -2\pi = 18\pi.$$

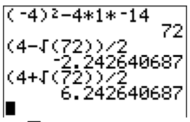
$$I(L_1) = \int_2^a \pi(\sqrt{x-2})^2\, dx = \tfrac{1}{2}\cdot 18\pi \text{ geeft } \left[\pi(\tfrac{1}{2}x^2 - 2x)\right]_2^a = 9\pi$$

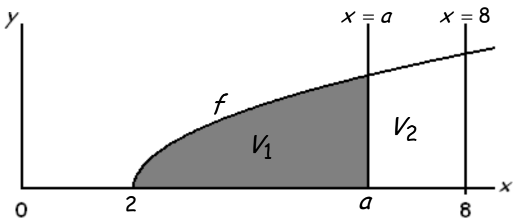
$\pi(\tfrac{1}{2}a^2 - 2a) - \pi(2-4) = 9\pi \Rightarrow \tfrac{1}{2}a^2 - 2a + 2 = 9$

$a^2 - 4a + 4 = 18 \Rightarrow a^2 - 4a - 14 = 0$

$D = 16 - 4\cdot 1\cdot -14 = 72$

$a = \frac{4 - \sqrt{72}}{2}$ (vold. niet) $\vee\ a = \frac{4 + \sqrt{72}}{2}$ (voldoet) $= \frac{4 + 6\sqrt{2}}{2} = 2 + 3\sqrt{2}.$





50a $F(x) = a(\boxed{3x+1})^6 \Rightarrow F'(x) = f(x) = 6a(3x+1)^5\cdot 3 = 18a(3x+1)^5.$

50b $(3x+1)^5 = 18a(3x+1)^5 \Rightarrow 1 = 18a \Rightarrow \frac{1}{18} = a.$

51 $G(x) = \frac{1}{a}F(\boxed{ax+b}) \Rightarrow G'(x) = g(x) = \frac{1}{a}F'(ax+b)\cdot a = F'(ax+b) = f(ax+b).$

52a $f(x) = (\boxed{2x-1})^6 \Rightarrow F(x) = \frac{1}{2}\cdot\frac{1}{7}\cdot(2x-1)^7 + c = \frac{1}{14}(2x-1)^7 + c.$

52b $g(x) = \frac{1}{(3x+4)^3} = (\boxed{3x+4})^{-3} \Rightarrow G(x) = \frac{1}{3}\cdot\frac{1}{-2}\cdot(3x+4)^{-2} + c = -\frac{1}{6}\cdot\frac{1}{(3x+4)^2} + c = \frac{-1}{6(3x+4)^2} + c.$

52c $h(x) = 4\sqrt{3-2x} = 4\cdot(\boxed{3-2x})^{\frac{1}{2}} \Rightarrow H(x) = 4\cdot\frac{1}{-2}\cdot\frac{1}{1\frac{1}{2}}\cdot(3-2x)^{1\frac{1}{2}} + c = -\frac{4}{3}\cdot(3-2x)\cdot\sqrt{3-2x} + c.$

52d $j(x) = \frac{2}{\sqrt{1-x}} = 2\cdot(\boxed{1-x})^{-\frac{1}{2}} \Rightarrow J(x) = 2\cdot\frac{1}{-1}\cdot\frac{1}{\frac{1}{2}}\cdot(1-x)^{\frac{1}{2}} + c = -4\cdot(1-x)^{\frac{1}{2}} + c = -4\cdot\sqrt{1-x} + c.$

53a $f(x) = \frac{1}{\boxed{x-1}} \Rightarrow F(x) = \frac{1}{1}\cdot\ln|x-1| + c = \ln|x-1| + c.$

53b $f(x) = \frac{3}{2x-5} = 3\cdot\frac{1}{\boxed{2x-5}} \Rightarrow F(x) = 3\cdot\frac{1}{2}\cdot\ln|2x-5| + c = \frac{3}{2}\cdot\ln|2x-5| + c.$

53c $f(x)=e^{\boxed{4x-1}} \Rightarrow F(x)=\frac{1}{4}\cdot e^{4x-1}+c.$

53d $f(x)=\ln(\boxed{4x-1}) \Rightarrow F(x)=\frac{1}{4}\cdot((4x-1)\cdot\ln(4x-1)-(4x-1))+c=(x-\frac{1}{4})\cdot\ln(4x-1)-x+\frac{1}{4}+c.$

53e $f(x)=(2x+1)\cdot\sqrt{2x+1}=(\boxed{2x+1})^{1\frac{1}{2}} \Rightarrow F(x)=\frac{1}{2}\cdot\frac{1}{2\frac{1}{2}}(2x+1)^{2\frac{1}{2}}+c=\frac{1}{5}\cdot(2x+1)^2\cdot\sqrt{2x+1}+c.$

53f $f(x)=2^{\boxed{3x}} \Rightarrow F(x)=\frac{1}{3}\cdot\frac{1}{\ln(2)}\cdot 2^{3x}+c=\frac{2^{3x}}{3\ln(2)}+c.$

53g $f(x)=3^{\boxed{2-5x}} \Rightarrow F(x)=\frac{1}{-5}\cdot\frac{1}{\ln(3)}\cdot 3^{2-5x}+c=-\frac{3^{2-5x}}{5\ln(3)}+c.$

53h $f(x)={}^2\log(5x+3)=\frac{\ln(5x+3)}{\ln(2)}=\frac{1}{\ln(2)}\cdot\ln(\boxed{5x+3}) \Rightarrow F(x)=\frac{1}{5}\cdot\frac{1}{\ln(2)}\cdot((5x+3)\cdot\ln(5x+3)-(5x+3))+c.$

54a $f(x)=g(x) \Rightarrow \frac{5}{2x+1}=\frac{5}{10-4x} \Rightarrow 5\cdot(2x+1)=5\cdot(10-4x) \Rightarrow 2x+1=10-4x \Rightarrow 6x=9 \Rightarrow x=\frac{9}{6}=1\frac{1}{2}.$

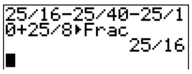
$$O(V)=\int_0^{1\frac{1}{2}}\frac{5}{\boxed{10-4x}}\,dx+\int_{1\frac{1}{2}}^{2}\frac{5}{\boxed{2x+1}}\,dx=\left[5\cdot\frac{1}{-4}\cdot\ln|10-4x|\right]_0^{1\frac{1}{2}}+\left[5\cdot\frac{1}{2}\cdot\ln|2x+1|\right]_{1\frac{1}{2}}^{2}$$

$$=-\frac{5}{4}\cdot\ln(4)+\frac{5}{4}\cdot\ln(10)+\frac{5}{2}\cdot\ln(5)-\frac{5}{2}\cdot\ln(4)=\frac{5}{4}\cdot\ln(10)+\frac{5}{2}\cdot\ln(5)-\frac{15}{4}\cdot\ln(4).$$

54b $$I(L)=\int_0^{1\frac{1}{2}}\pi\cdot\left(\frac{5}{10-4x}\right)^2dx+\int_{1\frac{1}{2}}^{2}\pi\cdot\left(\frac{5}{2x+1}\right)^2dx=\int_0^{1\frac{1}{2}}25\pi\cdot(\boxed{10-4x})^{-2}\,dx+\int_{1\frac{1}{2}}^{2}25\pi\cdot(\boxed{2x+1})^{-2}\,dx$$

$$=\left[25\pi\cdot\frac{1}{-4}\cdot\frac{1}{-1}\cdot(10-4x)^{-1}\right]_0^{1\frac{1}{2}}+\left[25\pi\cdot\frac{1}{2}\cdot\frac{1}{-1}\cdot(2x+1)^{-1}\right]_{1\frac{1}{2}}^{2}=\left[\frac{25\pi}{4\cdot(10-4x)}\right]_0^{1\frac{1}{2}}+\left[-\frac{25\pi}{2\cdot(2x+1)}\right]_{1\frac{1}{2}}^{2}$$

$$=\frac{25\pi}{4\cdot4}-\frac{25\pi}{4\cdot10}-\frac{25\pi}{2\cdot5}+\frac{25\pi}{2\cdot4}=\frac{25}{16}\pi.$$



55a De regel gaat over functies van de vorm $f(ax+b)$ en g is van de vorm $f(ax^2+b)$.

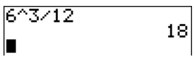
55b $G(x)=a(\boxed{4x^2-1})^{1\frac{1}{2}} \Rightarrow G'(x)=a\cdot 1\frac{1}{2}\cdot(4x^2-1)^{\frac{1}{2}}\cdot 8x=12ax\cdot\sqrt{4x^2-1}.$

Er is geen waarde van a waarvoor $G'(x)=\sqrt{4x^2-1}$.

55c $H(x)=a\cdot(4x^2-1)^{1\frac{1}{2}}+c \Rightarrow h(x)=12ax\cdot\sqrt{4x^2-1}$ (zie 55b). Dus $12ax=x \Rightarrow a=\frac{1}{12}$.

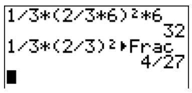
Dus $h(x)=x\cdot\sqrt{4x^2-1}$ heeft als primitieven $H(x)=\frac{1}{12}\cdot(4x^2-1)^{1\frac{1}{2}}+c=\frac{1}{12}\cdot(4x^2-1)\cdot\sqrt{4x^2-1}+c.$

56a $I(L)=\int_0^6\pi\cdot(\frac{1}{2}x)^2\,dx=\int_0^6\pi\cdot\frac{1}{4}x^2\,dx=\left[\pi\cdot\frac{1}{4}\cdot\frac{1}{3}\cdot x^3\right]_0^6=\frac{1}{12}\pi\cdot 6^3-\frac{1}{12}\pi\cdot 0=18\pi.$



56b L is een kegel. De straal van de grondcirkel is 3 en de hoogte is 6.

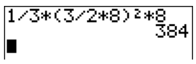
57 $I(\text{kegel})=\frac{1}{3}Gh=\frac{1}{3}\pi r^2h=\frac{1}{3}\cdot\pi\cdot(\frac{2}{3}\cdot 6)^2\cdot 6=\frac{1}{3}\cdot\pi\cdot\frac{4}{9}\cdot 36\cdot 6=32\pi.$



$I(\text{kegeltop})=\frac{1}{3}\cdot\pi\cdot(\frac{2}{3}p)^2\cdot p=\frac{1}{3}\cdot\pi\cdot\frac{4}{9}p^2\cdot p=\frac{4}{27}\pi p^3.$

$K_p=24\pi \Rightarrow 32\pi-\frac{4}{27}\pi p^3=24\pi \Rightarrow -\frac{4}{27}\pi p^3=-8\pi \Rightarrow \frac{4}{27}p^3=8 \Rightarrow p^3=8\cdot\frac{27}{4}=54 \Rightarrow p=\sqrt[3]{54}=\sqrt[3]{2\cdot3\cdot3\cdot3}=3\cdot\sqrt[3]{2}.$

58 $I(\text{kegel})=\frac{1}{3}Gh=\frac{1}{3}\cdot\pi\cdot(\frac{3}{2}\cdot 8)^2\cdot 8=\frac{1}{3}\cdot\pi\cdot\frac{9}{4}\cdot 64\cdot 8=384\pi.$



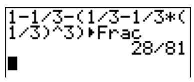
$I(\text{kegeltop})=\frac{1}{3}\cdot\pi\cdot(\frac{3}{2}p)^2\cdot p=\frac{1}{3}\cdot\pi\cdot\frac{9}{4}p^2\cdot p=\frac{3}{4}\pi p^3.$

$K_p=378\pi \Rightarrow 384\pi-\frac{3}{4}\pi p^3=378\pi \Rightarrow -\frac{3}{4}\pi p^3=-6\pi \Rightarrow \frac{3}{4}p^3=6 \Rightarrow p^3=6\cdot\frac{4}{3}=8 \Rightarrow p=\sqrt[3]{8}=\sqrt[3]{2\cdot2\cdot2}=2.$

59 De cirkel met straal r en middelpunt (0, 0) heeft vergelijking $x^2+y^2=r^2$ ofwel $y^2=r^2-x^2$.

$$I(\text{bol})=\int_{-r}^{r}\pi y^2\,dx=2\cdot\int_0^r\pi y^2\,dx=2\cdot\int_0^r\pi(r^2-x^2)\,dx=2\cdot\left[\pi(r^2x-\frac{1}{3}x^3)\right]_0^r=2\cdot(\pi(r^3-\frac{1}{3}r^3))=2\pi\cdot\frac{2}{3}r^3=\frac{4}{3}\pi r^3.$$

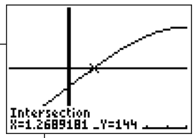
60 $$I=\int_{\frac{1}{3}r}^{r}\pi y^2\,dx=\int_{\frac{1}{3}r}^{r}\pi(r^2-x^2)\,dx=\left[\pi(r^2x-\frac{1}{3}x^3)\right]_{\frac{1}{3}r}^{r}=\pi(r^3-\frac{1}{3}r^3)-\pi(\frac{1}{3}r^3-\frac{1}{3}\cdot(\frac{1}{3}r)^3)=\pi\cdot\frac{2}{3}r^3-\pi\cdot\frac{26}{81}r^3=\frac{28}{81}\pi r^3.$$



```
4/3*6^3
                288
```

```
Plot1 Plot2 Plot3
\Y1■36X-1/3X^3-(
-108-1/3*(-3)^3)
\Y2■144
\Y3=
```

```
WINDOW
 Xmin=-3
 Xmax=6
 Xscl=1
 Ymin=0
 Ymax=144*2
 Yscl=1
 Xres=1
```



61 $I(\text{bol}) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 6^3 = 288\pi$. Nu is $I(S_p) = \frac{1}{2} \cdot 288\pi = 144\pi \Rightarrow$

$\int_{-3}^{p} \pi y^2 \, dx = 144\pi \Rightarrow \int_{-3}^{p} \pi(36 - x^2) \, dx = 144\pi \Rightarrow \left[\pi(36x - \frac{1}{3}x^3)\right]_{-3}^{p} = 144\pi \Rightarrow$

$\pi(36p - \frac{1}{3}p^3) - \pi(-108 - \frac{1}{3} \cdot -27) = 144\pi$. Intersect geeft dan $p \approx 1{,}27$.

62 k snijden met de x-as $(y = 0) \Rightarrow -\frac{3}{4}x + 6\frac{1}{4} = 0 \Rightarrow -\frac{3}{4}x = -\frac{25}{4} \Rightarrow x = -\frac{25}{4} \cdot -\frac{4}{3} = \frac{25}{3}$.

$I(L) = \int_{3}^{\frac{25}{3}} \pi\left(\boxed{-\frac{3}{4}x + 6\frac{1}{4}}\right)^2 dx - \int_{3}^{5} \pi(25 - x^2)\, dx = \left[\pi \cdot -\frac{4}{3} \cdot \frac{1}{3} \cdot (-\frac{3}{4}x + 6\frac{1}{4})^3\right]_3^{\frac{25}{3}} - \left[\pi(25x - \frac{1}{3}x^3)\right]_3^5$

$= -\frac{4}{9}\pi \cdot 0^3 - -\frac{4}{9}\pi \cdot (-\frac{3}{4} \cdot 3 + 6\frac{1}{4})^3 - \left(\pi \cdot (25 \cdot 5 - \frac{1}{3} \cdot 5^3) - \pi \cdot (25 \cdot 3 - \frac{1}{3} \cdot 3^3)\right) = \frac{100}{9}\pi = 11\frac{1}{9}\pi.$

```
4/9*(-3/4*3+25/4
)^3-((25*5-1/3*5
^3)-(25*3-1/3*3^
3))▸Frac
              100/9
```

63a $A(2, 2)$ en $B(4, 8) \Rightarrow AB = \sqrt{(4-2)^2 + (8-2)^2} = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} \approx 6{,}32.$

63b $C(3;\ 4{,}5) \Rightarrow AC + CB = \sqrt{1^2 + 2{,}5^2} + \sqrt{1^2 + 3{,}5^2} \approx 6{,}33.$
Deze tweede benadering is beter omdat $AB < AC + CB <$ boog AB.

```
1/2*2²
                  2
1/2*4²
                  8
1/2*3²
                4.5
```

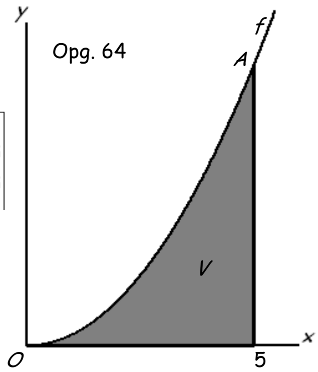
```
√(2²+6²)
         6.32455532
√(1²+2.5²)+√(1²+
3.5²)
        6.332637348
```

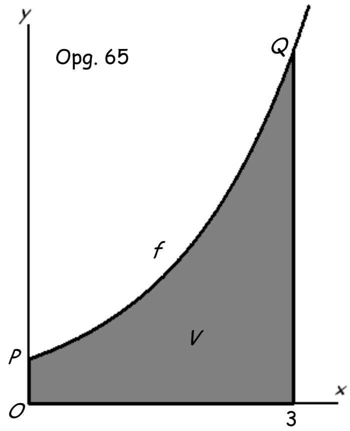
64 $f(x) = \frac{1}{2}x^2 \Rightarrow f'(x) = x.$

boog $OA = \int_0^5 \sqrt{1 + x^2}\, dx$ (fnInt) $\approx 13{,}904.$

omtrek $\approx 13{,}904 + 5 + \frac{1}{2} \cdot 5^2 \approx 31{,}40.$

```
fnInt(√(1+X²),X,
0,5)
        13.90376795
Ans+5+1/2*5²
        31.40376795
```





65 $f(x) = 2^x \Rightarrow f'(x) = 2^x \cdot \ln(2).$

boog $PQ = \int_0^3 \sqrt{1 + (2^x \cdot \ln(2))^2}\, dx$ (fnInt) $\approx 7{,}792.$

omtrek $\approx 7{,}792 + 1 + 3 + 2^3 \approx 19{,}79.$

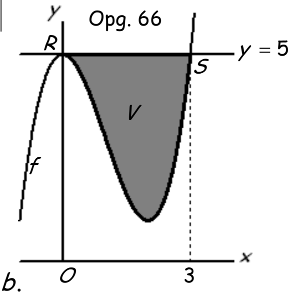
```
fnInt(√(1+(2^X*l
n(2))²),X,0,3)
        7.792085817
Ans+1+3+2^3
        19.79208582
```

66 $x^3 - 3x^2 + 5 = 5 \Rightarrow x^2(x - 3) = 0 \Rightarrow x = 0 \ \vee\ x = 3.$
$f(x) = x^3 - 3x^2 + 5 \Rightarrow f'(x) = 3x^2 - 6x.$

boog $RS = \int_0^3 \sqrt{1 + (3x^2 - 6x)^2}\, dx$ (fnInt) $\approx 8{,}81.$

omtrek $\approx 8{,}81 + 3 \approx 11{,}81.$

```
fnInt(√(1+(3X²-6
X)²),X,0,3)
        8.814564192
Ans+3
        11.81456419
```



67 De formule van de kabel is van de vorm $h(x) = ax^2 + b.$
$(0, 5)$ op de parabool $\Rightarrow b = 5.$

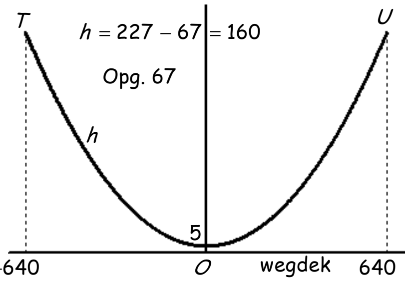
$\left.\begin{array}{l} h(x) = ax^2 + 5 \\ \text{door } (640, 160) \end{array}\right\} \Rightarrow 160 = a \cdot 640^2 + 5 \Rightarrow a = \frac{155}{640^2} = \frac{31}{81920}.$

$h(x) = \frac{31}{81920}x^2 + 5 \Rightarrow h'(x) = \frac{31}{40960}x.$

boog $TU = \int_{-640}^{640} \sqrt{1 + (\frac{31}{40960}x)^2}\, dx$ (fnInt) ≈ 1328 (m).

```
160-5
                155
640²
             409600
Ans/5
              81920
```

```
Ans/2
              40960
fnInt(√(1+(31/40
960X)²),X,-640,6
40)
        1328.423343
```




```
Plot1 Plot2 Plot3
\Y1■4(e^(0.062X)
+e^(-0.062X))
\Y2=
```

X	Y1
-20	14.98
0	8
20	14.98

68a $y(x) = y(-x) \Rightarrow$ hoogte van bevestiging aan de palen is $y(20) = y(-20) \approx 15{,}0$ (m).
Niet gevraagd wordt: de laagste hoogte van de kabel boven het wegdek $y(0) = 4 \cdot (e^0 + e^0) = 4 \cdot (1 + 1) = 4 \cdot 2 = 8$ (m).

68b $y = 4(e^{\boxed{0{,}062x}} + e^{\boxed{-0{,}062x}}) \Rightarrow y' = 4(e^{0{,}062x} \cdot 0{,}062 + e^{-0{,}062x} \cdot -0{,}062) = 0{,}248(e^{0{,}062x} - e^{-0{,}062x}).$

Lengte van één kabel: $\int_{-20}^{20} \sqrt{1 + (0{,}248(e^{0{,}062x} - e^{-0{,}062x}))^2}\, dx$ (fnInt) $\approx 43{,}2$ (m).

```
Plot1 Plot2 Plot3
\Y1■4(e^(0.062X)
+e^(-0.062X))
\Y2■4*0.062(e^(0
.062X)-e^(-0.062
X))
\Y3=
```

```
fnInt(√(1+Y2²),X
,-20,20)
        43.17476802
fnInt(Y1,X,-20,2
0)
        408.5457093
```

68c Oppervlakte van één net: $\int_{-20}^{20} 4(e^{0{,}062x} + e^{-0{,}062x})\, dx$ (fnInt) ≈ 409 (m²).

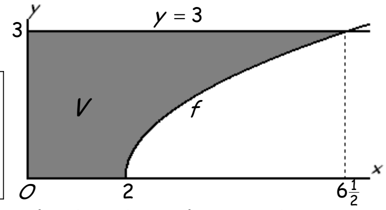
69 Voor de straal van de onderste cilinder moet je de grafiek van f snijden met de lijn $y=\frac{1}{2}$.
$f(x)=\frac{1}{2} \Rightarrow \sqrt{x}=\frac{1}{2}$ (kwadrateren) $\Rightarrow x=\frac{1}{4}$. Dus de straal van de onderste cilinder is $\frac{1}{4}$.
Voor de straal van de bovenste cilinder moet je de grafiek van f snijden met de lijn $y=1\frac{1}{2}$.
$f(x)=1\frac{1}{2} \Rightarrow \sqrt{x}=1\frac{1}{2}$ (kwadrateren) $\Rightarrow x=2\frac{1}{4}$. Dus de straal van de bovenste cilinder is $2\frac{1}{4}$.

```
0.5²
                .25
1.5²
               2.25
```

70a $y=\sqrt{2x-4} \Rightarrow y^2=2x-4 \Rightarrow 2x=y^2+4 \Rightarrow x=\frac{1}{2}y^2+2 \Rightarrow x^2=(\frac{1}{2}y^2+2)^2=\frac{1}{4}y^4+2y^2+4.$

$$I(L)=\int_0^3 \pi x^2\,dy=\int_0^3 \pi(\tfrac{1}{4}y^4+2y^2+4)\,dy$$
$$=\left[\pi\cdot(\tfrac{1}{4}\cdot\tfrac{1}{5}y^5+2\cdot\tfrac{1}{3}y^3+4y)\right]_0^3=\left[\pi\cdot(\tfrac{1}{20}y^5+\tfrac{2}{3}y^3+4y)\right]_0^3$$
$$=\pi\cdot(\tfrac{1}{20}\cdot 3^5+\tfrac{2}{3}\cdot 3^3+4\cdot 3)-\pi\cdot 0=\tfrac{843}{20}\pi=42\tfrac{3}{20}\pi.$$

```
3^5/20+2/3*3*3^3+4
*3▸Frac
             843/20
843/20
              42.15
843-42*20
                  3
```



70b $y=3 \Rightarrow \sqrt{2x-4}=3 \Rightarrow 2x-4=9 \Rightarrow 2x=13 \Rightarrow x=6\frac{1}{2}$; $y=0 \Rightarrow \sqrt{2x-4}=0 \Rightarrow 2x-4=0 \Rightarrow 2x=4 \Rightarrow x=2.$

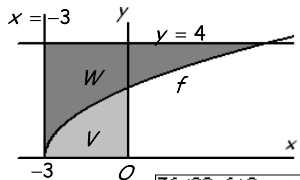
$$I(M)=I(\text{cilinder})-\int_2^{6\frac{1}{2}} \pi y^2\,dx=\pi\cdot 3^2\cdot 6\tfrac{1}{2}-\int_2^{6\frac{1}{2}} \pi(2x-4)\,dx=58\tfrac{1}{2}\pi-\left[\pi\cdot(x^2-4x)\right]_2^{6\frac{1}{2}}$$
$$=58\tfrac{1}{2}\pi-\left(\pi\cdot(6\tfrac{1}{2}^2-4\cdot 6\tfrac{1}{2})-\pi\cdot(2^2-4\cdot 2)\right)=38\tfrac{1}{4}\pi.$$

```
9*6.5
               58.5
Ans-((6.5²-4*6.5
)-(2²-4*2))
              38.25
```

71a $$I(L)=\int_0^4 \pi x^2\,dy=\int_0^4 \pi y\,dy=\left[\pi\cdot\tfrac{1}{2}y^2\right]_0^4=\pi\cdot\tfrac{1}{2}\cdot 4^2-\pi\cdot 0=8\pi.$$

71b $$I(L_q)=\int_0^q \pi y\,dy=\left[\pi\cdot\tfrac{1}{2}y^2\right]_0^q=\tfrac{1}{2}q^2\pi \text{ en } I(L_p)=\int_0^p \pi y^2\,dx=\int_0^p \pi x^4\,dx=\left[\pi\cdot\tfrac{1}{5}x^5\right]_0^p=\pi\cdot\tfrac{1}{5}\cdot p^5-\pi\cdot 0=\tfrac{1}{5}p^5\pi.$$
$P(p,q)$ ligt op de grafiek van de parabool $y=x^2 \Rightarrow q=p^2$.
$I(L_q)=I(L_p)$ met $q=p^2 \Rightarrow \frac{1}{2}q^2\pi=\frac{1}{5}p^5\pi$ met $q=p^2 \Rightarrow \frac{1}{2}p^4\pi=\frac{1}{5}p^5\pi \Rightarrow \frac{1}{2}p^4=\frac{1}{5}p^5 \Rightarrow p=0 \vee \frac{1}{2}=\frac{1}{5}p.$
$p=0$ geeft geen omwentelingslichaam (dus voldoet niet) en $\frac{1}{2}=\frac{1}{5}p$ geeft $p=\frac{5}{2}$ en $q=p^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$.

72a $$I(L)=\int_{-3}^0 \pi y^2\,dx=\int_{-3}^0 \pi\cdot(\sqrt{2x+6})^2\,dx=\int_{-3}^0 \pi(2x+6)\,dx=\left[\pi(x^2+6x)\right]_{-3}^0=\pi\cdot 0-\pi\cdot(9-18)=--9\pi=9\pi.$$



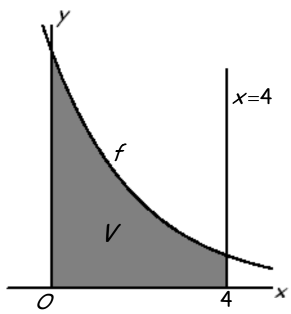
72b $f(0)=\sqrt{6}$.
$y=\sqrt{2x+6} \Rightarrow y^2=2x+6 \Rightarrow 2x=y^2-6 \Rightarrow x=\frac{1}{2}y^2-3 \Rightarrow x^2=\frac{1}{4}y^4-3y^2+9.$
$$I(M)=\int_0^{\sqrt{6}} \pi x^2\,dy=\int_0^{\sqrt{6}} \pi(\tfrac{1}{4}y^4-3y^2+9)\,dy=\left[\pi(\tfrac{1}{20}y^5-y^3+9y)\right]_0^{\sqrt{6}}$$
$$=\pi\cdot(\tfrac{1}{20}\cdot\sqrt{6}^5-\sqrt{6}^3+9\cdot\sqrt{6})-\pi\cdot 0=\pi\cdot(\tfrac{1}{20}\cdot 6^2\cdot\sqrt{6}-6\cdot\sqrt{6}+9\cdot\sqrt{6})=\tfrac{24}{5}\pi\cdot\sqrt{6}=4\tfrac{4}{5}\pi\cdot\sqrt{6}.$$

```
36/20-6+9
                4.8
Ans▸Frac
               24/5
```

72c $y=\sqrt{2x+6} \xrightarrow{\text{translatie }(3,0)} y=\sqrt{2(x-3)+6}=\sqrt{2x-6+6}=\sqrt{2x}.$
$y=\sqrt{2x} \Rightarrow y^2=2x \Rightarrow x=\frac{1}{2}y^2 \Rightarrow x^2=\frac{1}{4}y^4.$

```
4^5/20▸Frac
              256/5
```

$$I(N)=\int_0^4 \pi x^2\,dy=\int_0^4 \pi\cdot\tfrac{1}{4}y^4\,dy=\left[\pi\cdot\tfrac{1}{20}y^5\right]_0^4=\pi\cdot\tfrac{1}{20}\cdot 4^5-\pi\cdot 0=\tfrac{256}{5}\pi=51\tfrac{1}{5}\pi.$$



73a $$O(V)=\int_0^4 2e^{\boxed{-\frac{1}{2}x+1}}\,dx=\left[2\cdot\tfrac{1}{-\frac{1}{2}}\cdot e^{-\frac{1}{2}x+1}\right]_0^4=\left[-4\cdot e^{-\frac{1}{2}x+1}\right]_0^4=-4\cdot e^{-1}--4\cdot e^1=4e-\tfrac{4}{e}.$$

```
Plot1 Plot2 Plot3
\Y1■2*e^(-1/2X+1
)
\Y2■-e^(-1/2X+1)
```

```
fnInt(√(1+Y2²),X
,0,4)
        6.392397027
Ans+Y1(0)+4+Y1(4
)
        16.56471957
```

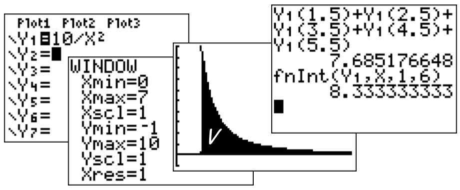
73b $f(x)=2e^{\boxed{-\frac{1}{2}x+1}} \Rightarrow f'(x)=2\cdot-\frac{1}{2}\cdot e^{-\frac{1}{2}x+1}=-e^{-\frac{1}{2}x+1}.$
$$\int_0^4 \sqrt{1+(-e^{-\frac{1}{2}x+1})^2}\,dx \text{ (fnInt)} \approx 6{,}392 \Rightarrow \text{omtrek} \approx 6{,}392+2e+4+2e^{-1} \approx 16{,}56.$$

73c $$I(L)=\pi\cdot 4^2\cdot\tfrac{2}{e}+\int_{\frac{2}{e}}^{2e} \pi x^2\,dy=\tfrac{32\pi}{e}+\int_{\frac{2}{e}}^{2e} \pi\cdot\left(2-2\ln(\tfrac{1}{2}y)\right)^2 \text{ (GA DIT ZELF NA!!!) } dy \text{ (fnInt)} \approx 81{,}16.$$

```
Plot1 Plot2 Plot3
\Y1■2*e^(-1/2X+1
)
\Y2■2-2ln(1/2X)
\Y3=
```

```
π*4²*Y1(4)+fnInt
(π*Y2²,X,2/e,2e)
        81.16083477
```

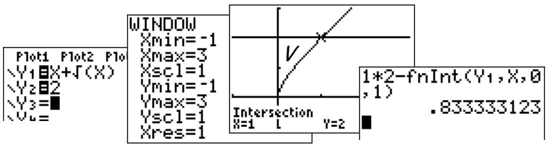
Diagnostische toets



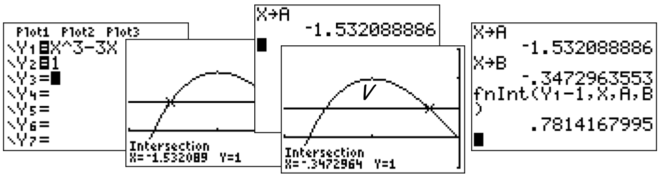
D1a De middens van de intervallen zijn 1,5, 2,5, 3,5, 4,5 en 5,5.
$O(V) \approx f(1{,}5)\cdot 1 + f(2{,}5)\cdot 1 + f(3{,}5)\cdot 1 + f(4{,}5)\cdot 1 + f(5{,}5)\cdot 1 \approx 7{,}69.$

D1b $O(V) = \int_1^6 f(x)\,dx$ (fnInt) $\approx 8{,}3333.$

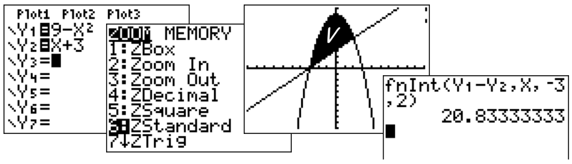
D2 $f(x) = 2$ (intersect) $\Rightarrow x = 1.$
$O(V) = 1\cdot 2 - \int_0^1 f(x)\,dx$ (fnInt) $\approx 0{,}8333.$



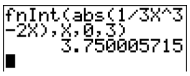
D3 $f(x) = 1$ (intersect) $\Rightarrow x \approx -1{,}53 \vee x \approx -0{,}35.$
$O(V) = \int_{-1{,}53}^{-0{,}35} (f(x) - 1)\,dx$ (fnInt) $\approx 0{,}78.$



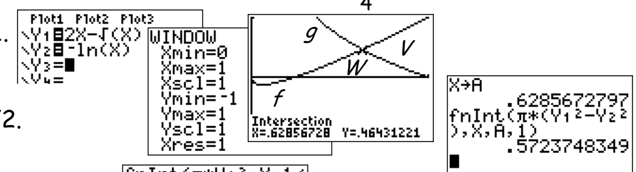
D4 $9 - x^2 = x + 3$ (intersect of)
$x^2 + x - 6 = 0 \Rightarrow (x+3)(x-2) = 0 \Rightarrow x = -3 \vee x = 2.$
$O(V) = \int_{-3}^{2} (f(x) - g(x))\,dx$ (fnInt) $\approx 20{,}83.$



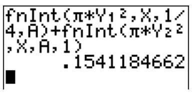
D5 $O(V) = \int_0^3 |f(x)|\,dx$ (fnInt) $\approx 3{,}75.$



D6a $f(x) = 0 \Rightarrow 2x - \sqrt{x} = 0 \Rightarrow 2x = \sqrt{x} \Rightarrow 4x^2 = x \Rightarrow x = 0 \vee 4x = 1 \Rightarrow x = 0 \vee x = \frac{1}{4}.$
$g(x) = 0 \Rightarrow -\ln(x) = 0 \Rightarrow \ln(x) = 0 \Rightarrow x = e^0 = 1.$
$f(x) = g(x)$ (intersect) $\Rightarrow x \approx 0{,}63.$
$I = \int_{0{,}63}^{1} \pi\cdot((f(x))^2 - (g(x))^2)\,dx$ (fnInt) $\approx 0{,}572.$



D6b $I = \int_{0{,}25}^{0{,}63} \pi\cdot(f(x))^2\,dx + \int_{0{,}63}^{1} \pi\cdot(g(x))^2\,dx$ (fnInt) $\approx 0{,}154.$



D7a $F(x) = (\boxed{x^3+5})^4 + 2 \Rightarrow F'(x) = 4(x^3+5)^3\cdot 3x^2 = 12x^2(x^3+5)^3.$ Dus F is een primitieve van f.

D7b $G(x) = e^{\boxed{x^3}} - 3 \Rightarrow G'(x) = e^{x^3}\cdot 3x^2 = 3x^2 e^{x^3}.$ Dus G is een primitieve van g.

D8a $f(x) = \frac{6}{x^4} = 6\cdot x^{-4} \Rightarrow F(x) = 6\cdot\frac{1}{-3}x^{-3} + c = -2x^{-3} + c = -\frac{2}{x^3} + c.$

D8b $f(x) = \frac{2x+6}{x^2} = \frac{2x}{x^2} + \frac{6}{x^2} = \frac{2}{x} + \frac{6}{x^2} = 2\cdot\frac{1}{x} + 6\cdot x^{-2} \Rightarrow F(x) = 2\cdot\ln|x| + 6\cdot\frac{1}{-1}\cdot x^{-1} + c = 2\cdot\ln|x| - \frac{6}{x} + c.$

D8c $f(x) = 3\cdot 2^x \Rightarrow F(x) = 3\cdot\frac{1}{\ln(2)}\cdot 2^x + c = \frac{3\cdot 2^x}{\ln(2)} + c.$

D8d $f(x) = 6e^x + x^2 \Rightarrow F(x) = 6\cdot e^x + \frac{1}{3}x^3 + c.$

D8e $f(x) = \ln(x^2) = 2\cdot\ln(x) \Rightarrow F(x) = 2\cdot(x\ln(x) - x) + c = 2x\ln(x) - 2x + c.$

D8f $f(x) = {}^2\log(4\cdot x) = {}^2\log(4) + {}^2\log(x) = {}^2\log(2^2) + \frac{\ln(x)}{\ln(2)} = 2 + \frac{\ln(x)}{\ln(2)} \Rightarrow F(x) = 2x + \frac{1}{\ln(2)}\cdot(x\ln(x) - x) + c.$

D9a $\left.\begin{array}{l} f(x) = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x} \Rightarrow F(x) = \frac{1}{2}x^2 + \ln|x| + c \\ \text{door } (x,y) = (1,e) \end{array}\right\} \Rightarrow e = \frac{1}{2}\cdot 1^2 + \ln(1) + c \Rightarrow e = \frac{1}{2} + 0 + c \Rightarrow e - \frac{1}{2} = c.$
Dus $F(x) = \frac{1}{2}x^2 + \ln|x| + e - \frac{1}{2}.$

D9b $O(V) = \int_1^2 \frac{x^2+1}{x}\,dx = \left[\frac{1}{2}x^2 + \ln|x|\right]_1^2 = (2 + \ln(2)) - \left(\frac{1}{2} - \ln(1)\right) = 1\frac{1}{2} + \ln(2).$

D9c $f(x)=g(x)\Rightarrow \frac{x^2+1}{x}=\frac{2}{x}\Rightarrow x^2+1=2\Rightarrow x^2=1\Rightarrow x=-1$ (vold. niet) $\vee\ x=1$ (voldoet).

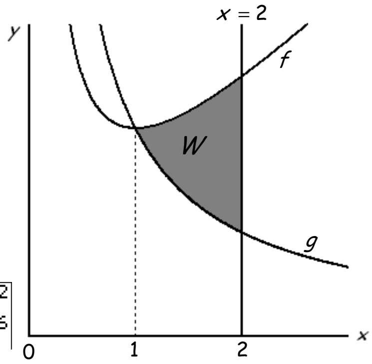
$$I=\int_1^2 \pi\cdot\left(\frac{x^2+1}{x}\right)^2\,dx-\int_1^2 \pi\cdot\left(\frac{2}{x}\right)^2\,dx=\int_1^2 \pi\cdot\frac{x^4+2x^2+1}{x^2}\,dx-\int_1^2 \pi\cdot\frac{4}{x^2}\,dx$$

$$=\int_1^2 \pi\cdot\left(\frac{x^4}{x^2}+\frac{2x^2}{x^2}+\frac{1}{x^2}\right)dx-\int_1^2 \pi\cdot\frac{4}{x^2}\,dx=\int_1^2 \pi\cdot(x^2+2+x^{-2})\,dx-\int_1^2 \pi\cdot 4x^{-2}\,dx$$

$$=\left[\pi\cdot(\tfrac{1}{3}x^3+2x-x^{-1})\right]_1^2-\left[\pi\cdot -4x^{-1}\right]_1^2=\left[\pi\cdot(\tfrac{1}{3}x^3+2x-\tfrac{1}{x})\right]_1^2+\left[\pi\cdot\tfrac{4}{x}\right]_1^2$$

$$=\pi\cdot(\tfrac{1}{3}\cdot 8+2\cdot 2-\tfrac{1}{2})-\pi\cdot(\tfrac{1}{3}\cdot 1+2\cdot 1-\tfrac{1}{1})+\pi\cdot\tfrac{4}{2}-\pi\cdot\tfrac{4}{1}=\tfrac{17}{6}\pi=2\tfrac{5}{6}\pi.$$

```
8/3+4-1/2-(1/3+2
-1)+2-4▸Frac
            17/6
```



D10a $f(x)=(\boxed{2x+6})^5\Rightarrow F(x)=\frac{1}{2}\cdot\frac{1}{6}\cdot(2x+6)^6+c=\frac{1}{12}(2x+6)^6+c.$

D10b $f(x)=\frac{10}{(3x-1)^2}=10\cdot(\boxed{3x-1})^{-2}\Rightarrow F(x)=\frac{1}{3}\cdot\frac{1}{-1}\cdot 10\cdot(3x-1)^{-1}+c=\frac{-10}{3(3x-1)}+c.$

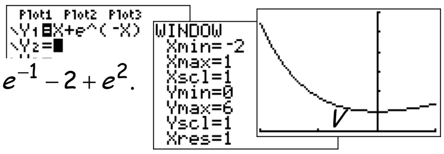
D10c $f(x)=(5x+2)^2\cdot\sqrt{5x+2}=(\boxed{5x+2})^{2\frac{1}{2}}\Rightarrow F(x)=\frac{1}{5}\cdot\frac{1}{3\frac{1}{2}}\cdot(5x+2)^{3\frac{1}{2}}+c=\frac{2}{35}\cdot(5x+2)^3\cdot\sqrt{5x+2}+c.$

D10d $f(x)=4\cdot e^{\boxed{2x+3}}\Rightarrow F(x)=\frac{1}{2}\cdot 4\cdot e^{2x+3}+c=2e^{2x+3}+c.$

D10e $f(x)=8\cdot 2^{\boxed{2x-1}}\Rightarrow F(x)=\frac{1}{2}\cdot 8\cdot\frac{1}{\ln(2)}\cdot 2^{2x-1}+c=\frac{4}{\ln(2)}\cdot 2^{2x-1}+c.$

D10f $f(x)=\ln(\boxed{2x+3})\Rightarrow F(x)=\frac{1}{2}\cdot((2x+3)\cdot\ln(2x+3)-(2x+3))+c=(x+1\frac{1}{2})\cdot\ln(2x+3)-x-1\frac{1}{2}+c.$

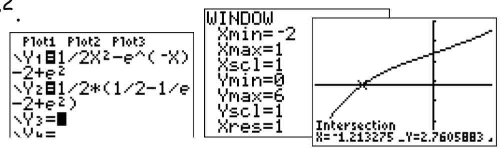
D11 $O(V)=\int_{-2}^{1}(x+e^{\boxed{-x}})\,dx=\left[\frac{1}{2}x^2-e^{-x}\right]_{-2}^{1}=\left(\frac{1}{2}-e^{-1}\right)-\left(\frac{1}{2}\cdot(-2)^2-e^2\right)=\frac{1}{2}-e^{-1}-2+e^2.$



$O(\text{deel I})=\int_{-2}^{p}(x+e^{-x})\,dx=\left[\frac{1}{2}x^2-e^{-x}\right]_{-2}^{p}=\frac{1}{2}p^2-e^{-p}-2+e^2.$

$O(\text{deel I})=\frac{1}{2}O(V)\Rightarrow\frac{1}{2}p^2-e^{-p}-2+e^2=\frac{1}{2}\cdot\left(\frac{1}{2}-e^{-1}-2+e^2\right).$

Intersect geeft dan $p\approx -1{,}213$.



D12a $I(\text{bol})=\frac{4}{3}\cdot\pi\cdot 4^3=\frac{256}{3}\pi.$

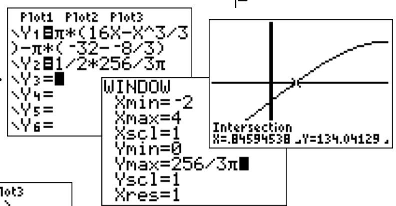
```
4/3*4^3▸Frac
           256/3
```

D12b $I=\int_{-4}^{-2}\pi y^2\,dx=\int_{-4}^{-2}\pi(16-x^2)\,dx=\left[\pi(16x-\frac{1}{3}x^3)\right]_{-4}^{-2}=\pi(-32-\frac{-8}{3})-\pi(-64-\frac{-64}{3})=\frac{40}{3}\pi=13\frac{1}{3}\pi.$

```
-32+8/3-(-64+64/
3)▸Frac
            40/3
```

D12c $I(L_p)=\int_{-2}^{p}\pi y^2\,dx=\left[\pi(16x-\frac{1}{3}x^3)\right]_{-2}^{p}=\pi(16p-\frac{p^3}{3})-\pi(-32-\frac{-8}{3})=\frac{1}{2}\cdot\frac{256}{3}\pi.$

Intersect geeft $p\approx 0{,}85$.

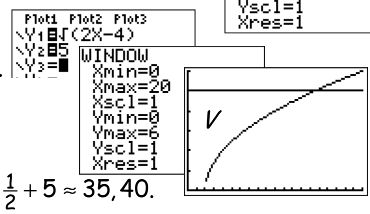


D13a $f(x)=0\Rightarrow\sqrt{2x-4}=0\Rightarrow 2x-4=0\Rightarrow 2x=4\Rightarrow x=2.$

$f(x)=5\Rightarrow\sqrt{2x-4}=5\Rightarrow 2x-4=25\Rightarrow 2x=29\Rightarrow x=\frac{29}{2}=14\frac{1}{2}.$

$f(x)=\sqrt{\boxed{2x-4}}\Rightarrow f'(x)=\frac{1}{2\cdot\sqrt{2x-4}}\cdot 2=\frac{1}{\sqrt{2x-4}}.$

$\int_2^{14\frac{1}{2}}\sqrt{1+\left(\frac{1}{\sqrt{2x-4}}\right)^2}\,dx$ (fnInt) $\approx 13{,}903\Rightarrow$ omtrek $=2+13{,}903+14\frac{1}{2}+5\approx 35{,}40.$




```
fnInt(√(1+1/(2X-
4)),X,2,14.5)
     13.90376303
Ans+2+14.5+5
     35.40376303
```

D13b $\int_2^{14\frac{1}{2}}\pi y^2\,dx=\int_2^{14\frac{1}{2}}\pi\cdot(2x-4)\,dx=\left[\pi(x^2-4x)\right]_2^{14\frac{1}{2}}=\pi(14\frac{1}{2}^2-4\cdot 14\frac{1}{2})-\pi(2^2-4\cdot 2)=156\frac{1}{4}\pi.$

$I=\pi\cdot 5^2\cdot 14\frac{1}{2}-156\frac{1}{4}\pi=206\frac{1}{4}\pi.$

```
14.5²-4*14.5-(2²
-4*2)
          156.25
5²*14.5-Ans
          206.25
```

D13c $\int_0^5\pi x^2\,dy=\int_0^5\pi\cdot(\frac{1}{4}y^4+2y^2+4)\,dy=\left[\pi(\frac{1}{20}y^5+\frac{2}{3}y^3+4y)\right]_0^5=\pi(\frac{1}{20}\cdot 5^5+\frac{2}{3}\cdot 5^3+4\cdot 5)=259\frac{7}{12}\pi.$

Gebruik: $y=\sqrt{2x-4}\Rightarrow 2x-4=y^2\Rightarrow 2x=y^2+4\Rightarrow x=\frac{1}{2}y^2+2\Rightarrow x^2=\frac{1}{4}y^4+2y^2+4.$

```
1/20*5^5+2/3*5^3
+4*5▸Frac
         3115/12
3115/12
     259.5833333
3115-259*12
               7
```

Gemengde opgaven 10. Integraalrekening

G13a $f(x)=7\cdot\log(\boxed{5x})\Rightarrow F(x)=7\cdot\frac{1}{\ln(10)}\cdot\frac{1}{5}\cdot(5x\cdot\ln(5x)-5x)+c=7\cdot\frac{x\cdot\ln(5x)-x}{\ln(10)}+c.$

G13b $f(x)=e^{\boxed{\frac{1}{2}x-1}}\Rightarrow F(x)=\frac{1}{\frac{1}{2}}\cdot e^{\frac{1}{2}x-1}+c=2e^{\frac{1}{2}x-1}+c.$

G13c $f(x)=5\cdot\ln(2x-4)^2=10\cdot\ln(\boxed{2x-4})\Rightarrow F(x)=\frac{1}{2}\cdot 10\cdot((2x-4)\ln(2x-4)-(2x-4))+c$
$=5(2x-4)\ln(2x-4)-5(2x-4)+c.$

G13d $f(x)=\ln(x^2-6x+9)=\ln(x-3)^2=2\cdot\ln(\boxed{x-3})\Rightarrow F(x)=\frac{1}{1}\cdot 2\cdot((x-3)\ln(x-3)-(x-3))+c$
$=2(x-3)\ln(x-3)-2(x-3)+c.$

G13e $f(x)=\frac{x^4-6x^2\cdot\sqrt{x}+8x}{x^3}=x-6x^{-\frac{1}{2}}+8x^{-2}\Rightarrow F(x)=\frac{1}{2}\cdot x^2-6\cdot\frac{1}{\frac{1}{2}}\cdot x^{\frac{1}{2}}+8\cdot\frac{1}{-1}\cdot x^{-1}+c=\frac{1}{2}x^2-12\cdot\sqrt{x}-\frac{8}{x}+c.$

G13f $f(x)=(x^2+3)^2=x^4+6x^2+9\Rightarrow F(x)=\frac{1}{5}\cdot x^5+6\cdot\frac{1}{3}\cdot x^3+9x+c=\frac{1}{5}x^5+2x^3+9x+c.$

G14a $f(x)=\sqrt{6x+3}=(\boxed{6x+3})^{\frac{1}{2}}\Rightarrow F(x)=\frac{1}{6}\cdot\frac{1}{\frac{3}{2}}\cdot(6x+3)^{\frac{3}{2}}+c=\frac{1}{6}\cdot\frac{2}{3}\cdot(6x+3)^{1\frac{1}{2}}+c=\frac{1}{9}\cdot(6x+3)\cdot\sqrt{6x+3}+c.$

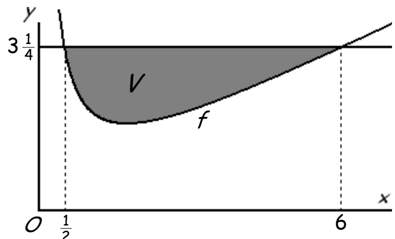
G14b $f(x)=\frac{10}{2x-1}=10\cdot\frac{1}{\boxed{2x-1}}\Rightarrow F(x)=10\cdot\frac{1}{2}\cdot\ln|2x-1|+c=5\cdot\ln|2x-1|+c.$

G14c $f(x)=(\boxed{3x-6})^{-2}\Rightarrow F(x)=\frac{1}{3}\cdot\frac{1}{-1}\cdot(3x-6)^{-1}+c=-\frac{1}{3}\cdot\frac{1}{3x-6}+c=\frac{1}{3(3x-6)}+c.$

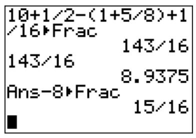
G14d $f(x)=(2x+5)^{-1}=\frac{1}{\boxed{2x+5}}\Rightarrow F(x)=\frac{1}{2}\cdot\ln|2x+5|+c.$

G14e $f(x)=10^{\boxed{2x-3}}\Rightarrow F(x)=\frac{1}{2}\cdot\frac{1}{\ln(10)}\cdot 10^{2x-3}+c=\frac{10^{2x-3}}{2\cdot\ln(10)}+c.$

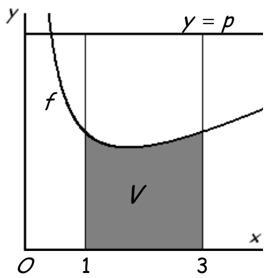
G14f $f(x)=\frac{8^x-1}{2^x}=\frac{8^x}{2^x}-\frac{1}{2^x}=4^x-2^{\boxed{-x}}\Rightarrow F(x)=\frac{1}{\ln(4)}\cdot 4^x-\frac{1}{-1}\cdot\frac{1}{\ln(2)}\cdot 2^{-x}+c=\frac{4^x}{\ln(4)}+\frac{2^{-x}}{\ln(2)}+c.$



G15a $f(x)=3\frac{1}{4}\Rightarrow\frac{x^2+3}{2x}=3\frac{1}{4}\Rightarrow x^2+3=6\frac{1}{2}x\Rightarrow$
$x^2-6\frac{1}{2}x+3=0\Rightarrow(x-\frac{1}{2})(x-6)=0\Rightarrow x=\frac{1}{2}\vee x=6.$

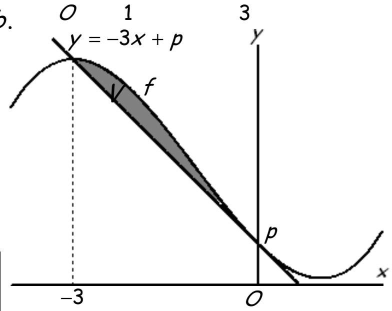


$O(V)=\int_{\frac{1}{2}}^{6}\left(3\frac{1}{4}-\frac{x^2+3}{2x}\right)dx=\int_{\frac{1}{2}}^{6}\left(3\frac{1}{4}-\frac{1}{2}x-\frac{3}{2}\cdot\frac{1}{x}\right)dx$
$=\left[3\frac{1}{4}x-\frac{1}{4}x^2-\frac{3}{2}\cdot\ln(x)\right]_{\frac{1}{2}}^{6}=\left(19\frac{1}{2}-9-1\frac{1}{2}\ln(6)\right)-\left(1\frac{5}{8}-\frac{1}{16}-1\frac{1}{2}\ln(\frac{1}{2})\right)$
$=10\frac{1}{2}-1\frac{5}{8}+\frac{1}{16}-1\frac{1}{2}\ln(6)+1\frac{1}{2}\ln(\frac{1}{2})=8\frac{15}{16}-1\frac{1}{2}\ln(12).$

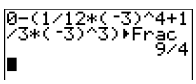


G15b $O(V)=\int_{1}^{3}\left(\frac{x^2+3}{2x}\right)dx=\left[\frac{1}{4}x^2+\frac{3}{2}\cdot\ln(x)\right]_{1}^{3}=\left(2\frac{1}{4}+1\frac{1}{2}\ln(3)\right)-\left(\frac{1}{4}+1\frac{1}{2}\ln(1)\right)=2+1\frac{1}{2}\ln(3).$
$O(V)=\frac{1}{2}\cdot O(\text{rechthoek})\Rightarrow 2+1\frac{1}{2}\ln(3)=\frac{1}{2}\cdot 2\cdot p\Rightarrow p=2+1\frac{1}{2}\ln(3).$

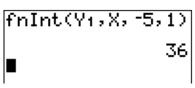
G16a $f_p(x)=\frac{1}{3}x^3+x^2-3x+p\Rightarrow f_p'(x)=x^2+2x-3.$ Stel de raaklijn is $y=ax+b$.
$a=f_p'(0)=-3$ en $b=f_p(0)=p\Rightarrow y=-3x+p.$
$f_p(x)=-3x+p\Rightarrow\frac{1}{3}x^3+x^2-3x+p=-3x+p\Rightarrow$
$\frac{1}{3}x^3+x^2=0\Rightarrow x^2\cdot(\frac{1}{3}x+1)=0\Rightarrow x=0\ \vee\ \frac{1}{3}x=-1\Rightarrow x=0\ \vee\ x=-3.$

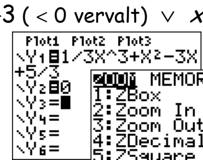


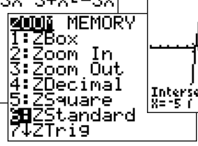
$O(V)=\int_{-3}^{0}\left(f_p(x)-(-3x+p)\right)dx=\int_{-3}^{0}\left(\frac{1}{3}x^3+x^2-3x+p+3x-p\right)dx$
$=\int_{-3}^{0}\left(\frac{1}{3}x^3+x^2\right)dx=\left[\frac{1}{12}x^4+\frac{1}{3}x^3\right]_{-3}^{0}=0-\left(\frac{81}{12}-9\right)=2\frac{1}{4}.$

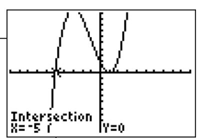


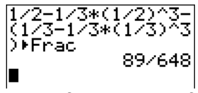
G16b $f_p'(x)=0\Rightarrow x^2+2x-3=0\Rightarrow(x+3)(x-1)=0\Rightarrow x=-3$ (< 0 vervalt) $\vee\ x=1.$
$f_p(1)=0\Rightarrow\frac{1}{3}\cdot 1^3+1^2-3\cdot 1+p=0\Rightarrow p=1\frac{2}{3}.$
$f_{1\frac{2}{3}}(x)=0\Rightarrow\frac{1}{3}x^3+x^2-3x+1\frac{2}{3}=0$ (intersect) $\Rightarrow x=-5.$
$O(W)=\int_{-5}^{1}\left(f_{1\frac{2}{3}}(x)\right)dx$ (fnInt) $=36.$









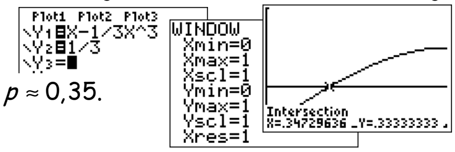


G17a $I = \int_{\frac{1}{3}r}^{\frac{1}{2}r} \pi y^2 \, dx = \int_{\frac{1}{3}r}^{\frac{1}{2}r} \pi(r^2 - x^2) \, dx = \left[\pi(r^2x - \frac{1}{3}x^3)\right]_{\frac{1}{3}r}^{\frac{1}{2}r} = \pi(r^2 \cdot \frac{1}{2}r - \frac{1}{3} \cdot (\frac{1}{2}r)^3) - \pi(r^2 \cdot \frac{1}{3}r - \frac{1}{3} \cdot (\frac{1}{3}r)^3) = \frac{89}{649}\pi r^3.$

G17b $I(L) = \int_{-pr}^{pr} \pi y^2 \, dx = 2 \cdot \int_{0}^{pr} \pi(r^2 - x^2) \, dx = 2 \cdot \left[\pi(r^2x - \frac{1}{3}x^3)\right]_0^{pr} = 2\pi(r^2 \cdot pr - \frac{1}{3} \cdot (pr)^3) - 2\pi \cdot 0 = 2\pi r^3 \cdot (p - \frac{1}{3}p^3).$

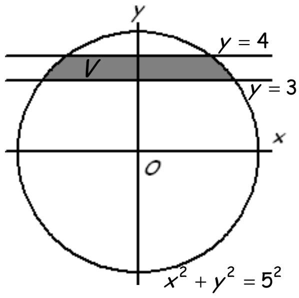
$I(B) = \frac{4}{3}\pi r^3.$

$I(L) = \frac{1}{2}I(B) \Rightarrow 2\pi r^3 \cdot (p - \frac{1}{3}p^3) = \frac{2}{3}\pi r^3 \Rightarrow p - \frac{1}{3}p^3 = \frac{1}{3}$ (intersect) $\Rightarrow p \approx 0{,}35.$



G18 $y = 3$ invullen in $x^2 + y^2 = 25$ geeft $x^2 + 9 = 25 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4.$

$y = 4$ invullen in $x^2 + y^2 = 25$ geeft $x^2 + 16 = 25 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3.$



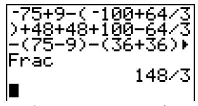
$I(L) = \int_{-4}^{-3} \pi y^2 \, dx + \int_{-3}^{3} \pi \cdot 4^2 \, dx + \int_{3}^{4} \pi y^2 \, dx - \int_{-4}^{4} \pi \cdot 3^2 \, dx$

$= \int_{-4}^{-3} \pi \cdot (25 - x^2) \, dx + \int_{-3}^{3} 16\pi \, dx + \int_{3}^{4} \pi \cdot (25 - x^2) \, dx - \int_{-4}^{4} 9\pi \, dx$

$= \left[\pi(25x - \frac{1}{3}x^3)\right]_{-4}^{-3} + [16\pi x]_{-3}^{3} + \left[\pi(25x - \frac{1}{3}x^3)\right]_{3}^{4} - [9\pi x]_{-4}^{4}$

$= \pi(25 \cdot -3 - \frac{1}{3} \cdot (-3)^3) - \pi(25 \cdot -4 - \frac{1}{3} \cdot (-4)^3) + 48\pi - -48\pi + \pi(25 \cdot 4 - \frac{1}{3} \cdot 4^3) - \pi(25 \cdot 3 - \frac{1}{3} \cdot 3^3) - (36\pi - -36\pi)$

$= \pi(-75 + 9) - \pi(-100 + \frac{64}{3}) + 48\pi + 48\pi + \pi(100 - \frac{64}{3}) - \pi(75 - 9) - (36\pi + 36\pi) = \frac{148}{3}\pi = 49\frac{1}{3}\pi.$

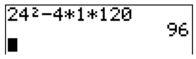


G19a $f_0(x) = 0 \Rightarrow -\frac{1}{3}x^3 + 4x = 0 \Rightarrow x^3 - 12x = 0 \Rightarrow x \cdot (x^2 - 12) = 0 \Rightarrow x = 0 \ \vee \ x^2 = 12 \Rightarrow x = 0 \ \vee \ x = \pm\sqrt{12} = \pm 2\sqrt{3}.$

$O(V) = \int_0^p \left(-\frac{1}{3}x^3 + 4x\right) dx = \left[-\frac{1}{12}x^4 + 2x^2\right]_0^p = \left(-\frac{1}{12}p^4 + 2p^2\right) - 0 = -\frac{1}{12}p^4 + 2p^2.$

$O(V) = 10 \Rightarrow -\frac{1}{12}p^4 + 2p^2 = 10$ (stel $p^2 = t$) $\Rightarrow -\frac{1}{12}t^2 + 2t - 10 = 0 \Rightarrow t^2 - 24t + 120 = 0.$

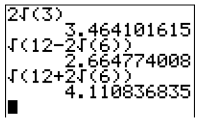
$D = (24)^2 - 4 \cdot 1 \cdot 120 = 96 \Rightarrow \sqrt{D} = \sqrt{96} = \sqrt{16 \cdot 6} = 4 \cdot \sqrt{6}$

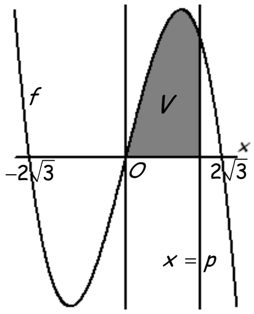


$t = p^2 = \frac{24 - 4 \cdot \sqrt{6}}{2} = 12 - 2\sqrt{6} \ \vee \ t = p^2 = \frac{24 + 4 \cdot \sqrt{6}}{2} = 12 + 2\sqrt{6}$

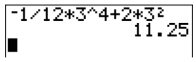
$p = \sqrt{12 - 2\sqrt{6}} \ \vee \ p = -\sqrt{12 - 2\sqrt{6}} \ \vee \ p = \sqrt{12 + 2\sqrt{6}} \ \vee \ p = -\sqrt{12 + 2\sqrt{6}}$

Er moet gelden $0 < p < 2\sqrt{3} = \sqrt{12} \Rightarrow \sqrt{12 - 2\sqrt{6}}.$





G19b $\int_0^3 \left(-\frac{1}{3}x^3 + 4x + a\right) dx = \left[-\frac{1}{12}x^4 + 2x^2 + ax\right]_0^3 = \left(-\frac{1}{12} \cdot 3^4 + 2 \cdot 3^2 + a \cdot 3\right) - 0 = 3a + 11\frac{1}{4}.$



$O(W) = 10 \Rightarrow 3a + 11\frac{1}{4} = 10 \vee 3a + 11\frac{1}{4} = -10$ (grafiek onder de x-as)

$3a = -1\frac{1}{4} = -\frac{5}{4} \vee 3a = -21\frac{1}{4} \Rightarrow a = -\frac{5}{12}$ (voldoet niet, want er ontstaat niet HET vlakdeel W) $\vee\ a = -7\frac{1}{12}$ (deze voldoet).

G20a $f(x) = \frac{1}{4}x^2 \Rightarrow f'(x) = \frac{1}{2}x.$

$f'(x) = 1 \Rightarrow \frac{1}{2}x = 1 \Rightarrow x = 2.$ Dus raakpunt $R(2, f(2)) = R(2, 1).$

Raaklijn door $R(2,1)$ van de vorm $y = x + b$ met $1 = 2 + b \Rightarrow b = -1.$ Dus de raaklijn door $R(2,1)$ is $y = x - 1.$

$g(x) = -\frac{4}{x^2} = -4x^{-2} \Rightarrow g'(x) = 8x^{-3} = \frac{8}{x^3}.$

$g'(x) = 1 \Rightarrow \frac{8}{x^3} = 1 \Rightarrow x^3 = 8 \Rightarrow x = 2.$ Dus raakpunt $S(2, g(2)) = S(2, -1).$

Raaklijn door $S(2,-1)$ van de vorm $y = x + b$ met $-1 = 2 + b \Rightarrow b = -3.$ Dus de raaklijn door $S(2,-1)$ is $y = x - 3.$

De raaklijnen snijden de y-as in $(0,-1)$ en $(0,-3)$. De diagonaal van het vierkant is dus 2.

G20b $f(a) = \frac{1}{4}a^2 \Rightarrow C(a, \frac{1}{4}a^2)$ en $D(-a, \frac{1}{4}a^2)$; $g(a) = -\frac{4}{a^2} \Rightarrow B(a, -\frac{4}{a^2})$ en $A(-a, -\frac{4}{a^2}).$

$O(ABCD) = 2a \cdot (\frac{1}{4}a^2 + \frac{4}{a^2}) = \frac{1}{2}a^3 + \frac{8}{a}.$

O(vlakdeel boven de grafiek van f) $= 2 \cdot \int_0^a \left(\frac{1}{4}a^2 - \frac{1}{4}x^2\right) dx = 2 \cdot \left[\frac{1}{4}a^2x - \frac{1}{12}x^3\right]_0^a = 2 \cdot \left(\frac{1}{4}a^3 - \frac{1}{12}a^3\right) = \frac{1}{3}a^3.$

O(vlakdeel boven de grafiek van f) $= \frac{1}{2} \cdot O(ABCD)$

$\frac{1}{3}a^3 = \frac{1}{4}a^3 + \frac{4}{a} \Rightarrow \frac{1}{12}a^3 = \frac{4}{a} \Rightarrow a^4 = 48 \Rightarrow a = \sqrt[4]{48}.$

G21a $A(0,\, e^0) = A(0,1)$ en $B(1,\, e^{-1}) = B(1,\, \frac{1}{e})$.

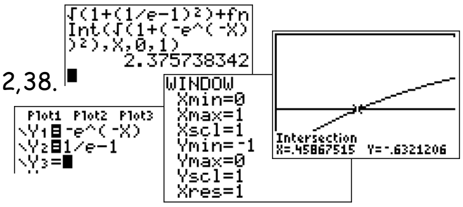
AB: $y = \left(\frac{1}{e} - 1\right)x + b$ door $A(0,1) \Rightarrow b = 1$. Dus AB: $y = \left(\frac{1}{e} - 1\right)x + 1$.

$$O(V) = \int_0^1 \left(\left(\tfrac{1}{e} - 1\right)x + 1\right)\,dx - \int_0^1 e^{\boxed{-x}}\,dx = \left[\tfrac{1}{2}\cdot\left(\tfrac{1}{e} - 1\right)x^2 + x\right]_0^1 - \left[-e^{-x}\right]_0^1$$

$$= \tfrac{1}{2}\cdot\left(\tfrac{1}{e} - 1\right)\cdot 1^2 + 1 - 0 - \left(-e^{-1} - -e^0\right) = \tfrac{1}{2e} - \tfrac{1}{2} + 1 + e^{-1} - 1 = \tfrac{1}{2e} - \tfrac{1}{2} + \tfrac{1}{e} = \tfrac{3}{2e} - \tfrac{1}{2}.$$

G21b $f(x) = e^{\boxed{-x}} \Rightarrow f'(x) = -e^{-x}$.

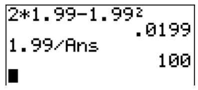
Omtrek $= AB + \text{boog } AB = \sqrt{1^2 + \left(\frac{1}{e} - 1\right)^2} + \int_0^1 \sqrt{1 + \left(-e^{-x}\right)^2}\,dx$ (fnInt) $\approx 2{,}38$.



G21c $f'(x) = rc_{AB} \Rightarrow -e^{-x} = \frac{1}{e} - 1$ (intersect) $\Rightarrow x \approx 0{,}46$.

G22a $O = \int_0^2 3(2x - x^2)\,dx - \int_0^2 2(2x - x^2)\,dx = \left[3(x^2 - \frac{1}{3}x^3)\right]_0^2 - \left[2(x^2 - \frac{1}{3}x^3)\right]_0^2 = 3(4 - \frac{8}{3}) - 2(4 - \frac{8}{3}) = 4 - \frac{8}{3} = 1\frac{1}{3}$.

G22b $x = 1{,}99$ invullen in $n(2x - x^2) = x$ geeft $n\cdot(2\cdot 1{,}99 - 1{,}99^2) = 1{,}99 \Rightarrow n\cdot 0{,}0199 = 1{,}99 \Rightarrow n = 100$.
Dus voor $n > 100$ is $x_{S_n} > 1{,}99$.



G22c $y = n(2x - x^2) = 2nx - nx^2 \Rightarrow y' = \left[\frac{dy}{dx}\right] = 2n - 2nx$. Dus $\left[\frac{dy}{dx}\right]_{x=0} = 2n \Rightarrow$ raaklijn in O is $y = 2nx$.

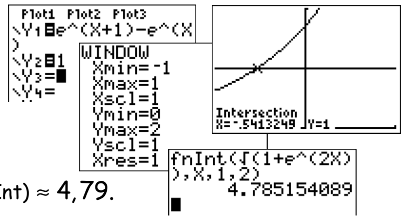
$x = 1$ invullen in $y = 2nx$ geeft $y = 2n \Rightarrow R_n(1,\, 2n)$
$x = 1$ invullen in $y = n(2x - x^2)$ geeft $y = n\cdot(2\cdot 1 - 1^2) = n\cdot 1 = n \Rightarrow T_n(1,\, n)$
$\Rightarrow T_n$ is het midden van AR_n.

G23a $O = (a + 1 - a)\cdot e^{a+1} - \int_a^{a+1} e^x\,dx = e^{a+1} - \left[e^x\right]_a^{a+1} = e^{a+1} - (e^{a+1} - e^a) = e^{a+1} - e^{a+1} + e^a = e^a$.

$O = 3 \Rightarrow e^a = 3 \Rightarrow a = \ln(3)$.

G23b $A(a,\, e^a)$ en $B(a+1,\, e^{a+1}) \Rightarrow \text{rc}_{AB} = \frac{e^{a+1} - e^a}{a + 1 - a} = e^{a+1} - e^a$.

$\text{rc}_{AB} < 1 \Rightarrow e^{a+1} - e^a < 1$ (intersect) $\Rightarrow a < -0{,}54$.



G23c $f(x) = e^x \Rightarrow f'(x) = e^x$. Boog $AB = \int_1^2 \sqrt{1 + \left(e^x\right)^2}\,dx$ (fnInt) $\approx 4{,}79$.

G23d $OPAQ$ wentelen om de x-as geeft: $I_1 = \int_0^1 \pi e^2\,dx = \left[\pi e^2 x\right]_0^1 = \pi e^2$ of $I_1 = G\cdot h = \pi\cdot e^2\cdot 1 = \pi e^2$.

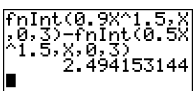
Het deel onder de grafiek van f wentelen om de x-as geeft:

$I_2 = \int_0^1 \pi(e^x)^2\,dx = \int_0^1 \pi\cdot e^{\boxed{2x}}\,dx = \left[\frac{1}{2}\pi e^x\right]_0^1 = \frac{1}{2}\pi e^2 - \frac{1}{2}\pi$.

Het deel boven de grafiek van f wentelen om de x-as geeft: $I_3 = I_1 - I_2$.

Het verschil tussen de inhouden is: $I_3 - I_2 = (I_1 - I_2) - I_2 = I_1 - 2I_2 = \pi e^2 - 2\cdot(\frac{1}{2}\pi e^2 - \frac{1}{2}\pi) = \pi$.

G24a $O = \int_0^3 0{,}9x^{1\frac{1}{2}}\,dx - \int_0^3 0{,}5x^{1\frac{1}{2}}\,dx$ (fnInt) $\approx 2{,}49$.



G24b $I = \int_0^3 \pi\cdot(2x^{1\frac{1}{2}})^2\,dx = \int_0^3 4\pi x^3\,dx = \left[\frac{1}{4}\cdot 4\pi x^4\right]_0^3 = \left[\pi x^4\right]_0^3 = 81\pi$.

G24c $f_{\frac{2}{3}}(x) = \frac{2}{3}\cdot x^{1\frac{1}{2}} \Rightarrow f_{\frac{2}{3}}'(x) = \frac{2}{3}\cdot\frac{3}{2}\cdot x^{\frac{1}{2}} = \sqrt{x}$.

$$L(\tfrac{2}{3}) = \int_0^3 \sqrt{1 + (\sqrt{x})^2}\,dx = \int_0^3 \sqrt{1 + x}\,dx = \int_0^3 (\boxed{1 + x})^{\frac{1}{2}}\,dx$$

$$= \left[\frac{1}{1\frac{1}{2}}(1 + x)^{1\frac{1}{2}}\right]_0^3 = \left[\tfrac{2}{3}(1 + x)^{1\frac{1}{2}}\right]_0^3 = \left[\tfrac{2}{3}\cdot(1 + x)\cdot\sqrt{(1 + x)}\right]_0^3 = \tfrac{2}{3}\cdot 4\cdot\sqrt{4} - \tfrac{2}{3}\cdot 1\cdot\sqrt{1} = \tfrac{16}{3} - \tfrac{2}{3} = \tfrac{14}{3}.$$

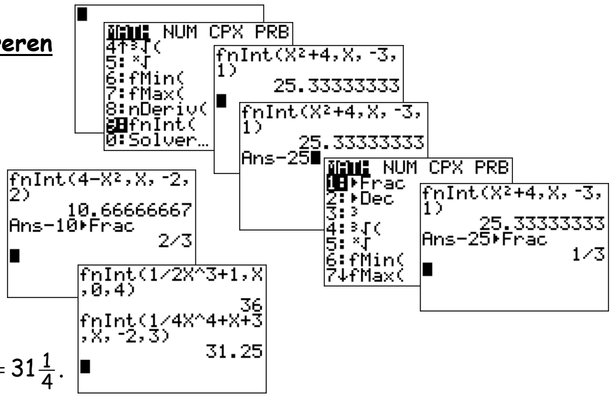
TI-84 10. Integreren

1a $\int_{-3}^{1} (x^2+4)\,\mathrm{d}x$ (fnInt) $= 25\frac{1}{3}$.

1b $\int_{-2}^{2} (4-x^2)\,\mathrm{d}x$ (fnInt) $= 10\frac{2}{3}$.

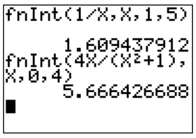
1c $\int_{0}^{4} (\frac{1}{2}x^3+1)\,\mathrm{d}x$ (fnInt) $= 36$.

1d $\int_{-2}^{3} (\frac{1}{4}x^4+x+3)\,\mathrm{d}x$ (fnInt) $= 31\frac{1}{4}$.



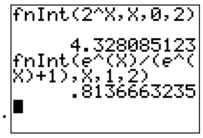
2a $\int_{1}^{5} \frac{1}{x}\,\mathrm{d}x$ (fnInt) $\approx 1{,}61$.

2b $\int_{0}^{4} \frac{4x}{x^2+1}\,\mathrm{d}x$ (fnInt) $\approx 5{,}67$.



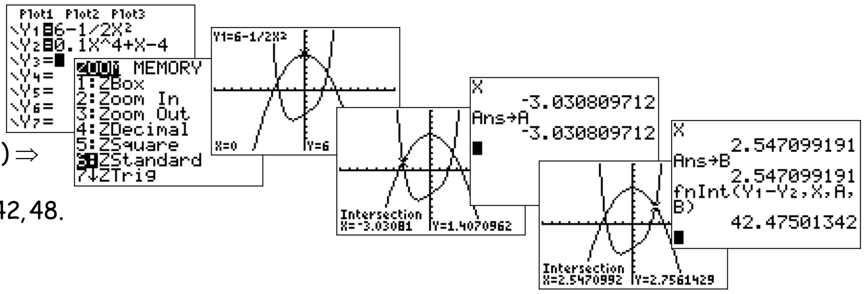
2c $\int_{0}^{2} 2^x\,\mathrm{d}x$ (fnInt) $\approx 4{,}33$.

2d $\int_{1}^{2} \frac{e^x}{e^x+1}\,\mathrm{d}x$ (fnInt) $\approx 0{,}81$.



TI-84 11. Integralen

1 $f(x) = g(x)$ (intersect) $\Rightarrow$
$x_A \approx -3{,}031$ en $x_B \approx 2{,}547$.
De grafiek van $f(x)$ loopt tussen de snijpunten boven de grafiek van $g(x) \Rightarrow$
$O(V) = \int_{x_A}^{x_B} (f(x)-g(x))\,\mathrm{d}x$ (fnInt) $\approx 42{,}48$.

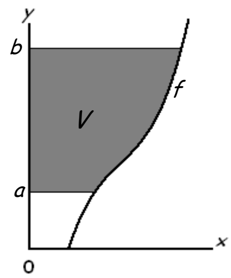


$f(x)$	primitieven $F(x)$
ax^n	$\frac{a}{n+1}\cdot x^{n+1}+c$
g^x	$\frac{1}{\ln(g)}\cdot g^x+c$
e^x	e^x+c
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
$\ln(x)$	$x\cdot\ln(x)-x+c$
${}^g\log(x)$	$\frac{1}{\ln(g)}\cdot(x\cdot\ln(x)-x)+c$
$f(ax+b)$	$\frac{1}{a}\cdot F(ax+b)+c$

$$\int_a^b f(x)\,\mathrm{d}x = \left[F(x)\right]_a^b = F(b)-F(a).$$

$O(\text{cirkel}) = \pi r^2$
$I(\text{cilinder}) = Gh = \pi r^2 h$
$I(\text{kegel}) = \frac{1}{3}Gh = \frac{1}{3}\pi r^2 h$
$I(\text{bol}) = \frac{4}{3}\pi r^3$

De lengte van de grafiek van f tussen $x=a$ en $x=b$ is $\int_a^b \sqrt{1+(f'(x))^2}\,\mathrm{d}x$.



Het vlakdeel V ligt rechts van de y-as en wordt ingesloten door de grafiek van de functie f, de y-as en de lijnen $y=a$ en $y=b$.

De inhoud van het lichaam L dat ontstaat als V om de y-as wentelt is $I(L) = \int_a^b \pi x^2\,\mathrm{d}y$.