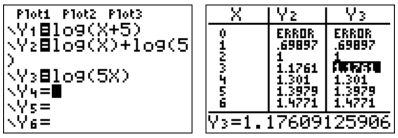
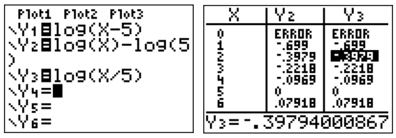
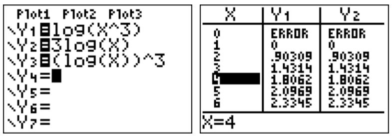


1a $y = \log(x) + \log(5)$ en $y = \log(5x)$ komen op hetzelfde neer.

1b $y = \log(x) - \log(5)$ en $y = \log(\frac{x}{5})$ komen op hetzelfde neer.



1c $y = \log(x^3)$ en $y = 3 \cdot \log(x)$ komen op hetzelfde neer.



2a $^2\log(6) + {}^2\log(10) = {}^2\log(6 \cdot 10) = {}^2\log(60)$.

2b $^3\log(30) - {}^3\log(6) = {}^3\log(\frac{30}{6}) = {}^3\log(5)$.

2c $2 \cdot {}^5\log(3) + {}^5\log(\frac{1}{2}) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9 \cdot \frac{1}{2}) = {}^5\log(4\frac{1}{2})$.

2d $^{\frac{1}{2}}\log(15) - 4 \cdot {}^{\frac{1}{2}}\log(3) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(3^4) = {}^{\frac{1}{2}}\log(\frac{15}{81}) = {}^{\frac{1}{2}}\log(\frac{5}{27})$.

2e $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36} \cdot 12) = {}^4\log(\frac{12}{36}) = {}^4\log(\frac{1}{3})$.

2f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(\frac{50}{25}) = \log(2)$.

3a $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$.

3b $3 - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log(\frac{1}{8} : 10) = {}^{\frac{1}{2}}\log(\frac{1}{80})$.

3c $2 - \log(5) = \log(10^2) - \log(5) = \log(\frac{100}{5}) = \log(20)$.

3d $^2\log(12) - {}^3\log(9) = {}^2\log(12) - {}^3\log(3^2) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(\frac{12}{4}) = {}^2\log(3)$.

3e $\frac{1}{2} \cdot {}^3\log(16) + {}^{\frac{1}{2}}\log(8) = {}^3\log(16^{\frac{1}{2}}) + {}^{\frac{1}{2}}\log(2^3) = {}^3\log(\sqrt{16}) + {}^{\frac{1}{2}}\log((\frac{1}{2})^{-3})$
$= {}^3\log(4) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log(\frac{4}{27})$.

3f $\log(500) - {}^5\log(125) = \log(500) - {}^5\log(5^3) = \log(500) - 3 = \log(500) - \log(10^3) = \log(\frac{500}{1000}) = \log(\frac{1}{2})$.

4a $^3\log(6) + {}^3\log(1\frac{1}{2}) = {}^3\log(6 \cdot 1\frac{1}{2}) = {}^3\log(9) = {}^3\log(3^2) = 2$.

4b $^5\log(2) - {}^5\log(50) = {}^5\log(\frac{2}{50}) = {}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$.

4c $^2\log(27) + 3 \cdot {}^2\log(\frac{1}{6}) = {}^2\log(27) + {}^2\log((\frac{1}{6})^3) = {}^2\log(27 \cdot \frac{1}{216}) = {}^2\log(\frac{27}{216}) = {}^2\log(\frac{1}{8}) = {}^2\log(2^{-3}) = -3$.

4d $2 \cdot {}^4\log(6) - 2 \cdot {}^4\log(3) = {}^4\log(6^2) - {}^4\log(3^2) = {}^4\log(36) - {}^4\log(9) = {}^4\log(\frac{36}{9}) = {}^4\log(4) = 1$.

5a $g^{{}^g\log(a) - {}^g\log(b)} = \dfrac{g^{{}^g\log(a)}}{g^{{}^g\log(b)}} = \dfrac{a}{b} = g^{{}^g\log(\frac{a}{b})}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log(\frac{a}{b})$.

5b $g^{n \cdot {}^g\log(a)} = \left(g^{{}^g\log(a)}\right)^n = a^n = g^{{}^g\log(a^n)}$, dus $n \cdot {}^g\log(a) = {}^g\log(a^n)$.

6a $3 + {}^2\log(3) = {}^2\log(2^3) + {}^2\log(3) = {}^2\log(8 \cdot 3) = {}^2\log(24)$.

6b $^2\log(x+1) = 3 + {}^2\log(3)$ BV = beginvoorwaarde: $x + 1 > 0 \Rightarrow x > -1$
$^2\log(x+1) = {}^2\log(24)$
$x + 1 = 24$
$x = 23$ (voldoet).

7a $^5\log(x) = 3 \cdot {}^5\log(2) - 2 \cdot {}^5\log(3)$ BV: $x > 0$
$^5\log(x) = {}^5\log(2^3) - {}^5\log(3^2)$
$^5\log(x) = {}^5\log(\frac{8}{9})$
$x = \frac{8}{9}$ (voldoet).

7b $^2\log(x) = 4 - {}^2\log(3)$ BV: $x > 0$
$^2\log(x) = {}^2\log(2^4) - {}^2\log(3)$
$^2\log(x) = {}^2\log(\frac{16}{3})$
$x = \frac{16}{3}$ (voldoet).

7c $^2\log(x+3) = 3 + {}^2\log(x)$ BV: $x > -3$ én $x > 0 \Rightarrow x > 0$
$^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$
$^2\log(x+3) = {}^2\log(8 \cdot x)$
$x + 3 = 8x$
$-7x = -3$
$x = \frac{3}{7}$ (voldoet).

7d $^3\log(2x) = 1 + {}^3\log(x+1)$ BV: $2x > 0$ én $x > -1 \Rightarrow x > 0$
$^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$
$^3\log(2x) = {}^3\log(3 \cdot (x+1))$
$2x = 3x + 3$
$-x = 3$
$x = -3$ (voldoet niet).

8a
$5 \cdot \log(x) = 5 - \log(3125)$ BV: $x > 0$
$\log(x^5) = \log(10^5) - \log(3125)$
$\log(x^5) = \log(\frac{100000}{3125})$
$x^5 = 32 \Rightarrow x = 2$ (voldoet).

```
100000/3125
                 32
Ans^(1/5)
                  2
```

8b
$^{\frac{1}{2}}\log(2x-1) = 2 + {^{\frac{1}{2}}\log(x+2)}$ BV: $x > \frac{1}{2}$
$^{\frac{1}{2}}\log(2x-1) = {^{\frac{1}{2}}\log((\frac{1}{2})^2)} + {^{\frac{1}{2}}\log(x+2)}$
$^{\frac{1}{2}}\log(2x-1) = {^{\frac{1}{2}}\log(\frac{1}{4} \cdot (x+2))}$
$2x - 1 = \frac{1}{4} \cdot (x+2)$ (links en rechts $\times 4$)
$8x - 4 = x + 2$
$7x = 6 \Rightarrow x = \frac{6}{7}$ (voldoet).

9a
$^{5}\log(x) = 2 + \frac{1}{2} \cdot {^{5}\log(3)}$ BV: $x > 0$
$^{5}\log(x) = {^{5}\log(5^2)} + {^{5}\log(3^{\frac{1}{2}})}$
$^{5}\log(x) = {^{5}\log(25 \cdot \sqrt{3})}$
$x = 25 \cdot \sqrt{3}$ (voldoet).

9b
$^{3}\log(x+4) + 1 = 2 \cdot {^{3}\log(x-2)}$ BV: $x > 2$
$^{3}\log(x+4) + {^{3}\log(3)} = {^{3}\log((x-2)^2)}$
$^{3}\log(3 \cdot (x+4)) = {^{3}\log((x-2)^2)}$
$3x + 12 = x^2 - 4x + 4$
$x^2 - 7x - 8 = 0$
$(x-8) \cdot (x+1) = 0$
$x = 8$ (voldoet) $\vee$ $x = -1$ (vold. niet).

9c
$^{2}\log(2x) - {^{2}\log(x+3)} = {^{2}\log(x)} - 2$ BV: $x > 0$
$^{2}\log(2x) - {^{2}\log(x+3)} = {^{2}\log(x)} - {^{2}\log(2^2)}$
$^{2}\log(\frac{2x}{x+3}) = {^{2}\log(\frac{x}{4})}$
$x \cdot (x+3) = 4 \cdot 2x$
$x^2 + 3x = 8x$
$x^2 - 5x = 0$
$x \cdot (x-5) = 0$
$x = 0$ (vold. niet) $\vee$ $x = 5$ (voldoet).

8c
$^{3}\log(x+2) = 1 - {^{3}\log(x)}$ BV: $x > -2$ én $x > 0 \Rightarrow x > 0$
$^{3}\log(x+2) = {^{3}\log(3)} - {^{3}\log(x)}$
$^{3}\log(x+2) = {^{3}\log(\frac{3}{x})}$
$x + 2 = \frac{3}{x}$
$x^2 + 2x = 3$
$x^2 + 2x - 3 = 0$
$(x+3) \cdot (x-1) = 0$
$x = -3$ (vold. niet) $\vee$ $x = 1$ (voldoet).

8d
$2 \cdot {^{3}\log(x)} + 1 = {^{3}\log(5x-2)}$ BV: $x > 0$ én $x > \frac{2}{5} \Rightarrow x > \frac{2}{5}$
$^{3}\log(x^2) + {^{3}\log(3)} = {^{3}\log(5x-2)}$
$^{3}\log(3x^2) = {^{3}\log(5x-2)}$
$3x^2 = 5x - 2$
$3x^2 - 5x + 2 = 0$ (*abc*-formule)
$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 25 - 24 = 1$
$x = \frac{5 \pm \sqrt{1}}{2 \cdot 3} = \frac{5 \pm 1}{6}$
$x = \frac{5+1}{6} = 1$ (voldoet) $\vee$ $x = \frac{5-1}{6} = \frac{2}{3}$ (voldoet).

9d
$^{3}\log(x) = 2 - {^{3}\log(x-1)}$ BV: $x > 1$
$^{3}\log(x) = {^{3}\log(3^2)} - {^{3}\log(x-1)}$
$^{3}\log(x) = {^{3}\log(\frac{9}{x-1})}$
$x \cdot (x-1) = 9$
$x^2 - x - 9 = 0$ (*abc*-formule)
$D = (-1)^2 - 4 \cdot 1 \cdot -9 = 1 + 36 = 37$
$x = \frac{1 \pm \sqrt{37}}{2 \cdot 1} = \frac{1 \pm \sqrt{37}}{2}$
$x = \frac{1 - \sqrt{37}}{2} < 0$ (vold. niet) $\vee$ $x = \frac{1 + \sqrt{37}}{2} > 1$ (voldoet).

10ab
$p^2 - 2p - 8 = 0$
$(p-4)(p+2) = 0$
$p = 4 \vee p = -2.$
$^{2}\log(x) = 4 \vee {^{2}\log(x)} = -2$ (BV: $x > 0$)
$x = 2^4 = 16$ (voldoet) $\vee$ $x = 2^{-2} = \frac{1}{4}$ (voldoet).

11
$^{3}\log(4) = \frac{\log(4)}{\log(3)} \approx 1{,}262$ en $^{\frac{1}{2}}\log(3) = \frac{\log(3)}{\log(\frac{1}{2})} \approx -1{,}585.$

```
log(4)/log(3)
        1.261859507
log(3)/log(1/2)
       -1.584962501
```

12a
$^{3}\log(3x-5) + {^{\frac{1}{3}}\log(x-1)} = 0$ BV: $x > \frac{5}{3}$
$^{3}\log(3x-5) - {^{3}\log(x-1)} = 0$
$^{3}\log(3x-5) = {^{3}\log(x-1)}$
$3x - 5 = x - 1$
$2x = 4$
$x = 2$ (voldoet).

12b
$^{5}\log(3x) + 2 \cdot {^{\frac{1}{5}}\log(x)} = 0$ BV: $x > 0$
$^{5}\log(3x) - {^{5}\log(x^2)} = 0$
$^{5}\log(3x) = {^{5}\log(x^2)}$
$3x = x^2$
$x^2 - 3x = 0$
$x \cdot (x-3) = 0$
$x = 0$ (vold. niet) $\vee$ $x = 3$ (voldoet).

12c
$2x \cdot {^{\frac{1}{3}}\log(3x+5)} = {^{\frac{1}{3}}\log(3x+5)}$ BV: $x > -\frac{5}{3}$
$2x = 1 \vee {^{\frac{1}{3}}\log(3x+5)} = 0$
$x = \frac{1}{2}$ (voldoet) $\vee$ $3x + 5 = (\frac{1}{3})^0 = 1$
$x = \frac{1}{2} \vee 3x = -4$
$x = \frac{1}{2} \vee x = -\frac{4}{3}$ (voldoet).

12d
$^{2}\log^2(x) = 2 \cdot {^{2}\log(x)} + 3$ BV: $x > 0$
Stel $^{2}\log(x)$ tijdelijk t
$t^2 = 2t + 3$
$t^2 - 2t - 3 = 0$
$(t-3) \cdot (t+1) = 0$
$t = {^{2}\log(x)} = 3 \vee t = {^{2}\log(x)} = -1$
$x = 2^3 = 8$ (voldoet) $\vee$ $x = 2^{-1} = \frac{1}{2}$ (voldoet).

13a $-2\cdot {}^{\frac{1}{2}}\log(x) = 2 + {}^{2}\log(3-x)$ BV: $0 < x < 3$
$2\cdot {}^{2}\log(x) = {}^{2}\log(2^2) + {}^{2}\log(3-x)$
$^{2}\log(x^2) = {}^{2}\log(4\cdot(3-x))$
$x^2 = 12 - 4x$
$x^2 + 4x - 12 = 0$
$(x+6)\cdot(x-2) = 0$
$x = -6$ (vold. niet) $\vee$ $x = 2$ (voldoet).

(BV: $x > 0$ en $3 - x > 0$; $x > 0$ en $-x > -3$; $x > 0$ en $x < 3$; $0 < x < 3$)

13b $^{9}\log(2x) = {}^{3}\log(x-4)$ BV: $x > 4$
$\frac{^{3}\log(2x)}{^{3}\log(3^2)} = {}^{3}\log(x-4)$
$^{3}\log(2x) = 2\cdot {}^{3}\log(x-4)$
$^{3}\log(2x) = {}^{3}\log((x-4)^2)$
$2x = x^2 - 8x + 16$
$x^2 - 10x + 16 = 0$
$(x-8)\cdot(x-2) = 0$
$x = 8$ (voldoet) $\vee$ $x = 2$ (vold. niet).

13c $4x\cdot {}^{4}\log(2x-1) + 3\cdot {}^{4}\log(2x-1) = 0$ BV: $x > \frac{1}{2}$
$(4x+3)\cdot {}^{4}\log(2x-1) = 0$
$4x + 3 = 0 \vee {}^{4}\log(2x-1) = 0$
$4x = -3 \vee 2x - 1 = 4^0 = 1$
$x = -\frac{3}{4}$ (vold. niet) $\vee$ $2x = 2$
$x = 1$ (voldoet).

13d $^{\frac{1}{2}}\log^2(x+2) + 3\cdot {}^{\frac{1}{2}}\log(x+2) = 0$ BV: $x > -2$
Stel $^{\frac{1}{2}}\log(x+2)$ tijdelijk t
$t^2 + 3t = 0$
$t\cdot(t+3) = 0$
$t = {}^{\frac{1}{2}}\log(x+2) = 0 \vee t = {}^{\frac{1}{2}}\log(x+2) = -3$
$x + 2 = (\frac{1}{2})^0 = 1 \vee x + 2 = (\frac{1}{2})^{-3} = (2^{-1})^{-3} = 2^3 = 8$
$x = -1$ (voldoet) $\vee$ $x = 6$ (voldoet).

14a $3x\cdot {}^{2}\log(x+1) = {}^{\frac{1}{2}}\log(x+1)$ BV: $x > -1$
$3x\cdot {}^{2}\log(x+1) = -{}^{2}\log(x+1)$
$3x = -1 \vee {}^{2}\log(x+1) = 0$
$x = -\frac{1}{3}$ (voldoet) $\vee$ $x + 1 = 2^0 = 1$
$x = -\frac{1}{3} \vee x = 0$ (voldoet).

14b $x^2\cdot {}^{5}\log(2x+1) + 9\cdot {}^{\frac{1}{5}}\log(2x+1) = 0$ BV: $x > -\frac{1}{2}$
$x^2\cdot {}^{5}\log(2x+1) - 9\cdot {}^{5}\log(2x+1) = 0$
$(x^2 - 9)\cdot {}^{5}\log(2x+1) = 0$
$x^2 - 9 = 0 \vee 2x + 1 = 5^0 = 1$
$x^2 = 9 \vee 2x = 0$
$x = 3$ (voldoet) $\vee$ $x = -3$ (vold. niet) $\vee$ $x = 0$ (voldoet).

14c $2\cdot {}^{3}\log^2(x) + 2 = 5\cdot {}^{3}\log(x)$ BV: $x > 0$
Stel $^{3}\log(x)$ tijdelijk t
$2t^2 + 2 = 5t$
$2t^2 - 5t + 2 = 0$ (*abc*-formule)
$D = (-5)^2 - 4\cdot 2\cdot 2 = 25 - 16 = 9 \Rightarrow t = \frac{5 \pm \sqrt{9}}{2\cdot 2} = \frac{5 \pm 3}{4}$
$t = {}^{3}\log(x) = \frac{5+3}{4} = \frac{8}{4} = 2 \vee t = {}^{3}\log(x) = \frac{5-3}{4} = \frac{2}{4} = \frac{1}{2}$
$x = 3^2 = 9$ (voldoet) $\vee$ $x = 3^{\frac{1}{2}} = \sqrt{3}$ (voldoet).

14d $^{5}\log^2(x) + 3\cdot {}^{\frac{1}{5}}\log(x) + 2 = 0$ BV: $x > 0$
$^{5}\log^2(x) - 3\cdot {}^{5}\log(x) + 2 = 0$
Stel $^{5}\log(x)$ tijdelijk t
$t^2 - 3t + 2 = 0$
$(t-2)\cdot(t-1) = 0$
$t = {}^{5}\log(x) = 2 \vee t = {}^{5}\log(x) = 1$
$x = 5^2 = 25$ (voldoet) $\vee$ $x = 5^1 = 5$ (voldoet).

15 $2^x = 7$ (gegeven) en $2^{^{2}\log(7)} = 7 \Rightarrow x = {}^{2}\log(7)$.
Of: $2^x = 7$ (gegeven) $\Rightarrow {}^{2}\log(2^x) = {}^{2}\log(7) \Rightarrow x = {}^{2}\log(7)$.

Onthoud: $g^{\cdots}$ en $^{g}\log\ldots$ heffen elkaar op

16ab $(2^x)^2 + 2\cdot 2^x = 8$ (stel $2^x = p$)
$p^2 + 2p = 8$
$p^2 + 2p - 8 = 0$
$(p+4)\cdot(p-2) = 0$
$p = 2^x = -4$ (kan niet) $\vee$ $p = 2^x = 2 = 2^1 \Rightarrow x = 1$.

17a $3^x - 2 = 8\cdot(\frac{1}{3})^x$
$3^x - 2 = 8\cdot\frac{1}{3^x}$ (stel $3^x = t$)
$t - 2 = \frac{8}{t}$ (links en rechts $\times t$)
$t^2 - 2t = 8$
$t^2 - 2t - 8 = 0$
$(t-4)\cdot(t+2) = 0$
$t = 3^x = 4 \vee t = 3^x = -2$ (kan niet)
$x = {}^{3}\log(4)$.

17b $2^x = 6 - 5\cdot(\frac{1}{2})^x$
$2^x = 6 - 5\cdot\frac{1}{2^x}$ (stel $2^x = t$)
$t = 6 - \frac{5}{t}$ (links en rechts $\times t$)
$t^2 = 6t - 5$
$t^2 - 6t + 5 = 0$
$(t-5)\cdot(t-1) = 0$
$t = 2^x = 5 \vee t = 2^x = 1 = 2^0$
$x = {}^{2}\log(5) \vee x = 0$.

17c

$9^x = 4 + 3^{x+1}$

$(3^2)^x = 4 + 3^1 \cdot 3^x$

$(3^x)^2 = 4 + 3 \cdot 3^x$ (stel $3^x = t$)

$t^2 = 4 + 3t$

$t^2 - 3t - 4 = 0$

$(t-4)\cdot(t+1) = 0$

$t = 3^x = 4 \vee t = 3^x = -1$ (kan niet)

$x = {}^3\log(4)$.

17d

$2^x = 24 - 2^{2x-1}$

$2^x = 24 - 2^{-1}\cdot(2^x)^2$ (stel $2^x = t$)

$t = 24 - \frac{1}{2}t^2$

$\frac{1}{2}t^2 + t - 24 = 0$ (links en rechts $\times 2$)

$t^2 + 2t - 48 = 0$

$(t+8)\cdot(t-6) = 0$

$t = 2^x = -8$ (kan niet) $\vee t = 2^x = 6$

$x = {}^2\log(6)$.

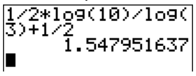
18a

$3^{2x-1} = 10$ (links en rechts ${}^3\log\ldots$)

$2x - 1 = {}^3\log(10)$ (links en rechts $+1$)

$2x = {}^3\log(10) + 1$ (links en rechts $:2$)

$x = \frac{1}{2}\cdot {}^3\log(10) + \frac{1}{2} \approx 1{,}55.$



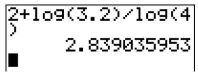
18b

$5\cdot 4^{x-2} = 16$ (links en rechts $:5$)

$4^{x-2} = \frac{16}{5} = \frac{32}{10} = 3{,}2$ (links en rechts ${}^4\log\ldots$)

$x - 2 = {}^4\log(3{,}2)$ (links en rechts $+2$)

$x = 2 + {}^4\log(3{,}2) \approx 2{,}84.$



18c

$9^x = 2\cdot 3^x + 6$

$(3^x)^2 = 2\cdot 3^x + 6$ (stel $3^x = t$)

$t^2 = 2t + 6$

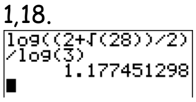
$t^2 - 2t - 6 = 0$ (abc-formule)

$D = (-2)^2 - 4\cdot 1\cdot -6 = 4 + 24 = 28 \Rightarrow t = \frac{2\pm\sqrt{28}}{2\cdot 1}$

$t = 3^x = \frac{2+\sqrt{28}}{2} \vee t = 3^x = \frac{2-\sqrt{28}}{2} < 0$ (kan niet)

(de tweede teller is neg en de noemer pos $\Rightarrow$ breuk neg)

(links en rechts ${}^3\log\ldots$) $x = {}^3\log(\frac{2+\sqrt{28}}{2}) \approx 1{,}18.$



18d

$2^x + 2^{-x} = 3$

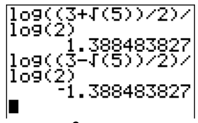
$2^x + \frac{1}{2^x} = 3$ (links en rechts $\times 2^x$)

$(2^x)^2 + 1 = 3\cdot 2^x$ (stel $2^x = t$)

$t^2 + 1 = 3t$

$t^2 - 3t + 1 = 0$ (abc-formule)

$D = (-3)^2 - 4\cdot 1\cdot 1 = 9 - 4 = 5 \Rightarrow t = \frac{3\pm\sqrt{5}}{2\cdot 1}$



$t = 2^x = \frac{3+\sqrt{5}}{2} \vee t = 2^x = \frac{3-\sqrt{5}}{2}$ (links en rechts ${}^2\log\ldots$)

$x = {}^2\log(\frac{3+\sqrt{5}}{2}) \approx 1{,}39 \vee x = {}^2\log(\frac{3-\sqrt{5}}{2}) \approx -1{,}39.$

19a

$3^{x+2} + 3^x = 600$

$3^x\cdot 3^2 + 3^x = 600$ (stel $3^x = t$)

$9t + t = 600$

$10t = 600$

$t = 3^x = 60$ (links en rechts ${}^3\log\ldots$)

$x = {}^3\log(60)$.

19b

$5^{x-1} + 5^{2x-1} = 4$ (links en rechts $\times 5$)

$5^x + 5^{2x} = 20$ (stel $5^x = t$)

$t + t^2 = 20$

$t^2 + t - 20 = 0$

$(t+5)\cdot(t-4) = 0$

$t = 5^x = -5$ (kan niet) $\vee t = 5^x = 4$ (links en rechts ${}^5\log\ldots$)

$x = {}^5\log(4)$.

19c

$3^x + 5\cdot(\frac{1}{3})^{x-2} = 18$

$3^x + 5\cdot(\frac{1}{3^x})\cdot(\frac{1}{3})^{-2} = 18$ (links en rechts $\times 3^x$)

$(3^x)^2 + 5\cdot 1\cdot(3^{-1})^{-2} = 18\cdot 3^x$ (stel $3^x = t$)

$t^2 + 5\cdot 9 = 18t$

$t^2 - 18t + 45 = 0$

$(t-15)\cdot(t-3) = 0$

$t = 3^x = 15 \vee t = 3^x = 3 = 3^1$

$x = {}^3\log(15) \vee x = 1.$

19d

$3^x + 2\cdot(\frac{1}{3})^{x-2} = 1$

$3^x + 2\cdot(\frac{1}{3^x})\cdot(\frac{1}{3})^{-2} = 1$ (links en rechts $\times 3^x$)

$(3^x)^2 + 2\cdot 1\cdot 9 = 1\cdot 3^x$ (stel $3^x = t$)

$t^2 + 18 = t$

$t^2 - t + 18 = 0$ (abc-formule)

$D = (-1)^2 - 4\cdot 1\cdot 18 < 0 \Rightarrow$ geen oplossing.

20a $y = 2^x \xrightarrow{\text{translatie } (-3,\,0)} f(x) = 2^{x+3}.$

20b $y = 2^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 8} f(x) = 8\cdot 2^x = 2^3\cdot 2^x = 2^{x+3}.$

21a $y = {}^2\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{8}} f(x) = {}^2\log(8\cdot x).$

21b $y = {}^2\log(x) \xrightarrow{\text{translatie } (0,\,3)} f(x) = {}^2\log(x) + 3 = {}^2\log(x) + {}^2\log(2^3) = {}^2\log(x) + {}^2\log(8) = {}^2\log(8\cdot x).$

22a $y = 2^x \xrightarrow{\text{translatie } (5,\,0)} f(x) = 2^{x-5} = 2^x\cdot 2^{-5} = \frac{1}{2^5}\cdot 2^x = \frac{1}{32}\cdot 2^x \xleftarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{32}} y = 2^x.$

22b $y = 4^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 2} f(x) = 2\cdot 4^x = \sqrt{4}\cdot 4^x = 4^{\frac{1}{2}}\cdot 4^x = 4^{x+\frac{1}{2}} \xleftarrow{\text{translatie } (-\frac{1}{2},\,0)} y = 4^x.$

22c $y = {}^2\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{32}} f(x) = {}^2\log(32\cdot x) = {}^2\log(x) + {}^2\log(32) = {}^2\log(x) + {}^2\log(2^5) = {}^2\log(x) + 5.$

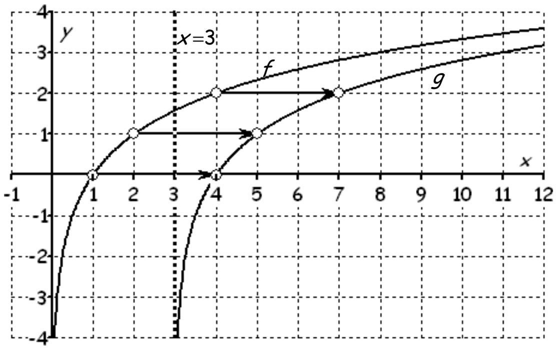
$y = {}^2\log(x) \xrightarrow{\text{translatie } (0,\,5)} f(x) = {}^2\log(x) + 5.$

22d $y = {}^4\log(x) \xrightarrow{\text{translatie } (0,\ \frac{1}{2})} f(x) = {}^4\log(x) + \frac{1}{2} = {}^4\log(x) + {}^4\log(4^{\frac{1}{2}}) = {}^4\log(x \cdot 4^{\frac{1}{2}}) = {}^4\log(x \cdot \sqrt{4}) = {}^4\log(2 \cdot x).$

$y = {}^4\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{2}} f(x) = {}^4\log(2 \cdot x).$

23a $f(x) = {}^2\log(x) \xrightarrow{\text{translatie } (3,\ 0)} g(x) = {}^2\log(x - 3).$

23b Zie de grafiek hiernaast.



23c Nee. (je kunt de vert. asymptoot $x = 0$ niet op $x = 3$ krijgen met een vermenigvuldiging t.o.v. x-as of een vermenigvuldiging t.o.v. y-as)

23d $g(x) = {}^2\log(x - 3) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{4}} h(x) = {}^2\log(4x - 3).$

$h(x) = {}^2\log(4x - 3) = {}^2\log(4 \cdot (x - \frac{3}{4})) = {}^2\log(4) + {}^2\log(x - \frac{3}{4}).$

Dus $p = -\frac{3}{4}$ en $q = {}^2\log(4) = {}^2\log(2^2) = 2.$

24a $f(x) = (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 4} g(x) = 4 \cdot (\frac{1}{2})^x.$

24b Er geldt: $4 \cdot (\frac{1}{2})^x = 2^2 \cdot (\frac{1}{2})^x = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^x = (\frac{1}{2})^{x-2}$. Dus $f(x) = (\frac{1}{2})^x \xrightarrow{\text{translatie } (2,\ 0)} g(x) = (\frac{1}{2})^{x-2}.$

24cd $g(x) = 4 \cdot (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{4}} f(x) = (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } -\frac{1}{2}} h(x) = (\frac{1}{2})^{-2x} = \left((2^{-1})^{-2}\right)^x = 4^x.$

24e $g(x) = 4 \cdot (\frac{1}{2})^x \xrightarrow{\text{translatie } (3,\ 4)} j(x) = 4 \cdot (\frac{1}{2})^{x-3} + 4.$

$j(x) = 4 \cdot (\frac{1}{2})^{x-3} + 4 = 4 \cdot (\frac{1}{2})^x \cdot (\frac{1}{2})^{-3} + 4 = 4 \cdot (\frac{1}{2})^x \cdot 2^3 + 4 = 32 \cdot (\frac{1}{2})^x + 4.$ Dus $a = 32$ en $b = 4.$

25 $|AB| = g(1) - f(1) = 8 - 2^1 - 2^{1-2} = 8 - 2 - 2^{-1} = 6 - \frac{1}{2} = 5\frac{1}{2}.$

26a $x = {}^2\log(6\frac{2}{5}) \Rightarrow y_S = g({}^2\log(6\frac{2}{5})) = 8 - 2^{{}^2\log(6\frac{2}{5})} = 8 - 6\frac{2}{5} = 1\frac{3}{5}.$

26b Links van het snijpunt S is de afstand altijd kleiner dan 8, omdat de lijnen $y = 0$ en $y = 8$ asymptoten zijn van respectievelijk de grafieken van f en g. Rechts van het snijpunt S kan de afstand wel gelijk worden aan 10. Dus er precies één waarde van p waarvoor de grafieken een lijnstuk met lengte 10 afsnijden van de lijn $x = p$.

26c Er zijn twee waarden van p voor $0 < a < 8$.

27a $f(x) = g(x)$ geeft:

$3^{x-1} - 2 = 4 - 3^x$

$3^x \cdot 3^{-1} - 2 = 4 - 3^x$

$\frac{1}{3} \cdot 3^x - 2 = 4 - 3^x$

$1\frac{1}{3} \cdot 3^x = 6$

$3^x = 6 \cdot \frac{3}{4} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}^{\bullet\bullet\bullet}$

$x_A = {}^3\log(4\frac{1}{2})$

$y_A = g({}^3\log(4\frac{1}{2})) = 4 - 4\frac{1}{2}^{\bullet\bullet\bullet} = -\frac{1}{2}.$

27b $f(p) - g(p) = 6$

(kan alleen rechts van A)

$3^{p-1} - 2 - (4 - 3^p) = 6$

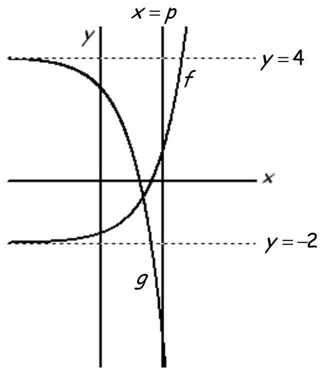
$3^p \cdot 3^{-1} - 2 - 4 + 3^p = 6$

$\frac{1}{3} \cdot 3^p - 6 + 3^p = 6$

$1\frac{1}{3} \cdot 3^p = 12$

$3^p = 12 \cdot \frac{3}{4} = 9 = 3^2$

$p = 2.$



28a $f(x) = {}^3\log(1 - 3x)$ (BV: $1 - 3x > 0 \Rightarrow -3x > -1 \Rightarrow x < \frac{1}{3} \Rightarrow$) met $D_f = \langle \leftarrow,\ \frac{1}{3} \rangle.$

$g(x) = {}^3\log(x + 5)$ (BV: $x + 5 > 0 \Rightarrow x > -5 \Rightarrow$) met $D_g = \langle -5,\ \rightarrow \rangle.$

28b $f(x) = g(x)$ geeft:

${}^3\log(1 - 3x) = {}^3\log(x + 5)$ BV: $-5 < x < \frac{1}{3}$

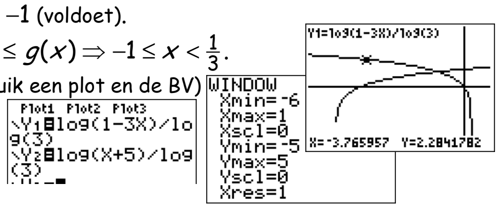
$1 - 3x = x + 5$

$-4x = 4$

$x_S = -1$ (voldoet).

$f(x) \le g(x) \Rightarrow -1 \le x < \frac{1}{3}.$

(gebruik een plot en de BV)



28c Links van $S \Rightarrow -5 < p < -1$ ∨ rechts van $S \Rightarrow -1 < p < \frac{1}{3}$

$f(p) - g(p) = 2$ BV: $-5 < p < \frac{1}{3}$ ∨ $g(p) - f(p) = 2$ BV: $-5 < p < \frac{1}{3}$

${}^3\log(1 - 3p) - {}^3\log(p + 5) = 2$ ∨ ${}^3\log(p + 5) - {}^3\log(1 - 3p) = 2$

${}^3\log(\frac{1 - 3p}{p + 5}) = {}^3\log(3^2)$ ∨ ${}^3\log(\frac{p + 5}{1 - 3p}) = {}^3\log(3^2)$

$\frac{1 - 3p}{p + 5} = 9$ ∨ $\frac{p + 5}{1 - 3p} = 9$

$1 - 3p = 9p + 45$ ∨ $p + 5 = 9 - 27p$

$-12p = 44$ ∨ $28p = 4$

$p = -\frac{44}{12} = -\frac{11}{3} = -3\frac{2}{3}$ (voldoet). ∨ $p = \frac{4}{28} = \frac{1}{7}$ (voldoet).

29a $f(x) = g(x)$ geeft:

$(\frac{3}{2})^{x+2} = 3 \cdot (\frac{2}{3})^{x} + 3$

$(\frac{3}{2})^{x} \cdot (\frac{3}{2})^{2} = 3 \cdot (\frac{3}{2})^{-x} + 3$

$\frac{9}{4} \cdot (\frac{3}{2})^{x} = 3 \cdot (\frac{3}{2})^{-x} + 3$ (stel $(\frac{3}{2})^{x} = t$)

$\frac{9}{4}t = 3 \cdot \frac{1}{t} + 3$ (links en rechts $\times t$)

$\frac{9}{4}t^2 = 3 \cdot 1 + 3t$ (links en rechts $\times 4$)

$9t^2 = 12 + 12t$ (links en rechts $: 3$)

$3t^2 = 4 + 4t$

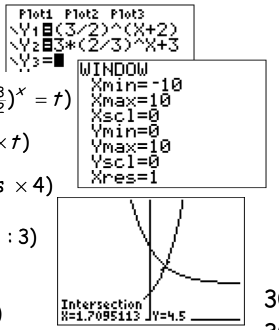
$3t^2 - 4t - 4 = 0$ (*abc*-formule)

$D = (-4)^2 - 4 \cdot 3 \cdot -4 = 16 + 48 = 64$, dus $\sqrt{D} = 8$

$t = (\frac{3}{2})^{x} = \frac{4+8}{2 \cdot 3} = 2^{\bullet\bullet\bullet} \vee t = (\frac{3}{2})^{x} = \frac{4-8}{6} < 0$ (kan niet)

$x_S = {}^{\frac{3}{2}}\log(2)$ en $y_S = g({}^{\frac{3}{2}}\log(2)) = 3 \cdot (\frac{3}{2})^{-{}^{\frac{3}{2}}\log(2)} + 3$

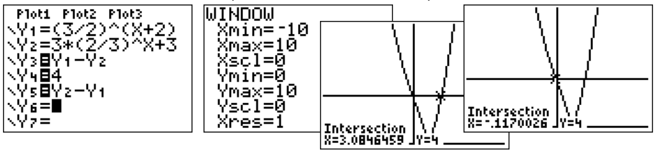
$= 3 \cdot (\frac{3}{2})^{{}^{\frac{3}{2}}\log(2^{-1})} + 3 = 3 \cdot \frac{1}{2}^{\bullet\bullet\bullet} + 3 = 4\frac{1}{2}$.



29b $f(p) - g(p) = 4 \quad \vee \quad g(p) - f(p) = 4$

$(\frac{3}{2})^{x+2} - (3 \cdot (\frac{2}{3})^{x} + 3) = 4 \quad \vee \quad (3 \cdot (\frac{2}{3})^{x} + 3) - (\frac{3}{2})^{x+2} = 4$

Intersect $\Rightarrow x \approx 3{,}085 \quad \vee \quad x \approx -0{,}117$.



30a $x_B = x_A + AB = x_A + 6 = p + 6$.

30b $\left.\begin{array}{l} y_A = f(p) \\ y_B = g(p+6) \\ y_A = y_B \end{array}\right\} \Rightarrow f(p) = g(p+6)$.

$f(p) = g(p+6) \Rightarrow {}^{2}\log(4p) = {}^{2}\log(p+6)$ (BV: $p > 0$)

$4p = p + 6 \Rightarrow 3p = 6 \Rightarrow p = 2$ (voldoet).

30c $q = y_A = f(p) = f(2) = {}^{2}\log(8) = {}^{2}\log(2^3) = 3$.

31 Boven S: $g(p) = f(p + 1\frac{1}{8}) = q$

${2 + {}^{\frac{1}{2}}\log(p+2) = {}^{\frac{1}{2}}\log(2(p + 1\frac{1}{8}))}$

${}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(p+2) = {}^{\frac{1}{2}}\log(2p + 2\frac{1}{4})$

${}^{\frac{1}{2}}\log(\frac{1}{4} \cdot (p+2)) = {}^{\frac{1}{2}}\log(2p + 2\frac{1}{4})$

$\frac{1}{4} \cdot (p+2) = 2p + 2\frac{1}{4}$

$p + 2 = 8p + 9$

$-7p = 7$

$p = -1 \Rightarrow q = g(p) = 2 + {}^{\frac{1}{2}}\log(1) = 2 + {}^{\frac{1}{2}}\log((\frac{1}{2})^0) = 2 + 0 = 2$.

Onder S: $f(p) = g(p + 1\frac{1}{8}) = q$

${}^{\frac{1}{2}}\log(2p) = 2 + {}^{\frac{1}{2}}\log(p + 1\frac{1}{8} + 2)$

${}^{\frac{1}{2}}\log(2p) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(p + 3\frac{1}{8})$

${}^{\frac{1}{2}}\log(2p) = {}^{\frac{1}{2}}\log(\frac{1}{4} \cdot (p + 3\frac{1}{8}))$

$2p = \frac{1}{4} \cdot (p + 3\frac{1}{8})$

$8p = p + 3\frac{1}{8}$

$7p = 3\frac{1}{8} = \frac{25}{8}$

$p = \frac{25}{56} \Rightarrow q = f(p) = {}^{\frac{1}{2}}\log(\frac{50}{56}) = {}^{\frac{1}{2}}\log(\frac{25}{28})$.

32a Omdat $x = -2$ en $x = 0$ de asymptoten van de grafieken van f en g zijn, nadert de lengte van het lijnstuk dat de grafieken van de lijn $y = q$ (boven het snijpunt) afsnijden tot 2 als q steeds groter wordt.
Alleen van een lijn $y = q$ die onder het snijpunt ligt kunnen de grafieken een lijnstuk met lengte 3 afsnijden.

32b Voor $0 < a < 2$.

33 Onder S: $f(p) = g(p+2) = q$

$2^{p-2} = 8 - 2^{p+2}$

$2^p \cdot 2^{-2} = 8 - 2^p \cdot 2^2$

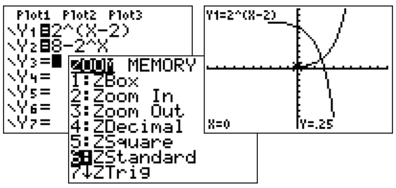
$\frac{1}{4} \cdot 2^p = 8 - 4 \cdot 2^p$

$4\frac{1}{4} \cdot 2^p = 8$

$17 \cdot 2^p = 32$

$2^p = \frac{32}{17} *$

$q = f(p) = \frac{1}{4} \cdot \frac{32}{17} * = \frac{8}{17}$.



Boven S: $g(p) = f(p+2) = q$

$8 - 2^p = 2^{p-2+2}$

$8 - 2^p = 2^p$

$8 = 2 \cdot 2^p$

$2^p = 4 *$

$q = g(p) = 8 - 4* = 4$.

34a ${}^{4}\log(x^2 - 1) = {}^{2}\log(x+3)$ BV: $|x| > 1$ en $x > -3 \Rightarrow -3 < x < -1$ of $x > 1$

$\frac{{}^{2}\log(x^2-1)}{{}^{2}\log(4)} = {}^{2}\log(x+3)$ (met ${}^{2}\log(4) = {}^{2}\log(2^2) = 2$)

${}^{2}\log(x^2 - 1) = 2 \cdot {}^{2}\log(x+3)$

${}^{2}\log(x^2 - 1) = {}^{2}\log((x+3)^2)$

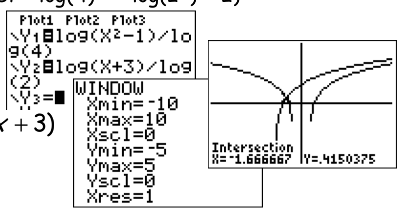
$x^2 - 1 = (x+3)^2 = (x+3) \cdot (x+3)$

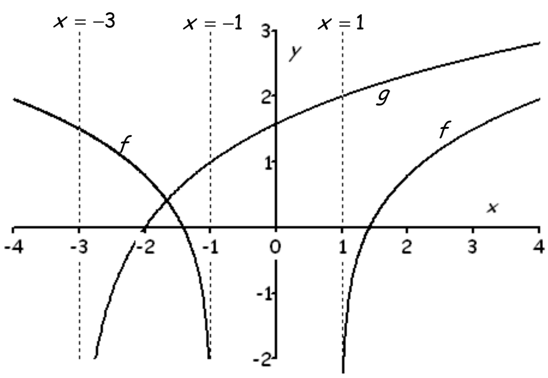
$x^2 - 1 = x^2 + 6x + 9$

$-6x = 10$

$x = -\frac{10}{6} = -\frac{5}{3}$ (voldoet).

$f(x) \le g(x)$ (gebruik een grafiek en de BV) $\Rightarrow -\frac{5}{3} \le x < -1 \vee x > 1$.





34b Links van S: (als f boven g loopt $\Rightarrow -3 < x < -\frac{5}{3}$)

$f(p) - g(p) = \frac{1}{2}$

${}^{4}\log(p^2-1) - {}^{2}\log(p+3) = \frac{1}{2}$

$\frac{{}^{2}\log(p^2-1)}{{}^{2}\log(4)} - {}^{2}\log(p+3) = \frac{1}{2}$

$\frac{{}^{2}\log(p^2-1)}{2} - {}^{2}\log(p+3) = \frac{1}{2}$ (links en rechts $\times 2$)

${}^{2}\log(p^2-1) - 2 \cdot {}^{2}\log(p+3) = 1$

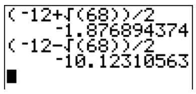
${}^{2}\log(p^2-1) - {}^{2}\log((p+3)^2) = {}^{2}\log(2)$

$\frac{p^2-1}{(p+3)^2} = 2$

$p^2 - 1 = 2(p^2+6p+9)$

$p^2 - 1 = 2p^2 + 12p + 18$

$p^2 + 12p + 19 = 0$ (abc-formule)



$D = 12^2 - 4 \cdot 1 \cdot 19 = 144 - 76 = 68 \Rightarrow \sqrt{D} = \sqrt{68} = 2 \cdot \sqrt{17}$

$p = \frac{-12+2\sqrt{17}}{2}$ (voldoet) $\vee$ $p = \frac{-12-2\sqrt{17}}{2}$ (vold. niet).

Rechts van S: (als g boven $f \Rightarrow -\frac{5}{3} < x < -1$ of $x > 1$)

$g(p) - f(p) = \frac{1}{2}$

${}^{2}\log(p+3) - {}^{4}\log(p^2-1) = \frac{1}{2}$

${}^{2}\log(p+3) - \frac{{}^{2}\log(p^2-1)}{2} = \frac{1}{2}$ (links en rechts $\times 2$)

$2 \cdot {}^{2}\log(p+3) - {}^{2}\log(p^2-1) = 1$

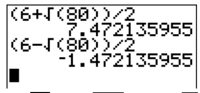
${}^{2}\log((p+3)^2) - {}^{2}\log(p^2-1) = {}^{2}\log(2)$

$\frac{(p+3)^2}{p^2-1} = 2$

$2 \cdot (p^2-1) = p^2 + 6p + 9$

$2p^2 - 2 = p^2 + 6p + 9$

$p^2 - 6p - 11 = 0$ (abc-formule)



$D = (-6)^2 - 4 \cdot 1 \cdot -11 = 36 + 44 = 80 \Rightarrow \sqrt{D} = \sqrt{80} = 4\sqrt{5}$

$p = \frac{6+4\sqrt{5}}{2}$ (voldoet) $\vee$ $p = \frac{6-4\sqrt{5}}{2}$ (voldoet).

Dus $p = -6 + \sqrt{17} \vee p = 3 + 2\sqrt{5} \vee p = 3 - 2\sqrt{5}$.

34c Boven S: $f(p) = g(p+1) = q$ (met $p < -\frac{5}{3}$)

${}^{4}\log(p^2-1) = {}^{2}\log(p+1+3)$

$\frac{1}{2} \cdot {}^{2}\log(p^2-1) = {}^{2}\log(p+4)$

${}^{2}\log(p^2-1) = 2 \cdot {}^{2}\log(p+4)$

${}^{2}\log(p^2-1) = {}^{2}\log((p+4)^2)$

$p^2 - 1 = p^2 + 8p + 16$

$-17 = 8p$

$p = -\frac{17}{8}$ (voldoet).

Onder S: $g(p) = f(p+1) = q$ (met $p < -\frac{5}{3}$)

${}^{2}\log(p+3) = {}^{4}\log((p+1)^2-1)$

${}^{2}\log(p+3) = \frac{1}{2} \cdot {}^{2}\log(p^2+2p)$

$2 \cdot {}^{2}\log(p+3) = {}^{2}\log(p^2+2p)$

${}^{2}\log((p+3)^2) = {}^{2}\log(p^2+2p)$

$p^2 + 6p + 9 = p^2 + 2p$

$4p = -9$

$p = -\frac{9}{4}$ (voldoet).

$p = -2\frac{1}{8} \Rightarrow q = f(p) = g(p+1) = g(-1\frac{1}{8}) = {}^{2}\log(-1\frac{1}{8}+3) = {}^{2}\log(1\frac{7}{8}) = {}^{2}\log(\frac{15}{8}) = {}^{2}\log(15) - {}^{2}\log(8) = {}^{2}\log(15) - 3.$

$p = -\frac{9}{4} \Rightarrow q = g(p) = g(-\frac{9}{4}) = {}^{2}\log(-\frac{9}{4}+3) = {}^{2}\log(\frac{3}{4}) = {}^{2}\log(3) - {}^{2}\log(4) = {}^{2}\log(3) - 2.$

35a $\left.\begin{array}{l} x_B = p \Rightarrow AB = p \\ AB : BC = 1 : 2 \Rightarrow BC = 2p \end{array}\right\} \Rightarrow AC = AB + BC = 3p \Rightarrow x_C = 3p.$

35b $f(x_B) = g(x_C) = q$ omdat B en C op de lijn $y = q$ liggen.

Dus $f(p) = g(3p)$ en hieruit volgt $2^p = 2^{3p-3}$.

$2^p = 2^{3p-3}$

$p = 3p - 3$

$-2p = -3$

$p = \frac{3}{2} = 1\frac{1}{2}$.

36 Stel $x_B = p$ dan $x_C = 2p$.

$f(p) = f(2p) = q$ geeft:

$6p \cdot 2^{-p} = 6 \cdot 2p \cdot 2^{-2p}$ (links en rechts $\times 2^{2p}$)

$6p \cdot 2^p = 12p$

$6p = 0 \vee 2^p = 2 = 2^1$

$p = 0$ (vold. niet) $\vee$ $p = 1$.

35c $q = f(x_B) = f(p) = f(1\frac{1}{2}) = 2^{1\frac{1}{2}} = 2^1 \cdot 2^{\frac{1}{2}} = 2 \cdot \sqrt{2}$.

37a Stel $x_B = p$ dan $x_C = 3p$.

$f(p) = g(3p) = q$ geeft:

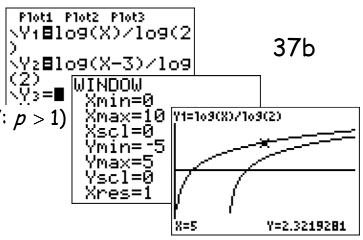
${}^{2}\log(p) = {}^{2}\log(3p-3)$ (BV: $p > 1$)

$p = 3p - 3$

$-2p = -3$

$p = \frac{3}{2}$ (voldoet).

$q = f(p) = f(\frac{3}{2}) = {}^{2}\log(\frac{3}{2})$.

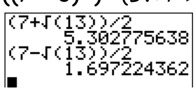


37b $y_F = 2 \cdot y_E \Rightarrow f(r) = 2 \cdot g(r)$.

${}^{2}\log(r) = 2 \cdot {}^{2}\log(r-3) = {}^{2}\log((r-3)^2)$ (BV: $r > 3$)

$r = r^2 - 6r + 9$

$r^2 - 7r + 9 = 0$ (abc-formule)



$D = (-7)^2 - 4 \cdot 1 \cdot 9 = 49 - 36 = 13 \Rightarrow \sqrt{D} = \sqrt{13}$

$r = \frac{7+\sqrt{13}}{2} \approx 5{,}303$ (voldoet) $\vee$ $r = \frac{7-\sqrt{13}}{2}$ (vold. niet).

38 Stel $x_B = p$ dan $x_C = 3p$.

$f(p) = f(3p) = q$

$8p \cdot (\frac{1}{3})^p = 8 \cdot 3p \cdot (\frac{1}{3})^{3p}$

$p = 0$ (vold. niet) $\vee$ $(\frac{1}{3})^p = 3 \cdot (\frac{1}{3})^{3p}$

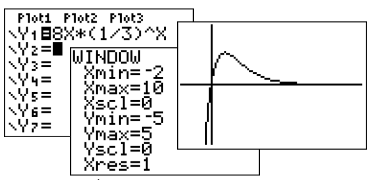
$(\frac{1}{3})^p = (\frac{1}{3})^{-1} \cdot (\frac{1}{3})^{3p}$

$(\frac{1}{3})^p = (\frac{1}{3})^{3p-1}$

$p = 3p - 1$

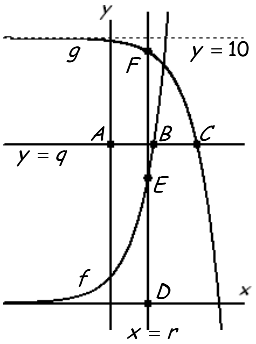
$-2p = -1$

$p = \frac{1}{2}$

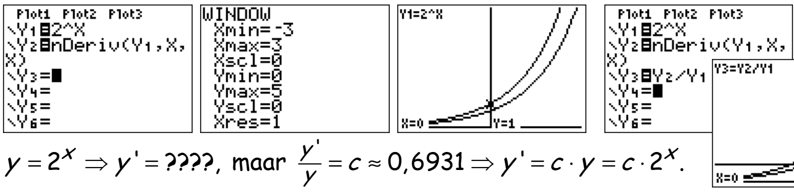


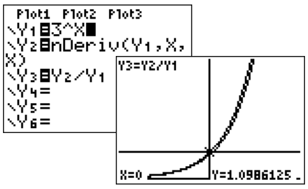
$q = f(p) = f(\frac{1}{2}) = 8 \cdot \frac{1}{2} \cdot (\frac{1}{3})^{\frac{1}{2}} = 4 \cdot \sqrt{\frac{1}{3}} = 4 \cdot \sqrt{\frac{1}{3} \cdot \frac{3}{3}} = \frac{4}{3} \cdot \sqrt{3}$.

39a Stel $x_B = p$ dan $x_C = 2p$.
$f(p) = g(2p) = q$
$3^p = 10 - 3^{2p-2}$ (links en rechts $\times 3^2$)
$9 \cdot 3^p = 90 - 3^{2p}$
$3^{2p} + 9 \cdot 3^p - 90 = 0$ (stel $3^p = t$)
$t^2 + 9t - 90 = 0$
$(t+15)(t-6) = 0$
$t = 3^p = -15$ (kan niet) $\vee$ $t = 3^p = 6$.
Dus $q = f(p) = 3^p = 6$.



39b $y_F = 2 \cdot y_E \Rightarrow g(r) = 2 \cdot f(r)$.
$10 - 3^{r-2} = 2 \cdot 3^r$ (links en rechts $\times 3^2$)
$90 - 3^r = 18 \cdot 3^r$
$-19 \cdot 3^r = -90$
$3^r = \frac{90}{19}$
$r = {}^3\log(\frac{90}{19})$.

40a




40b $y = 2^x \Rightarrow y' = ????$, maar $\frac{y'}{y} = c \approx 0{,}6931 \Rightarrow y' = c \cdot y = c \cdot 2^x$.

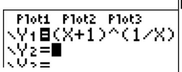
40c $y = 3^x \Rightarrow y' = ????$, maar $\frac{y'}{y} = c \approx 1{,}0986 \Rightarrow y' = c \cdot y = c \cdot 3^x$.

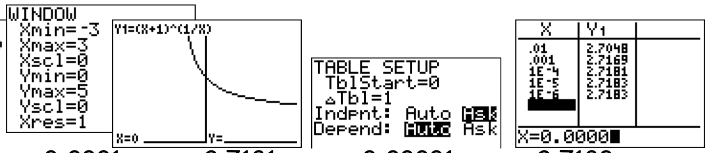
41a $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \to 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim_{h \to 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$.

41b $f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^0$ (zie 41a) $= \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 1 = \lim_{h \to 0} \frac{2^h - 1}{h}$.

41c $f(x) = 2^x \Rightarrow f'(x) = \lim_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$ (zie 41a) $= f'(0)$ (zie 41b) $\cdot 2^x$.

42a Zie de plot hiernaast.





42b De noemer van de breuk in de exponent wordt nul.
(een breuk waarvan de noemer nul is heeft geen waarde)

42c $x = 0{,}01 \Rightarrow y_1 \approx 2{,}7048$; $x = 0{,}001 \Rightarrow y_1 \approx 2{,}7169$; $x = 0{,}0001 \Rightarrow y_1 \approx 2{,}7181$ en $x = 0{,}00001 \Rightarrow y_1 \approx 2{,}7183$.

42d Voor het getal $a \approx 2{,}718$ geldt: $f(x) = a^x \Rightarrow f'(x) = a^x$.

43a $2e^2 - e^2 = 2e^2 - 1e^2 = 1e^2 = e^2$.
43b $4\sqrt{e} - \sqrt{e} = 4\sqrt{e} - 1\sqrt{e} = 3\sqrt{e}$.
43c $5e^2 \cdot 3e^3 = 15e^{2+3} = 15e^5$.
43d $\frac{12e^6}{4e^2} = 3e^{6-2} = 3e^4$.
43e $e^{5x} \cdot e^x = e^{5x+x} = e^{6x}$.
43f $e^x \cdot e^2 = e^{x+2}$.
43g $5e^x - 3e^x = 2e^x$.
43h $e^x \cdot (e^2 + 1) = e^{x+2} + e^x$.
43i $e^x \cdot (e^x + 1) = e^{2x} + e^x$.
43j $(e^x + 1)^2 = e^{2x} + 2e^x + 1$.
43k $(e^{3x} + 3)^2 = e^{6x} + 6e^{3x} + 9$.
43l $\frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$.

44a $(2 + 3e^{\frac{1}{2}x})^2 = 2^2 + 2 \cdot 2 \cdot 3e^{\frac{1}{2}x} + (3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$. Gebruik: $(\square + \Delta)^2 = (\square + \Delta) \cdot (\square + \Delta) = \square^2 + 2\square\Delta + \Delta^2$.
44b $(e^x + e^{-x})^2 = (e^x)^2 + 2 \cdot e^x \cdot e^{-x} + (e^{-x})^2 = e^{2x} + 2 + e^{-2x}$. $(e^0 = 1)$
44c $\frac{e^{2x} - 4}{e^x - 2} = \frac{(e^x + 2) \cdot (e^x - 2)}{e^x - 2} = e^x + 2$ $(e^x - 2 \neq 0 \Rightarrow e^x \neq 2)$ Gebruik: $(\square + \Delta) \cdot (\square - \Delta) = \square^2 - \square\Delta + \square\Delta - \Delta^2 = \square^2 - \Delta^2$.

45a $(2x + 4) \cdot e^x = 0$
$2x + 4 = 0 \vee e^x = 0$ (kan niet)
$2x = -4$
$x = -2$.

45b $x^2 \cdot e^x = 3x \cdot e^x$
$e^x = 0$ (kan niet) $\vee x^2 = 3x$
$x = 0 \vee x = 3$.

45c $x^2 \cdot e^x = 1 \cdot e^x$
$e^x = 0$ (kan niet) $\vee x^2 = 1$
$x = \pm 1$
$x = -1 \vee x = 1$.

45d $e^{3x} - e^x = 0$
$e^{3x} = e^x$
$3x = x$
$x = 0$.

45e $e^{4x} - 1 = 0$
$e^{4x} = 1 = e^0$
$4x = 0$
$x = 0$.

45f $e^x \cdot e^x = e^6$
$e^{2x} = e^6$
$2x = 6$
$x = 3$.

46a $e^x + e^x = 2e^6$
$2e^x = 2e^6$
$e^x = e^6$
$x = 6.$

46b $\frac{e^{5x}}{e^x} = e$
$e^{4x} = e^1$
$4x = 1$
$x = \frac{1}{4}.$

46c $2xe^x + e^x = 0$
$e^x \cdot (2x+1) = 0$
$e^x = 0$ (kan niet) $\vee\ 2x = -1$
$x = -\frac{1}{2}.$

46d $e^{x+2} - \sqrt{e} = 0$
$e^{x+2} = e^{\frac{1}{2}}$
$x + 2 = \frac{1}{2}$
$x = -1\frac{1}{2}.$

46e $e^{2x} + e^x = 2$ (stel $e^x = t$)
$t^2 + t - 2 = 0$
$(t+2)(t-1) = 0$
$t = e^x = -2$ (kan niet) $\vee\ t = e^x = 1 = e^0$
$x = 0.$

46f $e^{6x} + 1 = 2e^{3x}$ (stel $e^{3x} = t$)
$t^2 - 2t + 1 = 0$
$(t-1)(t-1) = 0$
$t = e^{3x} = 1 = e^0$
$3x = 0 \Rightarrow x = 0.$

47a $f(x) = x \cdot e^x \Rightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x \cdot (1+x).$

47b $f(x) = \frac{e^x}{x+1} \Rightarrow f'(x) = \frac{(x+1)\cdot e^x - e^x \cdot 1}{(x+1)^2} = \frac{xe^x + e^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}.$
Gebruik: $f(x) = \frac{t}{n} \Rightarrow f'(x) = \frac{nat - tan}{n^2} = \frac{\text{noemer} \cdot \text{afgeleide teller} - \text{teller} \cdot \text{afgeleide noemer}}{\text{noemer}^2}.$

48a $f(x) = e^x + 2 \Rightarrow f'(x) = e^x.$

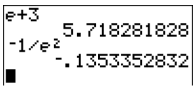
48b $f(x) = 2e^x + \frac{1}{x} = 2e^x + x^{-1} \Rightarrow f'(x) = 2e^x - 1x^{-2} = 2e^x - \frac{1}{x^2}.$

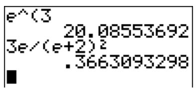
48c $f(x) = x \cdot e^x + 4 \Rightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x \cdot (1+x).$

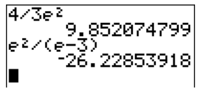
48d $f(x) = \frac{x}{e^x} \Rightarrow f'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2} = \frac{e^x \cdot (1-x)}{e^x \cdot e^x} = \frac{1-x}{e^x}.$

48e $f(x) = \frac{2e^x}{x-1} \Rightarrow f'(x) = \frac{(x-1)\cdot 2e^x - 2e^x \cdot 1}{(x-1)^2} = \frac{2xe^x - 2e^x - 2e^x}{(x-1)^2} = \frac{2xe^x - 4e^x}{(x-1)^2} = \frac{2e^x \cdot (x-2)}{(x-1)^2}.$

48f $f(x) = (2x-4) \cdot e^x \Rightarrow f'(x) = 2 \cdot e^x + (2x-4) \cdot e^x = e^x \cdot (2 + 2x - 4) = e^x \cdot (2x-2) = 2e^x \cdot (x-1).$

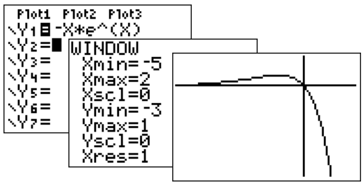
49a $e + 3 \approx 5{,}718.$
49b $-\frac{1}{e^2} \approx -0{,}135.$


49c $e^3 \approx 20{,}086.$
49d $\frac{3e}{(e+2)^2} \approx 0{,}366.$


49e $1\frac{1}{3}e^2 \approx 9{,}852.$
49f $\frac{e^2}{e-3} \approx -26{,}229.$


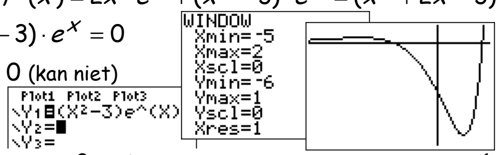
50a $f(x) = -x \cdot e^x \Rightarrow f'(x) = -1 \cdot e^x - x \cdot e^x = (-1-x) \cdot e^x.$
$f'(x) = 0 \Rightarrow (-1-x) \cdot e^x = 0 \Rightarrow -1 - x = 0 \vee e^x = 0$ (kan niet) $\Rightarrow x = -1.$
Maximum (zie plot) is $f(-1) = --1 \cdot e^{-1} = \frac{1}{e}.$

50b k: $y = ax$ met $a = f'(0) = (-1-0) \cdot e^0 = -1 \Rightarrow k$: $y = -x.$



51a $f(x) = 0$
$(x^2 - 3) \cdot e^x = 0$
$x^2 = 3 \vee e^x = 0$ (kan niet)
$x = -\sqrt{3} \vee x = \sqrt{3}.$

51b $f(x) = (x^2 - 3) \cdot e^x \Rightarrow f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3) \cdot e^x.$
$f'(x) = 0 \Rightarrow (x^2 + 2x - 3) \cdot e^x = 0$
$x^2 + 2x - 3 = 0 \vee e^x = 0$ (kan niet)
$(x+3) \cdot (x-1) = 0$
$x = -3 \vee x = 1.$
Max. (zie plot) is $f(-3) = 6 \cdot e^{-3} = \frac{6}{e^3}$ en min. (zie plot) is $f(1) = -2 \cdot e^1 = -2e.$



51c Voor grote negatieve waarden van x nadert $f(x) = (x^2 - 3) \cdot e^x$ naar nul $\Rightarrow y = 0$ (de x-as) is horizontale asymptoot.

51d $f(x) = p$ heeft precies twee oplossingen voor $p = \frac{6}{e^3} \vee -2e < p \le 0.$ (gebruik 51abc en de grafiek)

52 $f(x) = \frac{2e^x}{e^x + 1} \Rightarrow f'(x) = \frac{(e^x+1)\cdot 2e^x - 2e^x \cdot e^x}{(e^x+1)^2} = \frac{2e^{2x} + 2e^x - 2e^{2x}}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}.$
Stel k: $y = ax + b$ met $a = f'(1) = \frac{2e}{(e+1)^2}.$
k: $y = \frac{2e}{(e+1)^2}x + b$
door $P(1, \frac{2e}{e+1})$
$\Rightarrow \frac{2e}{e+1} = \frac{2e}{(e+1)^2} \cdot 1 + b \Rightarrow b = \frac{2e}{e+1} \cdot \frac{e+1}{e+1} - \frac{2e}{(e+1)^2} = \frac{2e^2 + 2e - 2e}{(e+1)^2} = \frac{2e^2}{(e+1)^2}.$
Dus k: $y = \frac{2e}{(e+1)^2}x + \frac{2e^2}{(e+1)^2}.$

k snijden met $y = 2$ geeft:

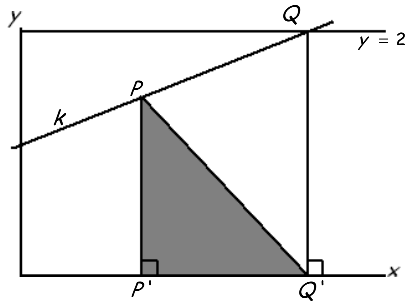
$\frac{2e}{(e+1)^2}x + \frac{2e^2}{(e+1)^2} = 2$

$\frac{2e}{(e+1)^2}x = 2 - \frac{2e^2}{(e+1)^2}$

$x = \frac{(e+1)^2}{2e} \cdot \left(2 - \frac{2e^2}{(e+1)^2}\right)$

$x = \frac{(e+1)^2}{2e} \cdot 2 - \frac{(e+1)^2}{2e} \cdot \frac{2e^2}{(e+1)^2}$

$x = \frac{(e+1)^2}{e} - \frac{2e^2}{2e} = \frac{e^2+2e+1}{e} - \frac{e^2}{e} = \frac{2e+1}{e} \Rightarrow Q(\frac{2e+1}{e},\ 2).$



$P'(1,\ 0)$ en $Q'(2+\frac{1}{e},\ 0)$.

$Opp.(\Delta PP'Q') = \frac{1}{2} \times basis \times hoogte$
$= \frac{1}{2} \cdot P'Q' \cdot PP'$
$= \frac{1}{2} \cdot P'Q' \cdot y_P$
$= \frac{1}{2} \cdot \left(2 + \frac{1}{e} - 1\right) \cdot \frac{2e}{e+1}$
$= \frac{1}{2} \cdot \left(1 + \frac{1}{e}\right) \cdot \frac{2e}{e+1}$
$= \frac{1}{2} \cdot \left(\frac{e}{e} + \frac{1}{e}\right) \cdot \frac{2e}{e+1}$
$= \frac{1}{2} \cdot \frac{e+1}{e} \cdot \frac{2e}{e+1} = 1.$

53 $f(x) = e^{ax+b} \Rightarrow f'(x) = e^{\boxed{ax+b}} \cdot a = a \cdot e^{ax+b}.$

54a $f(x) = e^{x^2+x} \Rightarrow f'(x) = e^{\boxed{x^2+x}} \cdot (2x+1) = (2x+1) \cdot e^{x^2+x}.$

54b $g(x) = x^2 + 2e^{3x} \Rightarrow g'(x) = 2x + 2e^{\boxed{3x}} \cdot 3 = 2x + 6e^{3x}.$

54c $h(x) = x \cdot e^{x^2} \Rightarrow h'(x) = 1 \cdot e^{x^2} + x \cdot e^{\boxed{x^2}} \cdot 2x = e^{x^2} + 2x^2 \cdot e^{x^2} = (1+2x^2) \cdot e^{x^2}.$

54d $j(x) = 3x \cdot e^{2x-1} \Rightarrow j'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{\boxed{2x-1}} \cdot 2 = 3 \cdot e^{2x-1} + 6x \cdot e^{2x-1} = (3+6x) \cdot e^{2x-1}.$

54e $k(x) = \frac{2e^{-x-1}}{x^2} \Rightarrow k'(x) = \frac{x^2 \cdot 2e^{\boxed{-x-1}} \cdot -1 - 2e^{-x-1} \cdot 2x}{(x^2)^2} = \frac{-2x^2e^{-x-1} - 4xe^{-x-1}}{x^4} = \frac{-2xe^{-x-1} \cdot (x+2)}{x^4} = \frac{-2e^{-x-1} \cdot (x+2)}{x^3}.$

54f $l(x) = \frac{e^{2x}}{e^{2x}+1} \Rightarrow l'(x) = \frac{(e^{2x}+1) \cdot e^{\boxed{2x}} \cdot 2 - e^{2x} \cdot e^{\boxed{2x}} \cdot 2}{(e^{2x}+1)^2} = \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x}+1)^2} = \frac{2e^{2x}}{(e^{2x}+1)^2}.$

55a $f(x) = \frac{1}{2}e^{2x} \Rightarrow f'(x) = \frac{1}{2}e^{\boxed{2x}} \cdot 2 = e^{2x}.$

Stel k: $y = ax + b$ met $a = f'(-1) = e^{-2}$.

$\left.\begin{array}{l} k\text{: } y = e^{-2}x + b \\ \text{door } A(-1,\ \frac{1}{2}e^{-2}) \end{array}\right\} \Rightarrow \frac{1}{2}e^{-2} = e^{-2} \cdot -1 + b \Rightarrow b = 1\frac{1}{2}e^{-2}.$

Dus k: $y = e^{-2}x + 1\frac{1}{2}e^{-2}$.

$g(x) = \frac{1}{e^{x+3}} = e^{-x-3} \Rightarrow g'(x) = e^{\boxed{-x-3}} \cdot -1 = -e^{-x-3}.$

Stel l: $y = ax + b$ met $a = g'(-1) = -e^{--1-3} = -e^{-2}$.

$\left.\begin{array}{l} l\text{: } y = -e^{-2}x + b \\ \text{door } B(-1,\ e^{-2}) \end{array}\right\} \Rightarrow e^{-2} = -e^{-2} \cdot -1 + b \Rightarrow b = 0.$

Dus l: $y = -e^{-2}x$.

k en l snijden geeft: $e^{-2}x + 1\frac{1}{2}e^{-2} = -e^{-2}x$

$2e^{-2}x = -1\frac{1}{2}e^{-2}$

$2x = -1\frac{1}{2}$

$x = -\frac{3}{4}.$

55b $h(x) = f(x) + g(x) \Rightarrow$

$h'(x) = f'(x) + g'(x) = e^{2x} - e^{-x-3}$ (zie 55a).

$h'(x) = 0 \Rightarrow e^{2x} - e^{-x-3} = 0$

$e^{2x} = e^{-x-3}$

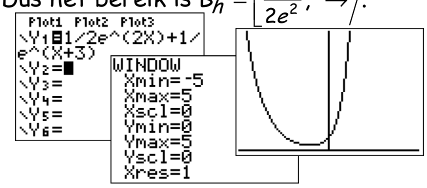
$2x = -x - 3$

$3x = -3$

$x = -1.$

Min. (zie plot) is $h(-1) = \frac{1}{2}e^{-2} + \frac{1}{e^2}$
$= \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}.$

Dus het bereik is $B_h = \left[\frac{3}{2e^2},\ \rightarrow\right\rangle$.

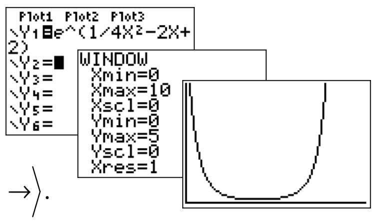


56a $f(x) = e^{\frac{1}{4}x^2-2x+2} \Rightarrow f'(x) = e^{\boxed{\frac{1}{4}x^2-2x+2}} \cdot (\frac{1}{2}x - 2) = (\frac{1}{2}x - 2) \cdot e^{\frac{1}{4}x^2-2x+2}.$

$f'(x) = 0 \Rightarrow (\frac{1}{2}x - 2) \cdot e^{\frac{1}{4}x^2-2x+2} = 0 \Rightarrow$

$\frac{1}{2}x - 2 = 0 \vee e^{\frac{1}{4}x^2-2x+2} = 0$ (kan niet) $\Rightarrow \frac{1}{2}x = 2 \Rightarrow x = 4.$

Minimum (zie plot) is $f(4) = e^{4-8+2} = e^{-2} = \frac{1}{e^2}$. Dus het bereik is $B_f = \left[\frac{1}{e^2},\ \rightarrow\right\rangle$.



56b $Q(p,\ f(p)) = Q(p,\ e^{\frac{1}{4}p^2-2p+2})$ en $R(0,\ e^{\frac{1}{4}p^2-2p+2}) \Rightarrow O$(rechthoek $PQRS$) $= O(p) = OP \cdot OR = p \cdot e^{\frac{1}{4}p^2-2p+2}.$

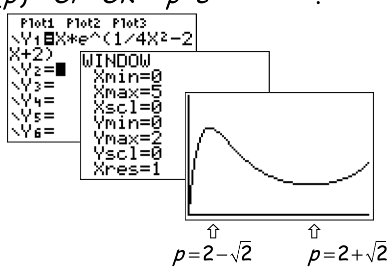
$O'(p) = 1 \cdot e^{\frac{1}{4}p^2-2p+2} + p \cdot e^{\frac{1}{4}p^2-2p+2} \cdot (\frac{1}{2}p - 2) = (\frac{1}{2}p^2 - 2p + 1) \cdot e^{\frac{1}{4}p^2-2p+2}$

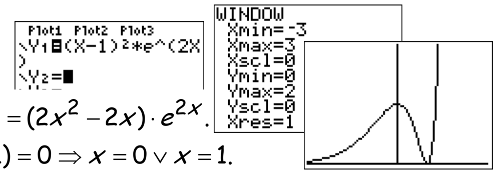
$O'(p) = 0 \Rightarrow (\frac{1}{2}p^2 - 2p + 1) \cdot e^{\frac{1}{4}p^2-2p+2} = 0$

$\frac{1}{2}p^2 - 2p + 1 = 0$ (abc-formule) $\vee\ e^{\frac{1}{4}p^2-2p+2} = 0$ (kan niet)

$D = (-2)^2 - 4 \cdot \frac{1}{2} \cdot 1 = 4 - 2 = 2 \Rightarrow p = \frac{2 \pm \sqrt{2}}{1} = 2 \pm \sqrt{2}.$

De oppervlakte is maximaal (zie de plot hiernaast) voor $p = 2 - \sqrt{2}$.





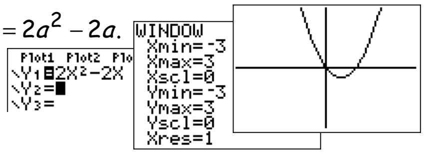
57a $f_1(x)=(x-1)^2\cdot e^{2x}=(x^2-2x+1)\cdot e^{2x}\Rightarrow$
$f_1{}'(x)=(2x-2)\cdot e^{2x}+(x^2-2x+1)\cdot e^{\boxed{2x}}\cdot 2=(2x-2+2x^2-4x+2)\cdot e^{2x}=(2x^2-2x)\cdot e^{2x}.$
$f_1{}'(x)=0\Rightarrow(2x^2-2x)\cdot e^{2x}\Rightarrow 2x^2-2x=0\vee e^{2x}=0$ (kan niet) $\Rightarrow 2x(x-1)=0\Rightarrow x=0\vee x=1.$
Maximum (zie de plot) is $f_1(0)=(-1)^2\cdot e^0=1\cdot 1=1$ en minimum (zie de plot) is $f_1(1)=(0)^2\cdot e^2=0.$

57b $f_a(x)=(x-a)^2\cdot e^{2x}=(x^2-2ax+a^2)\cdot e^{2x}\Rightarrow$
$f_a{}'(x)=(2x-2a)\cdot e^{2x}+(x^2-2ax+a^2)\cdot e^{\boxed{2x}}\cdot 2=(2x^2+(2-4a)x+2a^2-2a)\cdot e^{2x}.$
$f_a{}'(x)=0\Rightarrow 2x^2+(2-4a)x+2a^2-2a=0\vee e^{2x}=0$ (kan niet)
$x^2+(1-2a)x+a^2-a=0$ (*abc*-formule)
$D=(1-2a)^2-4\cdot 1\cdot(a^2-a)=1-4a+4a^2-4a^2+4a=1\Rightarrow x=\frac{-(1-2a)\pm\sqrt{1}}{2\cdot 1}=\frac{-1+2a\pm 1}{2}.$
$x=\frac{-1+2a+1}{2}=\frac{2a}{2}=a\vee x=\frac{-1+2a-1}{2}=\frac{2a-2}{2}=a-1.$ Dus $x_A=a-1$ en $x_B=a.$

57c $x_B=a\Rightarrow y_B=f(x_B)=f(a)=(a-a)^2\cdot e^{2a}=0\cdot e^{2a}=0.$ Dus de toppen van B liggen op de lijn $y=0$ (de x-as).

57d $x_A=a-1\Rightarrow y_A=f(x_A)=(a-1-a)^2\cdot e^{2(a-1)}=1\cdot e^{2(a-1)}=e^{2x_A}\Rightarrow$ de toppen van A liggen op de kromme $y=e^{2x}.$

57e C op de y-as $\Rightarrow x_C=0.$ $rc_{\text{raaklijn in }C}=f_a{}'(0)=(2a^2-2a)\cdot e^0=(2a^2-2a)\cdot 1=2a^2-2a.$
$rc_{\text{raaklijn in }C}=0\Rightarrow 2a^2-2a=0\Rightarrow 2a(a-1)=0\Rightarrow a=0\vee a=1.$
$rc_{\text{raaklijn in }C}$ (zie plot) $<0\Rightarrow 0<a<1.$



58a $2^x=\left(e^{{}^e\log(2)}\right)^x=e^{{}^e\log(2)\cdot x}$ of $2^x=e^{{}^e\log(2^x)}=e^{x\cdot{}^e\log(2)}=e^{{}^e\log(2)\cdot x}.$

58b $2^x=e^{{}^e\log(2)\cdot x}\Rightarrow[2^x]'=e^{\boxed{{}^e\log(2)\cdot x}}\cdot{}^e\log(2)={}^e\log(2)\cdot\left(e^{{}^e\log(2)}\right)^x={}^e\log(2)\cdot 2^x.$

$e^{\cdots}$ en $\ln\ldots={}^e\log\ldots$
heffen elkaar op
(zie ook de toetsen op de GR)

59a $\ln(e)=1.$

59b $\ln(e\cdot\sqrt{e})=\ln(e^1\cdot e^{\frac{1}{2}})=\ln(e^{1\frac{1}{2}})=1\frac{1}{2}.$

59c $\ln(\frac{1}{e})=\ln(e^{-1})=-1.$

59d $\ln(1)=\ln(e^0)=0.$

59e $3\cdot\ln(e\cdot\sqrt[3]{e})=3\cdot\ln(e^1\cdot e^{\frac{1}{3}})=3\cdot\ln(e^{1\frac{1}{3}})=\ln((e^{1\frac{1}{3}})^3)=\ln(e^4)=4.$

59f $\ln^2(e^3)=(\ln(e^3))^2=3^2=9.$

59g $\ln^3(e^2)=(\ln(e^2))^3=2^3=8.$

59h $e^{\ln(7)}+e^{2\cdot\ln(7)}=7+e^{\ln(7^2)}=7+7^2=7+49=56.$

59i $e^{\frac{1}{2}\cdot\ln(5)}=e^{\ln(5^{\frac{1}{2}})}=e^{\ln(\sqrt{5})}=\sqrt{5}.$

59j $e^{\ln(10)}\cdot e^{\ln(3)}=10\cdot 3=30.$

60a $e^{3x}=12$ (ln... nemen)
$3x=\ln(12)$ (: 3)
$x=\frac{1}{3}\ln(12).$

60b $5e^{2x}=60$ (: 5)
$e^{2x}=12$ (ln... nemen)
$2x=\ln(12)$
$x=\frac{1}{2}\ln(12).$

60c $6+e^{0,5x}=10$ (− 6)
$e^{0,5x}=4$ (ln... nemen)
$0,5x=\ln(4)$ (× 2)
$x=2\ln(4).$

60d $\frac{3}{e^{2x}}=10$ ($\times e^{2x}$)
$3=10e^{2x}$ (: 10)
$e^{2x}=\frac{3}{10}$ (ln... nemen)
$2x=\ln(\frac{3}{10})\Rightarrow x=\frac{1}{2}\ln(\frac{3}{10}).$

61a $2\cdot\ln(3)+\ln(4)=\ln(3^2)+\ln(4)=\ln(9\cdot 4)=\ln(36).$

61b $\ln(20)-3\cdot\ln(2)=\ln(20)-\ln(2^3)=\ln(\frac{20}{8})=\ln(2\frac{1}{2}).$

61c $4+\ln(3)=\ln(e^4)+\ln(3)=\ln(e^4\cdot 3)=\ln(3e^4).$

61d $1+\ln(10)=\ln(e^1)+\ln(10)=\ln(e\cdot 10)=\ln(10e).$

61e $\frac{1}{2}+2\ln(6)=\ln(e^{\frac{1}{2}})+\ln(6^2)=\ln(\sqrt{e}\cdot 36)=\ln(36\sqrt{e}).$

61f $e+\ln(2)=\ln(e^e)+\ln(2)=\ln(e^e\cdot 2)=\ln(2e^e).$

62a $\ln(x)=-1$ BV: $x>0$ ($e^{\cdots}$ nemen)
$x=e^{-1}=\frac{1}{e}$ (voldoet).

62b $4\ln(x)=2$ BV: $x>0$ (: 4)
$\ln(x)=\frac{1}{2}$ ($e^{\cdots}$ nemen)
$x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet).

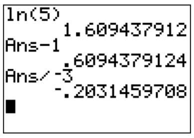
62c $\ln(3x)=3$ BV: $x>0$ ($e^{\cdots}$ nemen)
$3x=e^3$ (: 3)
$x=\frac{1}{3}e^3$ (voldoet).

62d $\ln(-x+2)=-2$ BV: $-x+2>0\Rightarrow -x>-2\Rightarrow x<2$ ($e^{\cdots}$ nemen)
$-x+2=e^{-2}=\frac{1}{e^2}$ (− 2)
$-x=-2+\frac{1}{e^2}$ ($\times -1$) $\Rightarrow x=2-\frac{1}{e^2}$ (voldoet).

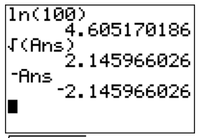
62e $\ln^2(x)=\frac{1}{4}$ BV: $x>0$
$\ln(x)=\pm\frac{1}{2}$ ($e^{\cdots}$ nemen)
$x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet) $\vee\ x=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$ (voldoet).

62f $\ln(x)=1+\ln(5)$ BV: $x>0$
$\ln(x)=\ln(e)+\ln(5)$
$\ln(x)=\ln(5e)$ ($e^{\cdots}$ nemen)
$x=5e$ (voldoet).

63a $4e^{1-3x}=20\ \ (:4)$
$e^{1-3x}=5$ (ln... nemen)
$1-3x=\ln(5)\ \ (-1)$
$-3x=-1+\ln(5)\ \ (:-3)$
$x=\frac{-1+\ln(5)}{-3}\approx -0{,}203.$



63b $e^{x^2}=100$ (ln... nemen)
$x^2=\ln(100)$
$x=\pm\sqrt{\ln(100)}$
$x=\sqrt{\ln(100)}\approx 2{,}146 \vee x=-\sqrt{\ln(100)}\approx -2{,}146.$



64a $3x\cdot\ln(x)=2\cdot\ln(x)$ BV: $x>0$
$3x\cdot\ln(x)-2\cdot\ln(x)=0$
$(3x-2)\cdot\ln(x)=0$
$3x=2\vee\ln(x)=0$ ($e^{\cdots}$ nemen)
$x=\frac{2}{3}$ (voldoet) $\vee\ x=e^0=1$ (voldoet).

64b $\ln^2(x)-\ln(x)=0$ BV: $x>0$ (stel $\ln(x)=t$)
$t^2-t=0$
$t\cdot(t-1)=0$
$t=\ln(x)=0\vee t=\ln(x)=1$ ($e^{\cdots}$ nemen)
$x=e^0=1$ (voldoet) $\vee\ x=e^1=e$ (voldoet).

64c $x^2\cdot\ln(x+1)=4\cdot\ln(x+1)$ BV: $x>-1$
$x^2\cdot\ln(x+1)-4\cdot\ln(x+1)=0$
$(x^2-4)\cdot\ln(x+1)=0$
$x^2=4\vee\ln(x+1)=0$ ($e^{\cdots}$ nemen)
$x=\pm2\vee x+1=e^0=1$
$x=2$ (voldoet) $\vee\ x=-2$ (vold. niet) $\vee\ x=0$ (voldoet).

64d $\ln^2(x)-2\ln(x)-3=0$ BV: $x>0$ (stel $\ln(x)=t$)
$t^2-2t-3=0$
$(t-3)\cdot(t+1)=0$
$t=\ln(x)=3\vee t=\ln(x)=-1$ ($e^{\cdots}$ nemen)
$x=e^3$ (voldoet) $\vee\ x=e^{-1}=\frac{1}{e}$ (voldoet).

64e $\ln(x+3)-\ln(x-1)=\ln(2)$ BV: $x>1$
$\ln(x+3)=\ln(x-1)+\ln(2)$
$\ln(x+3)=\ln(2\cdot(x-1))$ ($e^{\cdots}$ nemen)
$x+3=2x-2$
$-x=-5$
$x=5$ (voldoet).

64f $2\cdot\ln(x)=\ln(2)+\ln(x+4)$ BV: $x>0$
$\ln(x^2)=\ln(2\cdot(x+4))$ ($e^{\cdots}$ nemen)
$x^2=2x+8$
$x^2-2x-8=0$
$(x-4)\cdot(x+2)=0$
$x=4$ (voldoet) $\vee\ x=-2$ (vold. niet).

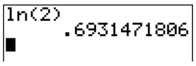
$f(x)=a^x\Rightarrow f'(x)=a^x\cdot\ln a.$

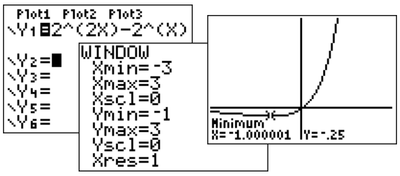
65a $f(x)=3^{4x-2}\Rightarrow f'(x)=3^{\boxed{4x-2}}\cdot\ln(3)\cdot4=4\cdot3^{4x-2}\cdot\ln(3).$

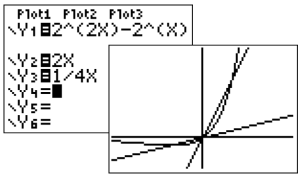
65b $g(x)=(2x-1)\cdot2^x\Rightarrow g'(x)=2\cdot2^x+(2x-1)\cdot2^x\cdot\ln(2)=(2+(2x-1)\cdot\ln(2))\cdot2^x.$

65c $h(x)=\frac{2^x+1}{2^x-1}\Rightarrow h'(x)=\frac{(2^x-1)\cdot2^x\cdot\ln(2)-(2^x+1)\cdot2^x\cdot\ln(2)}{(2^x-1)^2}=\frac{(2^x-1-2^x-1)\cdot2^x\cdot\ln(2)}{(2^x-1)^2}=\frac{-2\cdot2^x\cdot\ln(2)}{(2^x-1)^2}.$

66a $f(x)=2^{2x}-2^x\Rightarrow f'(x)=2^{\boxed{2x}}\cdot\ln(2)\cdot2-2^x\cdot\ln(2)=(2^{2x}\cdot2-2^x)\cdot\ln(2)=(2^{2x+1}-2^x)\cdot\ln(2).$
$f'(x)=0\Rightarrow(2^{2x+1}-2^x)\cdot\ln(2)=0$
$2^{2x+1}-2^x=0$
$2^{2x+1}=2^x$
$2x+1=x$
$x=-1.$







Minimum (zie plot) $f(-1)=2^{-2}-2^{-1}=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}\Rightarrow B_f=\left[-\frac{1}{4},\ \rightarrow\right\rangle.$

66b $y=ax$ (lijn door de oorsprong en ook $f(0)=1-1=0$) heeft twee oplossingen als $0<a<f'(0)\vee a>f'(0)$.
$f'(0)=(2^1-2^0)\cdot\ln(2)=(2-1)\cdot\ln(2)=\ln(2)$ dus $0<a<\ln(2)\vee a>\ln(2)$. ($y=ax$ met $a=\ln(2)$ raakt f in de oorsprong)

67a $f(x)=2^{x-1}+2^{-x-2}\Rightarrow f'(x)=2^{\boxed{x-1}}\cdot\ln(2)\cdot1+2^{\boxed{-x-2}}\cdot\ln(2)\cdot-1=(2^{x-1}-2^{-x-2})\cdot\ln(2).$
$f'(x)=0\Rightarrow(2^{x-1}-2^{-x-2})\cdot\ln(2)=0$
$2^{x-1}-2^{-x-2}=0$
$2^{x-1}=2^{-x-2}$
$x-1=-x-2$
$2x=-1$
$x=-\frac{1}{2}.$
Minimum (zie figuur 9.8) $f(-\frac{1}{2})=2^{-\frac{1}{2}-1}+2^{\frac{1}{2}-2}=2\cdot2^{-1\frac{1}{2}}=2^{-\frac{1}{2}}=\frac{1}{2^{\frac{1}{2}}}=\frac{1}{\sqrt{2}}\Rightarrow B_f=\left[\frac{1}{\sqrt{2}},\ \rightarrow\right\rangle.$

67b $f'(x) = -\frac{1}{4}\cdot\ln(2) \Rightarrow (2^{x-1} - 2^{-x-2})\cdot\ln(2) = -\frac{1}{4}\cdot\ln(2)$

$2^{x-1} - 2^{-x-2} = -\frac{1}{4}$ $(\times 4)$

$2^2 \cdot 2^{x-1} - 2^2 \cdot 2^{-x-2} = -1$

$2^{x+1} - 2^{-x} = -1$

$2 \cdot 2^x - \frac{1}{2^x} = -1$ (stel $2^x = t$)

$2t - \frac{1}{t} = -1$ $(\times t)$

$2t^2 - 1 = -t$

$2t^2 + t - 1 = 0$ (abc-formule)

$D = 1^2 - 4\cdot 2\cdot -1 = 1 + 8 = 9$

$t = 2^x = \frac{-1+3}{4} = \frac{1}{2} = 2^{-1} \vee t = 2^x = \frac{-1-3}{4} = -1$ (kan niet)

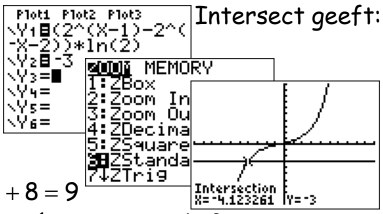
$x = -1 \Rightarrow y = f(-1) = 2^{-2} + 2^{-1} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$. Dus raakpunt $(-1, \frac{3}{4})$.

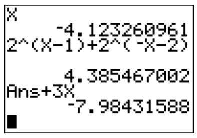
67c $f'(x) = -3 \Rightarrow (2^{x-1} - 2^{-x-2})\cdot\ln(2) = -3$.

Intersect geeft: $x_R \approx -4{,}1233$ en $y_R = f(x_R)$

Verder: $y_R = -3x_R + b \Rightarrow$

$y_R + 3x_R = b \approx -7{,}984$.





68a $e^{\ln(x)} = x$ (links en rechts de afgeleide nemen)

$e^{\boxed{\ln(x)}}\cdot[\ln(x)]' = 1$. Dus $e^{\ln(x)}\cdot[\ln(x)]' = 1$.

68b $\left.\begin{array}{l} e^{\ln(x)}\cdot[\ln(x)]' = 1 \\ e^{\ln(x)} = x \end{array}\right\} \Rightarrow x\cdot[\ln(x)]' = 1 \Rightarrow [\ln(x)]' = \frac{1}{x}$.

68c $g(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)}\cdot\ln(x) \Rightarrow g'(x) = \frac{1}{\ln(2)}\cdot\frac{1}{x} = \frac{1}{x\cdot\ln(2)}$.

69a $f(x) = \ln(6x) \Rightarrow f'(x) = \frac{1}{\boxed{6x}}\cdot 6 = \frac{1}{x}$

of $f(x) = \ln(6\cdot x) = \ln(6) + \ln(x) \Rightarrow f'(x) = 0 + \frac{1}{x} = \frac{1}{x}$.

69b $f(x) = \ln(2\cdot x) \Rightarrow f'(x) = \frac{1}{x}$.

$g(x) = \ln(x\cdot\sqrt{2}) \Rightarrow g'(x) = \frac{1}{x}$.

$h(x) = {}^2\log(3\cdot x) \Rightarrow h'(x) = \frac{1}{x\cdot\ln(2)}$.

70a $f(x) = \ln(x^6) \Rightarrow f'(x) = \frac{1}{\boxed{x^6}}\cdot 6x^5 = \frac{6x^5}{x^6} = \frac{6}{x}$

of $f(x) = \ln(x^6) = 6\cdot\ln(x) \Rightarrow f'(x) = 6\cdot\frac{1}{x} = \frac{6}{x}$.

70b $f(x) = \ln(x^2) \Rightarrow f'(x) = \frac{2}{x}$.

$g(x) = \ln(\frac{1}{x^3}) = \ln(x^{-3}) \Rightarrow g'(x) = \frac{-3}{x}$.

$h(x) = \ln(\frac{1}{x}) = \ln(x^{-1}) \Rightarrow h'(x) = \frac{-1}{x}$.

71a $f(x) = \frac{1-\ln(x)}{x} \Rightarrow f'(x) = \frac{x\cdot -\frac{1}{x} - (1-\ln(x))\cdot 1}{x^2} = \frac{-1-1+\ln(x)}{x^2} = \frac{\ln(x)-2}{x^2}$.

71b $f(x) = x\cdot\ln(x) \Rightarrow f'(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x) + 1$.

71c $f(x) = {}^2\log(4x-1) \Rightarrow f'(x) = \frac{1}{(\boxed{4x-1})\cdot\ln(2)}\cdot 4 = \frac{4}{(4x-1)\cdot\ln(2)}$.

71d $f(x) = \frac{\ln(3x)}{x} \Rightarrow f'(x) = \frac{x\cdot\frac{1}{x} - \ln(3x)\cdot 1}{x^2} = \frac{1-\ln(3x)}{x^2}$.

71e $f(x) = x\cdot\ln(x^3) \Rightarrow f'(x) = 1\cdot\ln(x^3) + x\cdot\frac{3}{x} = \ln(x^3) + 3$.

71f $f(x) = {}^3\log(x^2) \Rightarrow f'(x) = \frac{2}{x\cdot\ln(3)}$.

$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \ln(x^a) \Rightarrow f'(x) = \frac{a}{x}$

72a $f(x) = \ln(x^2 + x) \Rightarrow f'(x) = \frac{1}{\boxed{x^2+x}}\cdot(2x+1) = \frac{2x+1}{x^2+x}$.

72b $g(x) = \ln(2^x) \Rightarrow g'(x) = \frac{1}{\boxed{2^x}}\cdot 2^x\cdot\ln(2) = \ln(2)$ of $g(x) = \ln(2^x) = x\cdot\ln(2) = \ln(2)\cdot x \Rightarrow g'(x) = \ln(2)\cdot 1 = \ln(2)$.

72c $h(x) = {}^2\log(x^2+1) \Rightarrow h'(x) = \frac{1}{(\boxed{x^2+1})\cdot\ln(2)}\cdot 2x = \frac{2x}{(x^2+1)\cdot\ln(2)}$.

72d $j(x) = \log(4x^2) \Rightarrow j'(x) = \frac{1}{\boxed{4x^2}\cdot\ln(10)}\cdot 8x = \frac{2}{x\cdot\ln(10)}$ of $j(x) = \log(4x^2) = \log(4) + \log(x^2) \Rightarrow j'(x) = \frac{2}{x\cdot\ln(10)}$.

73a $f(x) = x\cdot\ln^2(x) \Rightarrow f'(x) = 1\cdot\ln^2(x) + x\cdot 2\boxed{\ln(x)}\cdot\frac{1}{x} = \ln^2(x) + 2\cdot\ln(x)$.

73b $g(x) = x^2\cdot{}^3\log(4x) \Rightarrow g'(x) = 2x\cdot{}^3\log(4x) + x^2\cdot\frac{1}{x\cdot\ln(3)} = 2x\cdot{}^3\log(4x) + \frac{x}{\ln(3)}$.

73c $h(x) = \log^2(4x) \Rightarrow h'(x) = 2\cdot\boxed{\log(4x)}\cdot\frac{1}{\boxed{4x}\cdot\ln(10)}\cdot 4 = \frac{2\cdot\log(4x)}{x\cdot\ln(10)}$.

73d $j(x) = \ln^2(4x^2+1) \Rightarrow j'(x) = 2\cdot\boxed{\ln(4x^2+1)}\cdot\frac{1}{\boxed{4x^2+1}}\cdot 8x = \frac{16x\cdot\ln(4x^2+1)}{4x^2+1}$.

74a $x^n = e^{\ln(x^n)} = e^{n\cdot\ln(x)}$.

74bc $f(x) = x^n = e^{n\cdot\ln(x)} \Rightarrow f'(x) = e^{\boxed{n\cdot\ln(x)}}\cdot n\cdot\frac{1}{x} = x^n\cdot n\cdot\frac{1}{x} = \frac{n\cdot x^n}{x} = nx^{n-1}$.

75a A op de x-as $(y=0) \Rightarrow f(x)=0$

$\frac{10\ln(x)}{x}=0$ ($\Rightarrow$ teller $=0) \Rightarrow \ln(x)=0 \Rightarrow x=1 \Rightarrow A(1,0)$

$f(x)=\frac{10\ln(x)}{x}$ (BV: $x>0) \Rightarrow f'(x)=\frac{x\cdot 10\cdot \frac{1}{x}-10\ln(x)\cdot 1}{x^2}=\frac{10-10\ln(x)}{x^2}$

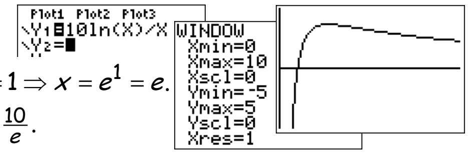
Stel de raaklijn: k: $y=ax+b$ met $a=f'(1)=\frac{10-10\ln(1)}{1^2}=\frac{10-0}{1}=10$

$y=10x+b$ door $A(1,0) \Rightarrow 0=10\cdot 1+b \Rightarrow b=-10$. Dus k: $y=10x-10$.

75b $f'(x)=0 \Rightarrow \frac{10-10\ln(x)}{x^2}=0$ ($\Rightarrow$ teller $=0$)

$10-10\ln(x)=0 \Rightarrow 10\ln(x)=10 \Rightarrow \ln(x)=1 \Rightarrow x=e^1=e.$

Maximum (zie plot) $f(e)=\frac{10\ln(e)}{e}=\frac{10\cdot 1}{e}=\frac{10}{e}$.



75c Stel $x_B=p$ dan is $x_C=2p$

$f(p)=f(2p)=q$ geeft

$\frac{10\ln(p)}{p}=\frac{10\ln(2p)}{2p}$

$10\ln(p)=5\ln(2p)$

$2\ln(p)=\ln(2p)$

$\ln(p^2)=\ln(2p)$

$p^2=2p$

$p=0$ (vold. niet) $\vee$ $p=2$ (voldoet)

$q=f(p)=f(2)=\frac{10\ln(2)}{2}=5\ln(2)$.

76a $x_A=\frac{1}{e} \Rightarrow y_A=f(\frac{1}{e})=\frac{\frac{1}{e}}{\ln(\frac{1}{e})}=\frac{\frac{1}{e}}{\ln(e^{-1})}=\frac{\frac{1}{e}}{-1}=-\frac{1}{e}$

$f(x)=\frac{x}{\ln(x)} \Rightarrow f'(x)=\frac{\ln(x)\cdot 1-x\cdot\frac{1}{x}}{\ln^2(x)}=\frac{\ln(x)-1}{\ln^2(x)}$

Stel k: $y=ax+b$ met $a=f'(\frac{1}{e})=\frac{\ln(\frac{1}{e})-1}{\ln^2(\frac{1}{e})}=\frac{-1-1}{(-1)^2}=-2$

$y=-2x+b$ door $A(\frac{1}{e},-\frac{1}{e}) \Rightarrow -\frac{1}{e}=-2\cdot\frac{1}{e}+b \Rightarrow b=-\frac{1}{e}+\frac{2}{e}=\frac{1}{e}$. Dus k: $y=-2x+\frac{1}{e}$.

76b $f'(x)=-6 \Rightarrow \frac{\ln(x)-1}{\ln^2(x)}=-6$ (BV: $x>0$)

$-6\cdot\ln^2(x)=\ln(x)-1$ (stel $\ln(x)=t$)

$-6t^2-t+1=0$ (abc-formule)

$D=(-1)^2-4\cdot -6\cdot 1=1+24=25 \Rightarrow t=\frac{1\pm\sqrt{25}}{2\cdot -6}$ ⇗

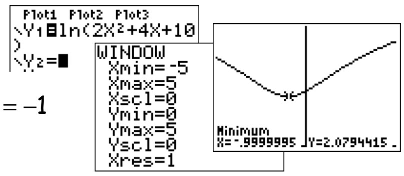
$t=\ln(x)=\frac{1+5}{-12}=-\frac{1}{2} \vee t=\ln(x)=\frac{1-5}{-12}=\frac{1}{3}$

$\begin{cases} x=e^{-\frac{1}{2}}=\frac{1}{e^{\frac{1}{2}}}=\frac{1}{\sqrt{e}} \text{ (voldoet)} \\ y=f(e^{-\frac{1}{2}})=\frac{e^{-\frac{1}{2}}}{\ln(e^{-\frac{1}{2}})}=\frac{\frac{1}{\sqrt{e}}}{-\frac{1}{2}}=-\frac{2}{\sqrt{e}} \end{cases} \vee \begin{cases} x=e^{\frac{1}{3}}=\sqrt[3]{e} \text{ (voldoet)} \\ y=f(e^{\frac{1}{3}})=\frac{e^{\frac{1}{3}}}{\ln(e^{\frac{1}{3}})}=\frac{\sqrt[3]{e}}{\frac{1}{3}}=3\cdot\sqrt[3]{e}. \end{cases}$

77a $f(x)=\ln(2x^2+4x+10) \Rightarrow f'(x)=\frac{1}{2x^2+4x+10}\cdot(4x+4)=\frac{4x+4}{2x^2+4x+10}$

$f'(x)=0 \Rightarrow \frac{4x+4}{2x^2+4x+10}=0$ ($\Rightarrow$ teller $=0) \Rightarrow 4x+4=0 \Rightarrow 4x=-4 \Rightarrow x=-1$

minimum (zie plot) $f(-1)=\ln(2-4+10)=\ln(8) \Rightarrow B_f=[\ln(8), \rightarrow\rangle$.



77b $f'(x)=\frac{2}{5} \Rightarrow \frac{4x+4}{2x^2+4x+10}=\frac{2}{5}$

$4x^2+8x+20=20x+20$

$4x^2-12x=0$

$4x\cdot(x-3)=0$

$\begin{cases} x=0 \\ y=f(0)=\ln(10) \end{cases} \vee \begin{cases} x=3 \\ y=f(3)=\ln(2\cdot 3^2+4\cdot 3+10)=\ln(40). \end{cases}$

77c $f'(x)=1 \Rightarrow \frac{4x+4}{2x^2+4x+10}=1$

$2x^2+4x+10=4x+4$

$2x^2=-6$

$x^2=-3$ (kan niet).

78a $f(x)=g(x) \Rightarrow \ln(2x)=\ln(\frac{4}{x})$ BV: $x>0$

$2x=\frac{4}{x}$

$2x^2=4$

$x^2=2$

$\begin{cases} x=\sqrt{2} \text{ (voldoet)} \vee x=-\sqrt{2} \text{ (vold. niet)} \\ y=f(\sqrt{2})=\ln(2\sqrt{2}). \end{cases}$

78b A op de x-as $(y=0) \Rightarrow g(x)=0$

$\ln(\frac{4}{x})=0$ BV: $x>0 \Rightarrow \frac{4}{x}=e^0=1 \Rightarrow x=4$ (voldoet) $\Rightarrow A(4,0)$

$g(x)=\ln(\frac{4}{x})=\ln(4)-\ln(x) \Rightarrow g'(x)=0-\frac{1}{x}=-\frac{1}{x}$

Stel de raaklijn: $y=ax+b$ met $a=g'(4)=-\frac{1}{4}$

$y=-\frac{1}{4}x+b$ door $A(4,0) \Rightarrow 0=-\frac{1}{4}\cdot 4+b \Rightarrow b=1$.

Dus $y=-\frac{1}{4}x+1$.

78c $f(p)-g(p)=2 \vee g(p)-f(p)=2$

$\ln(2p)-\ln(\frac{4}{p})=2 \vee \ln(\frac{4}{p})-\ln(2p)=2$

$\ln(2p:\frac{4}{p})=2 \vee \ln(\frac{4}{p}:2p)=2$

$\ln(2p\cdot\frac{p}{4})=2 \vee \frac{2}{p^2}=e^2$

$\frac{p^2}{2}=e^2 \vee p^2=\frac{2}{e^2}$

$p^2=2e^2 \vee p=\frac{1}{e}\cdot\sqrt{2}$

$p=e\cdot\sqrt{2} \vee p=\frac{1}{e}\cdot\sqrt{2}$

($p=-e\sqrt{2}$ vold. niet) ($p=-\frac{1}{e}\sqrt{2}$ vold. niet)

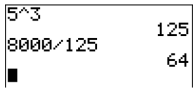
78d $y_B=f(p)=\ln(2p)$ en $y_C=g(p)=\ln(\frac{4}{p})$

$y_M=\frac{y_B+y_C}{2}=\frac{\ln(2p)+\ln(\frac{4}{p})}{2}=\frac{\ln(2p\cdot\frac{4}{p})}{2}=\frac{\ln(8)}{2}$.

$\frac{\ln(8)}{2}$ is onafhankelijk van p,

dus het midden M van BC is onafhankelijk van p.

Diagnostische toets



D1a $^3\log(5) + 2 \cdot {}^3\log(2) = {}^3\log(5) + {}^3\log(2^2) = {}^3\log(5 \cdot 4) = {}^3\log(20).$

D1b $3 - {}^2\log(5) = {}^2\log(2^3) - {}^2\log(5) = {}^2\log(8) - {}^2\log(5) = {}^2\log(\frac{8}{5}).$

D1c $^2\log(8000) + 3 \cdot {}^2\log(\frac{1}{5}) = {}^2\log(8000) + {}^2\log((\frac{1}{5})^3) = {}^2\log(8000) + {}^2\log(\frac{1}{125}) = {}^2\log(\frac{8000}{125}) = {}^2\log(64) = {}^2\log(2^6) = 6.$

D2a $2 \cdot {}^2\log(x-1) = 1 + {}^2\log(18)$ BV: $x > 1$
$^2\log((x-1)^2) = {}^2\log(2^1) + {}^2\log(18)$
$^2\log((x-1)^2) = {}^2\log(2 \cdot 18).$
$(x-1)^2 = 36$
$x - 1 = \pm 6$ (links en rechts $+1$)
$x = \pm 6 + 1$
$x = 6 + 1 = 7$ (voldoet) $\vee$ $x = -6 + 1 = -5$ (vold. niet).

D2b $^2\log(x) = 3 - {}^2\log(x+2)$ BV: $x > 0$
$^2\log(x) + {}^2\log(x+2) = {}^2\log(2^3)$
$^2\log(x \cdot (x+2)) = {}^2\log(8).$
$x \cdot (x+2) = 8.$
$x^2 + 2x - 8 = 0$
$(x+4) \cdot (x-2) = 0$
$x = -4$ (vold. niet) $\vee$ $x = 2$ (voldoet).

D3a $^2\log(x) - {}^{\frac{1}{2}}\log(x-1) = 3$ BV: $x > 1$
$^2\log(x) + {}^2\log(x-1) = {}^2\log(2^3)$
$^2\log(x \cdot (x-1)) = {}^2\log(8).$
$x \cdot (x-1) = 8$
$x^2 - x - 8 = 0$ (*abc*-formule)
$D = (-1)^2 - 4 \cdot 1 \cdot -8 = 1 + 32 = 33 \Rightarrow x = \frac{1 \pm \sqrt{33}}{2 \cdot 1}$
$x = \frac{1+\sqrt{33}}{2}$ (voldoet) $\vee$ $x = 3^x = \frac{1-\sqrt{33}}{2} < 1$ (vold. niet).

D3b $\log^2(x) - 5 \cdot \log(x) = 6$ BV: $x > 0$
Stel $\log(x)$ tijdelijk t
$t^2 - 5t = 6$
$t^2 - 5t - 6 = 0$
$(t-6) \cdot (t+1) = 0$
$t = \log(x) = 6 \vee t = \log(x) = -1$
$x = 10^6 = 1\,000\,000$ (voldoet) $\vee$ $x = 10^{-1} = \frac{1}{10}$ (voldoet).

D4a $3^x + 6 \cdot (\frac{1}{3})^x = 5$
$3^x + 6 \cdot (\frac{1}{3^x}) = 5$ (links en rechts $\times 3^x$)
$(3^x)^2 + 6 \cdot 1 = 5 \cdot 3^x$ (stel $3^x = t$)
$t^2 + 6 = 5t$
$t^2 - 5t + 6 = 0$
$(t-2) \cdot (t-3) = 0$
$t = 3^x = 2 \vee t = 3^x = 3 = 3^1 \Rightarrow x = {}^3\log(2) \vee x = 1.$

D4b $9^x = 3^x + 12$
$(3^2)^x = 3^x + 12$
$(3^x)^2 - 3^x - 12 = 0$ (stel $3^x = t$)
$t^2 - t - 12 = 0$
$(t-4) \cdot (t+3) = 0$
$t = 3^x = 4 \vee t = 3^x = -3$ (kan niet) $\Rightarrow x = {}^3\log(4).$

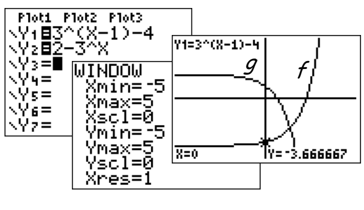
D4c $9^x = 3^{x+1} + 4$
$(3^2)^x = 3^x \cdot 3^1 + 4$
$(3^x)^2 = 3 \cdot 3^x + 4$
$(3^x)^2 - 3 \cdot 3^x - 4 = 0$ (stel $3^x = t$)
$t^2 - 3t - 4 = 0$
$(t-4) \cdot (t+1) = 0$
$t = 3^x = 4 \vee t = 3^x = -1$ (kan niet) $\Rightarrow x = {}^3\log(4).$

D4d $3^{x+2} + 3^{2x+1} = 12$
$3^x \cdot 3^2 + 3^{2x} \cdot 3^1 = 12$
$9 \cdot 3^x + 3 \cdot (3^x)^2 - 12 = 0$ (stel $3^x = t$ en deel door 3)
$t^2 + 3t - 4 = 0$
$(t+4) \cdot (t-1) = 0$
$t = 3^x = -4$ (kan niet) $\vee$ $t = 3^x = 1 = 3^0 \Rightarrow x = 0.$

D5a $y = 3^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{3}} f(x) = \frac{1}{3} \cdot 3^x = 3^{-1} \cdot 3^x = 3^{x-1} \xleftarrow{\text{translatie (1, 0)}} y = 3^x.$

D5b $y = {}^3\log(x) \xrightarrow{\text{translatie (0, –2)}} f(x) = {}^3\log(x) - 2 = {}^3\log(x) + {}^3\log(3^{-2}) = {}^3\log(x \cdot \frac{1}{9}) = {}^3\log(\frac{1}{9} \cdot x).$
$y = {}^3\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met 9}} f(x) = {}^3\log(\frac{1}{9} \cdot x).$

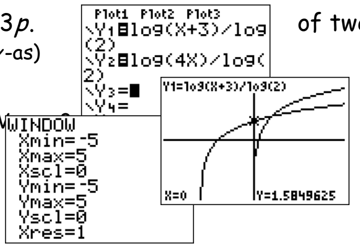
D6a
$f(p) - g(p) = 2$ $\vee$ $g(p) - f(p) = 2$
$3^{p-1} - 4 - (2 - 3^p) = 2$ $\vee$ $2 - 3^p - (3^{p-1} - 4) = 2$
$3^{p-1} - 4 - 2 + 3^p = 2$ $\vee$ $2 - 3^p - 3^{p-1} + 4 = 2$
$3^{p-1} - 6 + 3^p = 2$ (links en rechts $\times 3^1$) $\vee$ $6 - 3^p - 3^{p-1} = 2$ (links en rechts $\times 3^1$)
$3^p - 18 + 3 \cdot 3^p = 6$ $\vee$ $18 - 3 \cdot 3^p - 3^p = 6$
$4 \cdot 3^p = 24$ $\vee$ $-4 \cdot 3^p = -12$
$3^p = 6$ $\vee$ $3^p = 3$
$p = {}^3\log(6).$ $\vee$ $p = {}^3\log(3) = 1.$



D6b $f(p) = g(p+1) = q$
$3^{p-1} - 4 = 2 - 3^{p+1}$ (links en rechts $\times 3^1$)
$3^p - 12 = 6 - 3 \cdot 3^p \cdot 3^1$
$3^p - 12 = 6 - 9 \cdot 3^p$
$10 \cdot 3^p = 18$
$3^p = \frac{18}{10} = \frac{9}{5}$
$p = {}^3\log(\frac{9}{5}) \Rightarrow q = f(p) = 3^p \cdot 3^{-1} - 4 = \frac{9}{5} \cdot \frac{1}{3} - 4 = -3\frac{2}{5}$.

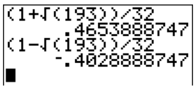
$\vee$ $g(p) = f(p+1) = q$
$\vee$ $2 - 3^p = 3^{p+1-1} - 4$
$\vee$ $2 - 3^p = 3^p - 4$
$\vee$ $-2 \cdot 3^p = -6$
$\vee$ $3^p = 3 = 3^1$
$\vee$ $p = 1 \Rightarrow q = g(p) = 2 - 3^1 = -1$.

D7a Stel $x_B = p > 0$ dan $x_C = 3p$.
(B en C beide rechts van de y-as)
$f(3p) = g(p) = q$
${}^2\log(3p+3) = {}^2\log(4p)$ BV
$3p + 3 = 4p$
$-p = -3$
$p = 3$ (voldoet)
$q = g(p) = g(3) = {}^2\log(12)$.
Dus $q = {}^2\log(12)$ of (zie hiernaast) $q = {}^2\log(\frac{12}{5})$.



of tweede mogelijkheid: Stel $x_B = -p < 0$ dan $x_C = p > 0$.
(B links en C rechts van de y-as)
$f(-p) = g(p) = q$
${}^2\log(-p+3) = {}^2\log(4p)$ BV: $0 < p < 3$
$-p + 3 = 4p$
$-5p = -3$
$p = \frac{3}{5}$ (voldoet)
$q = g(p) = g(\frac{3}{5}) = {}^2\log(\frac{12}{5})$.

D7b E op f is het midden van DF.
$f(p) = 2 \cdot g(p)$
${}^2\log(p+3) = 2 \cdot {}^2\log(4p)$ BV: $p > 0$
${}^2\log(p+3) = {}^2\log((4p)^2)$
$p + 3 = 16p^2$
$16p^2 - p - 3 = 0$ (abc-formule)



$D = (-1)^2 - 4 \cdot 16 \cdot -3 = 1 + 192 = 193 \Rightarrow p = \frac{1 \pm \sqrt{193}}{2 \cdot 16} \Rightarrow p = \frac{1+\sqrt{193}}{32} \approx 0{,}47$ (voldoet) $\vee$ $p = \frac{1-\sqrt{193}}{32}$ (vold. niet).

D8a $\frac{3e^3 - e^3}{e^2} = \frac{2e^3}{e^2} = 2e$ of $\frac{3e^3 - e^3}{e^2} = \frac{3e^3}{e^2} - \frac{e^3}{e^2} = 3e - e = 2e$.

D8b $(e^{3x} - 5)^2 = (e^{3x})^2 - 2 \cdot 5 \cdot e^{3x} + 5^2 = e^{6x} - 10e^{3x} + 25$.

D9a $3xe^x - e^x = 0$
$(3x - 1) \cdot e^x = 0$
$3x = 1 \vee e^x = 0$ (kan niet)
$x = \frac{1}{3}$.

D9c $e^{4x} - e^{x+1} = 0$
$e^{4x} = e^{x+1}$
$4x = x + 1$
$3x = 1$
$x = \frac{1}{3}$.

D9b $e^{2x-1} - \sqrt[3]{e^2} = 0$
$e^{2x-1} = \sqrt[3]{e^2} = e^{\frac{2}{3}}$
$2x - 1 = \frac{2}{3}$
$2x = 1\frac{2}{3} = \frac{5}{3}$
$x = \frac{5}{6}$.

D9d $e^{2x} + 2e^x = 3$ (stel $e^x = t$)
$t^2 + 2t - 3 = 0$
$(t+3) \cdot (t-1) = 0$
$t = e^x = -3$ (kan niet) $\vee$ $t = e^x = 1 = e^0$
$x = 0$.

D10a $f(x) = 2e^x - 3x^2 \Rightarrow f'(x) = 2e^x - 6x$.

D10b $f(x) = \frac{x^2+1}{e^x} \Rightarrow f'(x) = \frac{e^x \cdot 2x - (x^2+1) \cdot e^x}{(e^x)^2} = \frac{(2x - x^2 - 1) \cdot e^x}{e^x \cdot e^x} = \frac{-x^2 + 2x - 1}{e^x}$.

D10c $f(x) = (x^2+1) \cdot e^x \Rightarrow f'(x) = 2x \cdot e^x + (x^2+1) \cdot e^x = (x^2 + 2x + 1) \cdot e^x$.

D10d $f(x) = \frac{e^x}{x^2+1} \Rightarrow f'(x) = \frac{(x^2+1) \cdot e^x - e^x \cdot 2x}{(x^2+1)^2} = \frac{(x^2+2x+1) \cdot e^x}{(x^2+1)^2}$.

D10e $f(x) = x^2 \cdot e^{2x-1} \Rightarrow f'(x) = 2x \cdot e^{2x-1} + x^2 \cdot e^{\boxed{2x-1}} \cdot 2 = (2x^2 + 2x) \cdot e^{2x-1}$.

D10f $f(x) = e^{x^2+9} \Rightarrow f'(x) = e^{\boxed{x^2+9}} \cdot 2x = 2x \cdot e^{x^2+9}$.

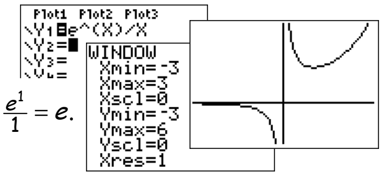
D11a $f(x)=\frac{e^x}{x}$ BV: $x\neq 0 \Rightarrow f'(x)=\frac{x\cdot e^x-e^x\cdot 1}{x^2}=\frac{(x-1)\cdot e^x}{x^2}$.

$f'(x)=0 \Rightarrow \frac{(x-1)\cdot e^x}{x^2}=0 \Rightarrow$ (teller = 0 en noemer ≠ 0)

$(x-1)\cdot e^x=0$

$x=1 \vee e^x=0$ (kan niet).

Minimum (zie plot) is $f(1)=\frac{e^1}{1}=e$.



D11b Stel l: $y=ax+b$ met $a=f'(2)=\frac{(2-1)\cdot e^2}{2^2}=\frac{e^2}{4}$.

l: $y=\frac{1}{4}e^2x+b$
door $A(2,\ \frac{1}{2}e^2)$ $\Big\} \Rightarrow \frac{1}{2}e^2=\frac{1}{4}e^2\cdot 2+b \Rightarrow b=0$.

Dus l: $y=\frac{1}{4}e^2x$.

D12a $\ln(e^3\cdot\sqrt{e})=\ln(e^3\cdot e^{\frac{1}{2}})=\ln(e^{3\frac{1}{2}})=3\frac{1}{2}$.

D12b $\ln(\frac{1}{e^2})=\ln(e^{-2})=-2$.

D13a $4+\ln(3)=\ln(e^4)+\ln(3)=\ln(e^4\cdot 3)=\ln(3e^4)$.

D13b $\ln(10)-4\ln(2)=\ln(10)-\ln(2^4)=\ln(\frac{10}{16})=\ln(\frac{5}{8})$.

D14a $2\ln(5x)=16$ BV: $x>0$
$\ln(5x)=8$
$5x=e^8$
$x=\frac{1}{5}\cdot e^8$ (voldoet).

D14b $\ln^2(5x)=16$ BV: $x>0$
$\ln(5x)=\pm 4$
$5x=e^4 \vee 5x=-e^4$
$x=\frac{1}{5}\cdot e^4$ (voldoet) $\vee\ x=\frac{1}{5}\cdot -e^4$ (vold. niet).

D14c $2\ln^2(x)-\ln(x)=0$ BV: $x>0$ (stel $\ln(x)=t$)
$2t^2-t=0$
$t\cdot(2t-1)=0$
$t=0 \vee 2t=1$
$t=\ln(x)=0 \vee t=\ln(x)=\frac{1}{2}$
$x=e^0=1$ (voldoet) $\vee\ x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet).

D14d $\ln(9x+1)-\ln(x+2)=\ln(4)$ BV: $x>-\frac{1}{9}$
$\ln(9x+1)=\ln(4)+\ln(x+2)$
$9x+1=4\cdot(x+2)$
$9x+1=4x+8$
$5x=7$
$x=\frac{7}{5}=1\frac{2}{5}$ (voldoet).

D15a $f(x)=2^{3x-4} \Rightarrow f'(x)=2^{\boxed{3x-4}}\cdot\ln(2)\cdot 3=3\cdot\ln(2)\cdot 2^{3x-4}$.

D15b $f(x)=x\cdot 3^x \Rightarrow f'(x)=1\cdot 3^x+x\cdot 3^x\cdot\ln(3)=3^x\cdot(1+x\cdot\ln(3))$.

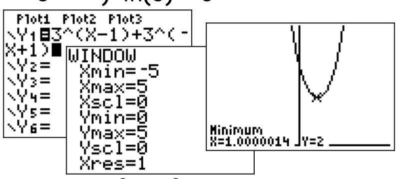
D15c $f(x)=\ln(x\cdot\sqrt[3]{x})=\ln(x\cdot x^{\frac{1}{3}})=\ln(x^{1\frac{1}{3}})=1\frac{1}{3}\cdot\ln(x) \Rightarrow f'(x)=1\frac{1}{3}\cdot\frac{1}{x}=\frac{4}{3x}$.

D15d $f(x)={}^2\log(4\cdot x)={}^2\log(4)+{}^2\log(x) \Rightarrow f'(x)=\frac{1}{x\cdot\ln(2)}$.

D15e $f(x)={}^3\log(5x-6) \Rightarrow f'(x)=\frac{1}{(\boxed{5x-6})\cdot\ln(3)}\cdot 5=\frac{5}{(5x-6)\cdot\ln(3)}$.

D15f $f(x)=\ln(3x^2+3) \Longrightarrow f'(x)=\frac{1}{(\boxed{3x^2+3})}\cdot 6x=\frac{6x}{3\cdot(x^2+1)}=\frac{2x}{x^2+1}$.

D16a $f(x)=3^{x-1}+3^{-x+1} \Rightarrow$
$f'(x)=3^{\boxed{x-1}}\cdot\ln(3)\cdot 1+3^{\boxed{-x+1}}\cdot\ln(3)\cdot -1=(3^{x-1}-3^{-x+1})\cdot\ln(3)$.
$f'(x)=0 \Rightarrow (3^{x-1}-3^{-x+1})\cdot\ln(3)=0$
$3^{x-1}-3^{-x+1}=0$
$3^{x-1}=3^{-x+1}$
$x-1=-x+1$
$2x=2$
$x=1$

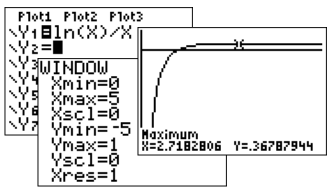


minimum (zie plot) is $f(1)=3^0+3^0=1+1=2 \Rightarrow B_f=[2,\ \rightarrow\rangle$.

D16b $f'(x)=(3^{x-1}-3^{-x+1})\cdot\ln(3)=\frac{8}{3}\cdot\ln(3)$
$3^{x-1}-3^{-x+1}=\frac{8}{3}$ $(\times 3)$
$3^x-3^{-x+2}=8$ $(\times 3^x)$
$3^{2x}-3^{+2}=8\cdot 3^x$ (stel $3^x=t$)
$t^2-8t-9=0$
$(t-9)\cdot(t+1)=0$
$t=3^x=9=3^2 \vee t=3^x=-1$ (kan niet)
$x=2 \Rightarrow y=f(2)=3^{2-1}+3^{-2+1}=3+\frac{1}{3}=3\frac{1}{3}$.

D17a $f(x)=0$ BV:$x>0 \Rightarrow \frac{\ln(x)}{x}=0$ (teller = 0 ⇒)
$\ln(x)=0 \Rightarrow x=e^0=1$ (voldoet)
$f(x)=\frac{\ln(x)}{x} \Rightarrow f'(x)=\frac{x\cdot\frac{1}{x}-\ln(x)\cdot 1}{x^2}=\frac{1-\ln(x)}{x^2}$.
k: $y=ax+b$ met $a=f'(1)=\frac{1-\ln(1)}{1^2}=\frac{1-0}{1}=1$.
$y=x+b$ door $(1,0) \Rightarrow 0=1+b \Rightarrow b=-1$
raaklijn: $y=x-1$.

D17b $f'(x)=0 \Rightarrow \frac{1-\ln(x)}{x^2}=0$ (teller = 0 ⇒) BV:$x>0$
$1-\ln(x)=0$
$\ln(x)=1$
$x=e^1=e$ (voldoet)
maximum (zie plot) is $f(e)=\frac{\ln(e)}{e}=\frac{\ln(e^1)}{e}=\frac{1}{e}$.
Dus $B_f=\langle\leftarrow,\ \frac{1}{e}]$.



Gemengde opgaven 9. Exponentiële en logaritmische functies

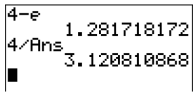
G1a
$9^x = 3^x + 2$
$3^{2x} = 3^x + 2$ (stel $3^x = t$)
$t^2 - t - 2 = 0$
$(t-2)\cdot(t+1) = 0$
$t = 3^x = 2 \vee t = 3^x = -1$ (kan niet)
$x = {}^3\log(2)$.

G1b
$\log^2(x) + 1 = 2\frac{1}{2}\log(x)$ BV: $x > 0$ (stel $\log(x) = t$)
$t^2 - 2\frac{1}{2}t + 1 = 0$
$(t-2)\cdot(t-\frac{1}{2}) = 0$
$t = \log(x) = 2 \vee t = \log(x) = \frac{1}{2}$
$x = 10^2 = 100$ (voldoet) $\vee\ x = 10^{\frac{1}{2}} = \sqrt{10}$ (voldoet).

G1c
$\frac{e^x}{e^x - 2} = 2$
$2e^x - 4 = e^x$
$e^x = 4$
$x = \ln(4)$.

G1d
$\ln(3x+2) = \frac{1}{2}$ BV: $x > -\frac{2}{3}$
$\ln(3x+2) = \ln(e^{\frac{1}{2}})$
$3x + 2 = \sqrt{e}$
$3x = -2 + \sqrt{e}$
$x = -\frac{2}{3} + \frac{1}{3}\cdot\sqrt{e}$ (voldoet).

G1e
$\ln(4x) - \ln(x+4) = 1$ BV: $x > 0$
$\ln(4x) = \ln(x+4) + \ln(e)$
$4x = e\cdot(x+4)$
$4x = ex + 4e$
$(4-e)\cdot x = 4e$
$x = \frac{4e}{4-e}$ (voldoet).



G1f
$\ln^2(x-2) = 4$ BV: $x > 2$ (stel $\ln(x-2) = t$)
$t^2 = 4$
$t = \ln(x-2) = 2 \vee t = \ln(x-2) = -2$
$x - 2 = e^2 \vee x - 2 = e^{-2} = \frac{1}{e^2}$
$x = 2 + e^2$ (voldoet) $\vee\ x = 2 + \frac{1}{e^2}$ (voldoet).

G1g
$3\cdot 2^{2x+1} + 1 = 5\cdot 2^x$
$3\cdot 2^{2x}\cdot 2^1 + 1 = 5\cdot 2^x$ (stel $2^x = t$)
$6t^2 - 5t + 1 = 0$ (abc-formule)
$D = (-5)^2 - 4\cdot 6\cdot 1 = 25 - 24 = 1$
$t = \frac{5\pm\sqrt{1}}{2\cdot 6}$
$t = 2^x = \frac{5+1}{12} = \frac{1}{2} = 2^{-1} \vee t = 2^x = \frac{5-1}{12} = \frac{1}{3}$
$x = -1 \vee x = {}^2\log(\frac{1}{3})$.

G1h
$x\cdot 2^{-x+1} = 4x\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 4\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 2^2\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 2^{-3x+3}$
$x = 0 \vee -x + 1 = -3x + 3$
$x = 0 \vee 2x = 2$
$x = 0 \vee x = 1$.

G2a $f(x) = x^2\cdot e^{x-1} \Rightarrow f'(x) = 2x\cdot e^{x-1} + x^2\cdot e^{\boxed{x-1}}\cdot 1 = (x^2+2x)\cdot e^{x-1}$.

G2b $g(x) = \ln^2(x) + \ln(x^2) \Rightarrow g'(x) = 2\cdot\boxed{\ln(x)}\cdot\frac{1}{x} + \frac{2}{x} = \frac{2\ln(x)+2}{x}$.

G2c $h(x) = {}^2\log(x^3 - x^2) \Rightarrow h'(x) = \frac{1}{(x^3-x^2)\cdot\ln(2)}\cdot(3x^2 - 2x) = \frac{3x^2-2x}{(x^3-x^2)\cdot\ln(2)}$.

G2d $j(x) = \ln(\ln(2x)) \Rightarrow j'(x) = \frac{1}{\boxed{\ln(2x)}}\cdot\frac{1}{x} = \frac{1}{x\cdot\ln(2x)}$.

$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(x^a) \Rightarrow f'(x) = \frac{a}{x}$
$f(x) = {}^g\log x \Rightarrow f'(x) = \frac{1}{x\cdot\ln(g)}$

G3a $f(x) = 2\cdot e^x \xrightarrow{\text{translatie } (3,0)} y = 2\cdot e^{x-3} = 2\cdot e^x\cdot e^{-3} = \frac{1}{e^3}\cdot 2\cdot e^x \xleftarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{e^3}} f(x) = 2\cdot e^x$.

G3b $g(x) = \ln(2x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } 3} y = \ln(\frac{1}{3}\cdot 2x) = \ln(\frac{1}{3}) + \ln(2x) \xleftarrow{\text{translatie } (0, \ln(\frac{1}{3}))} g(x) = \ln(2x)$.

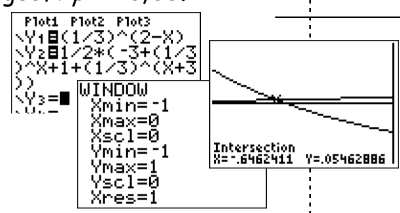
G4a Stel de snijpunten van de lijn $y = p$ met de grafieken van f, g en h zijn rescpectievelijk A, B en C.

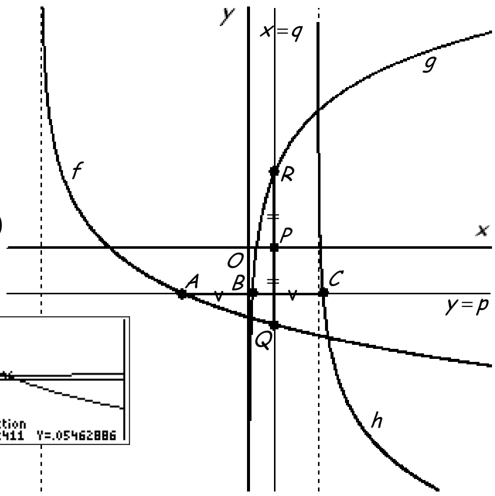
$f(x) = p \Rightarrow {}^{\frac{1}{3}}\log(x+3) = p$
BV: $x > -3$ $\quad x + 3 = (\frac{1}{3})^p$
$x_A = -3 + (\frac{1}{3})^p$ (voldoet)

$g(x) = p \Rightarrow 2 - {}^{\frac{1}{3}}\log(x) = p$
BV: $x > 0$ $\quad {}^{\frac{1}{3}}\log(x) = 2 - p$
$x_B = (\frac{1}{3})^{2-p}$ (voldoet)

$h(x) = p \Rightarrow -3 + {}^{\frac{1}{3}}\log(x-1) = p$
BV: $x > 1$ $\quad {}^{\frac{1}{3}}\log(x-1) = p + 3$
$x - 1 = (\frac{1}{3})^{p+3}$
$x_C = 1 + (\frac{1}{3})^{p+3}$ (voldoet)

$AB = BC \Rightarrow x_B = \frac{1}{2}\cdot(x_A + x_C)$
$(\frac{1}{3})^{2-p} = \frac{1}{2}\cdot(-3 + (\frac{1}{3})^p + 1 + (\frac{1}{3})^{p+3})$
intersect geeft $p \approx -0{,}65$.





G4b P het midden van $QR \Rightarrow y_Q + y_R = 0$
$f(q) + g(x) = 0$
${}^{\frac{1}{3}}\log(q+3) + 2 - {}^{\frac{1}{3}}\log(q) = 0$ BV: $q > 0$
${}^{\frac{1}{3}}\log(q+3) + {}^{\frac{1}{3}}\log((\frac{1}{3})^2) = {}^{\frac{1}{3}}\log(q)$
${}^{\frac{1}{3}}\log(\frac{1}{9}\cdot(q+3)) = {}^{\frac{1}{3}}\log(q) \Rightarrow \frac{1}{9}\cdot(q+3) = q$ $(\times 9) \Rightarrow q + 3 = 9q \Rightarrow -8q = -3 \Rightarrow q = \frac{3}{8}$ (voldoet).

G5a $f_2(x) = \frac{2+2\ln(x)}{x}$ BV: $x > 0$

$f_2{}'(x) = \frac{x \cdot 2 \cdot \frac{1}{x} - (2+2\ln(x)) \cdot 1}{x^2} = \frac{2-2-2\ln(x)}{x^2} = \frac{-2\ln(x)}{x^2}$

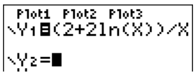
$f_2{}'(x) = 0 \Rightarrow \frac{-2\ln(x)}{x^2} = 0$ (teller = 0)

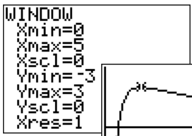
$-2\ln(x) = 0$

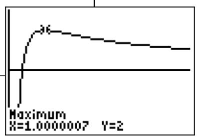
$\ln(x) = 0$

$x = e^0 = 1$ (voldoet)

$x_{top} = 1$ en $y_{top} = f_2(1) = \frac{2+2\ln(1)}{1} = \frac{2+0}{1} = 2.$







G5b $f_p(3) - f_{-p}(3) = 4$ ∨ $f_{-p}(3) - f_p(3) = 4$

$\frac{2+p\ln(3)}{3} - \frac{2-p\ln(3)}{3} = 4$ ∨ $\frac{2-p\ln(3)}{3} - \frac{2+p\ln(3)}{3} = 4$

$\frac{2+p\ln(3)-2+p\ln(3)}{3} = 4$ ∨ $\frac{2-p\ln(3)-2-p\ln(3)}{3} = 4$

$\frac{2p\ln(3)}{3} = 4$ ∨ $\frac{-2p\ln(3)}{3} = 4$

$2p\ln(3) = 12$ ∨ $-2p\ln(3) = 12$

$p\ln(3) = 6$ ∨ $p\ln(3) = -6$

$p = \frac{6}{\ln(3)}$ ∨ $p = -\frac{6}{\ln(3)}.$

G6a $f(x) = g(x) \Rightarrow e^{\frac12 x - 1} = e^{-x+1} \Rightarrow \frac12 x - 1 = -x + 1 \Rightarrow 1\frac12 x = 2 \Rightarrow x_A = 2 \cdot \frac23 = \frac43$ en $y_A = f(\frac43) = e^{\frac23 - 1} = e^{-\frac13} = \frac{1}{e^{\frac13}} = \frac{1}{\sqrt[3]{e}}.$

$f(x) = e^{\frac12 x - 1} \Rightarrow f'(x) = e^{\boxed{\frac12 x - 1}} \cdot \frac12 = \frac12 e^{\frac12 x - 1}$

k: $y = ax + b$ met $a = f'(\frac43) = \frac12 e^{\frac23 - 1} = \frac12 e^{-\frac13}$

k: $y = \frac12 e^{-\frac13} x + b$ door $A(\frac43, e^{-\frac13})$

$e^{-\frac13} = \frac12 e^{-\frac13} \cdot \frac43 + b$

$e^{-\frac13} = \frac23 e^{-\frac13} + b$

$b = \frac13 e^{-\frac13} \Rightarrow k$: $y = \frac12 e^{-\frac13} x + \frac13 e^{-\frac13}$

k snijden met de x-as ($y = 0$):

$\frac12 e^{-\frac13} x + \frac13 e^{-\frac13} = 0$

$\frac12 e^{-\frac13} x = -\frac13 e^{-\frac13}$

$x_B = -\frac23$

$g(x) = e^{-x+1} \Rightarrow g'(x) = e^{\boxed{-x+1}} \cdot -1 = -e^{-x+1}$

l: $y = ax + b$ met $a = g'(\frac43) = -e^{-\frac43 + 1} = -e^{-\frac13}$

l: $y = -e^{-\frac13} x + b$ door $A(\frac43, e^{-\frac13})$

$e^{-\frac13} = -e^{-\frac13} \cdot \frac43 + b$

$e^{-\frac13} = -\frac43 e^{-\frac13} + b$

$b = 2\frac13 e^{-\frac13} \Rightarrow l$: $y = -e^{-\frac13} x + 2\frac13 e^{-\frac13}$

l snijden met de x-as ($y = 0$):

$-e^{-\frac13} x + 2\frac13 e^{-\frac13} = 0$

$-e^{-\frac13} x = -2\frac13 e^{-\frac13}$

$x_C = 2\frac13.$

$O_{\Delta ABC} = \frac12 \cdot BC \cdot y_A = \frac12 \cdot (x_C - x_B) \cdot y_A = \frac12 \cdot (2\frac13 - -\frac23) \cdot y_A = \frac12 \cdot 3 \cdot \frac{1}{\sqrt[3]{e}} = \frac32 \cdot \frac{1}{\sqrt[3]{e}} = \frac{3}{2 \cdot \sqrt[3]{e}}.$

G6b Stel $x = r$ de x-coördinaat van het linker snijpunt met $y = p$. Nu is $PQ = 3$ als:

$f(r) = g(r+3) = p$ ∨ $g(r) = f(r+3) = p$

$e^{\frac12 r - 1} = e^{-(r+3)+1}$ ∨ $e^{-r+1} = e^{\frac12 (r+3) - 1}$

$\frac12 r - 1 = -r - 3 + 1$ ∨ $-r + 1 = \frac12 q + 1\frac12 - 1$

$\frac12 r - 1 = -r - 2$ ∨ $-r + 1 = \frac12 q + \frac12$

$1\frac12 r = -1$ ∨ $1\frac12 r = -\frac12$

$r = -1 \cdot \frac23 = \frac23$ ∨ $r = -\frac12 \cdot \frac23 = -\frac13$

$p = f(r) = f(\frac23) = e^{\frac13 - 1} = e^{-\frac23} = \frac{1}{e^{\frac23}} = \frac{1}{\sqrt[3]{e^2}}$ ∨ $p = g(r) = g(-\frac13) = e^{1\frac13} = e^1 \cdot e^{\frac13} = e \cdot \sqrt[3]{e}.$

$PQ < 3$ voor $\frac{1}{\sqrt[3]{e^2}} < p < e \cdot \sqrt[3]{e}.$ (gebruik de uitkomsten hieboven en figuur G.1 in het boek)

G6c Stel $x_F = s$ dan is $x_E = 4s$

$g(s) = f(4s) = q \Rightarrow e^{-s+1} = e^{2s-1} \Rightarrow -s + 1 = 2s - 1 \Rightarrow -3s = -2 \Rightarrow s = \frac23 \Rightarrow q = g(s) = g(\frac23) = e^{-\frac23 + 1} = e^{\frac13} = \sqrt[3]{e}.$

G7a $f(x) = g(x) \Rightarrow \ln(4x) = \ln(\frac1x)$ BV: $x > 0 \Rightarrow 4x = \frac1x \Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac14 \Rightarrow x_A = +\frac12$ en $y_A = f(\frac12) = \ln(2).$

$f(x) = \ln(4x) \Rightarrow f'(x) = \frac1x$

k: $y = ax + b$ met $a = f'(\frac12) = \frac{1}{\frac12} = 1 \cdot \frac21 = 2$

k: $y = 2x + b$ door $A(\frac12, \ln(2))$

$\ln(2) = 2 \cdot \frac12 + b$

$b = \ln(2) - 1$

k: $y = 2x + \ln(2) - 1.$

$g(x) = \ln(\frac1x) = \ln(x^{-1}) = -\ln x \Rightarrow g'(x) = -\frac1x.$

l: $y = ax + b$ met $a = g'(\frac12) = -\frac{1}{\frac12} = -1 \cdot \frac21 = -2$

l: $y = -2x + b$ door $A(\frac12, \ln(2))$

$\ln(2) = -2 \cdot \frac12 + b$

$b = \ln(2) + 1$

l: $y = -2x + \ln(2) + 1.$

$y = 2x + \ln(2) - 1$ snijdt de y-as ($x = 0$) in $(0, \ln(2) - 1)$ en $y = -2x + \ln(2) + 1$ snijdt de y-as ($x = 0$) in $(0, \ln(2) + 1).$

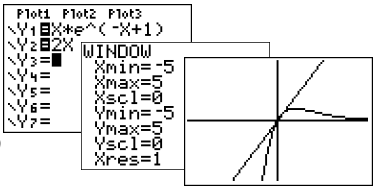
De lengte van het gevraagde lijnstuk is $\ln(2) + 1 - (\ln(2) - 1) = \ln(2) + 1 - \ln(2) + 1 = 2.$

G7b Stel $x_C = q$ dan is $x_D = 2q$

$g(q) = f(2q) = p \Rightarrow \ln(\frac{1}{q}) = \ln(8q)$ BV: $q > 0 \Rightarrow \frac{1}{q} = 8q \Rightarrow 8q^2 = 1 \Rightarrow q^2 = \frac{1}{8} \Rightarrow q = \sqrt{\frac{1}{8}}$ (voldoet) $\vee\ q = -\sqrt{\frac{1}{8}}$ (vold. niet)

$p = g(q) = g(\sqrt{\frac{1}{8}}) = \ln(\frac{1}{\sqrt{\frac{1}{8}}}) = \ln(\frac{1}{\sqrt{\frac{1}{8}}} \cdot \frac{\sqrt{8}}{\sqrt{8}}) = \ln(\sqrt{8})$.

G8a $f(x) = x \cdot e^{-x+1} = 2x$



$x = 0 \vee e^{-x+1} = 2$
$x = 0 \vee -x + 1 = \ln(2)$
$x = 0 \vee -x = -1 + \ln(2)$
$x = 0 \vee x = 1 - \ln(2)$
$f(x) \le 2x$ (zie plot) voor $x \le 0 \vee x \ge 1 - \ln(2)$.

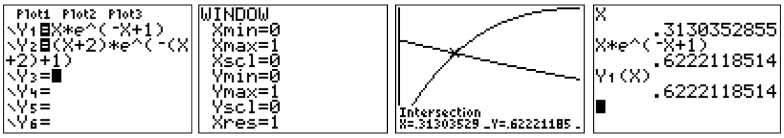
G8b $f(x) = x \cdot e^{-x+1} \Rightarrow f'(x) = 1 \cdot e^{-x+1} + x \cdot e^{\boxed{-x+1}} \cdot -1 = (1-x) \cdot e^{-x+1}$.

$f'(0) = (1-0) \cdot e^{-0+1} = 1 \cdot e^1 = e \Rightarrow y = ex$ is raaklijn in O

$x \cdot e^{-x+1} = ax$ heeft precies één oplossing (zie plot) $\Rightarrow a = e \vee a \le 0$.

G8c $f(q) = f(q+2) = p \Rightarrow q \cdot e^{-q+1} = (q+2) \cdot e^{-(q+2)+1}$

intersect geeft $q \approx ...$ en $p = f(q) = f(q+2) \approx 0{,}62$.

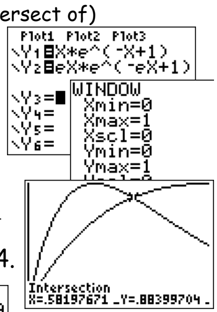


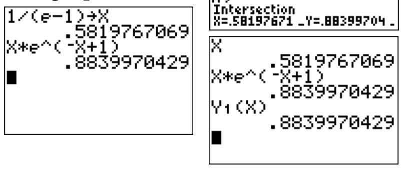
G8d Stel $x_B = p$ dan is $x_C = e \cdot p$

$f(p) = f(ep) = q$

$p \cdot e^{-p+1} = ep \cdot e^{-ep+1}$ (intersect of)

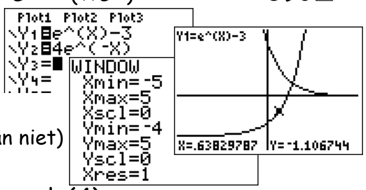
$p = 0 \vee e^{-p+1} = e \cdot e^{-ep+1}$
$p = 0 \vee e^{-p+1} = e^{-ep+2}$
$p = 0 \vee -p + 1 = -ep + 2$
$p = 0 \vee ep - p = 1$
$p = 0 \vee (e-1) \cdot p = 1$
$p = 0$ (vold. niet) $\vee\ p = \frac{1}{e-1}$
$q = f(p) = f(\frac{1}{e-1}) \approx 0{,}884$.





G9a $f(x) = g_4(x) \Rightarrow e^x - 3 = 4 \cdot e^{-x}$ $(\times e^x)$

$e^{2x} - 3e^x = 4$ (stel $e^x = t$)



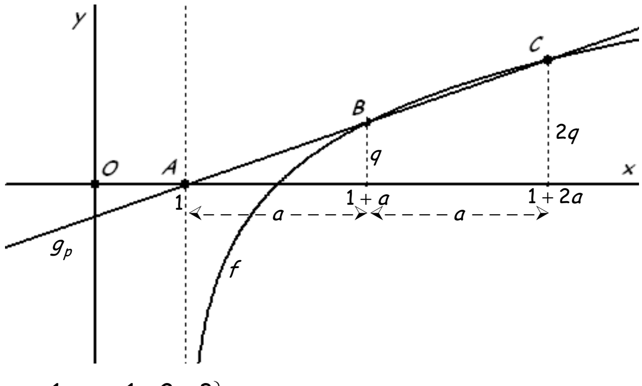
$t^2 - 3t - 4 = 0$
$(t-4) \cdot (t+1) = 0$
$t = e^x = 4 \vee t = e^x = -1$ (kan niet)
$x = \ln(4)$
$f(x) < g_4(x)$ (zie plot) voor $x < \ln(4)$.

G9b $f(x) = e^x - 3 \Rightarrow f'(x) = e^x$
$a = f'(x) = e^x = 2$
$x_{\text{raakp.}} = \ln(2) \Rightarrow$
$y_{\text{raakp.}} = f(\ln(2)) = e^{\ln(2)} - 3 = 2 - 3 = -1$
$-1 = 2 \cdot \ln(2) + b \Rightarrow b = -1 - 2 \cdot \ln(2)$.

G9c $g_p(x) = pe^{-x} \Rightarrow g_p'(x) = pe^{\boxed{-x}} \cdot -1 = -pe^{-x}$
$a = g_p'(x) = -pe^{-x} = 2$
$e^{-x} = -\frac{2}{p}$ BV: $p < 0$, want $e^{\cdots} > 0$
$-x = \ln(-\frac{2}{p})$
$x_{\text{raakp.}} = -\ln(-\frac{2}{p}) \Rightarrow$
$y_{\text{raakp.}} = g_p(-\ln(-\frac{2}{p})) = pe^{\ln(-\frac{2}{p})} = p \cdot -\frac{2}{p} = -2$
k: $y = 2x - 1 - \ln(2)$ (zie G9b) door raakpunt $\Rightarrow$
$-2 = 2 \cdot -\ln(-\frac{2}{p}) - 1 - 2\ln(2)$
$2 \cdot \ln(-\frac{2}{p}) = 1 - 2\ln(2)$
$\ln(-\frac{2}{p}) = \frac{1}{2} - \ln(2)$
$-\frac{2}{p} = e^{\frac{1}{2} - \ln(2)} = e^{\frac{1}{2}} \cdot e^{-\ln(2)} = \sqrt{e} \cdot e^{\ln(2^{-1})} = \sqrt{e} \cdot 2^{-1} = \frac{\sqrt{e}}{2}$
$p \cdot \sqrt{e} = -2 \cdot 2$
$p = -\frac{4}{\sqrt{e}}$ (voldoet).

G10a $f(x) = g_p(x) = 2$ BV: $x > 1$
$\ln(x-1) = p \cdot (x-1) = 2$ ✪.
$\ln(x-1) = 2$
$x - 1 = e^2$
$x = e^2 + 1$ (voldoet) (nu invullen in ✪)
$p \cdot (e^2 + 1 - 1) = 2$
$p \cdot e^2 = 2$
$p = \frac{2}{e^2}$.

G10b Snijden met de x-as ($y = 0$): $g_p(x) = 0 \Rightarrow p \cdot (x-1) = 0$
$x - 1 = 0$ (p is een of ander getal) $\Rightarrow x = 1 \Rightarrow A(1, 0)$.
Bekijk nu de grafiek van f en van de lijn g_p.
B is het midden van $AC \Rightarrow y_C = 2 \cdot y_B \Rightarrow$
$f(1+2a) = 2 \cdot f(1+a)$
$\ln(1+2a-1) = 2 \cdot \ln(1+a-1)$
$\ln(2a) = 2 \cdot \ln(a)$ BV: $a > 0$
$\ln(2a) = \ln(a^2)$
$a^2 = 2a$
$a = 0$ (vold. niet) $\vee\ a = 2$ (voldoet)



$\left.\begin{array}{l} a = 2 \Rightarrow x_B = 1 + a = 1 + 2 = 3 \\ y_B = f(x_B) = \ln(3-1) = \ln(2) \end{array}\right\} \Rightarrow B(3, \ln(2))$.

B op $y = p \cdot (x-1) \Rightarrow \ln(2) = p \cdot (3-1) \Rightarrow 2p = \ln(2) \Rightarrow p = \frac{\ln(2)}{2}$.

G11a $f_1(x)=\frac{\ln(x)}{x}$ BV: $x>0 \Rightarrow f_1'(x)=\frac{x\cdot\frac{1}{x}-\ln(x)\cdot 1}{x^2}=\frac{1-\ln(x)}{x^2}$.

$f_1'(x)=0$

$\frac{1-\ln(x)}{x^2}=0$ (teller = 0 ⇒)

$\ln(x)=1$

$x_{top}=e^1=e \Rightarrow y_{top}=f_1(e)=\frac{\ln(e)}{e}=\frac{1}{e} \Rightarrow$ top $(e,\frac{1}{e})$.

G11b $f_k(x)=\frac{\ln(kx)}{x}$ BV: $kx>0 \Rightarrow f_k'(x)=\frac{x\cdot\frac{1}{x}-\ln(kx)\cdot 1}{x^2}=\frac{1-\ln(kx)}{x^2}$.

$f_k'(x)=0$

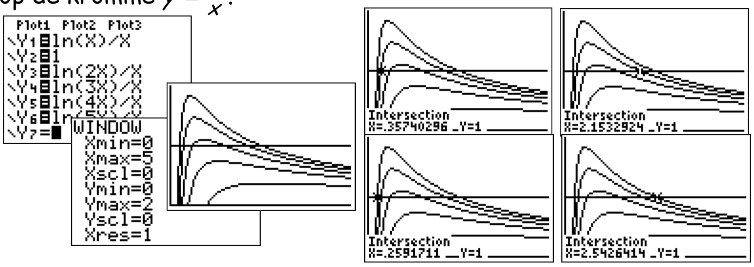
$\frac{1-\ln(kx)}{x^2}=0$ (teller = 0 ⇒)

$\ln(kx)=1 \Rightarrow f_k(x)=\frac{1}{x}$. Dus de top ligt op de kromme $y=\frac{1}{x}$.

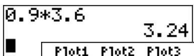
G11c $k \neq 0$ en k geheel ⇒ $k=1,\ 2,\ 3,\ \ldots$

k	y_1	y_2	x_A	x_B	AB
4	$\frac{\ln(4x)}{x}$	1	0,36	2,15	1,8
5	$\frac{\ln(5x)}{x}$	1	0,26	2,54	2,3

Dus vanaf $k=5$ is $AB>2$.



G12a 90% van 3,6 liter is 3,24 liter.



$L(t)=3{,}24$

$3{,}6\cdot(1-e^{-2{,}5t})=3{,}24$ (intersect of)

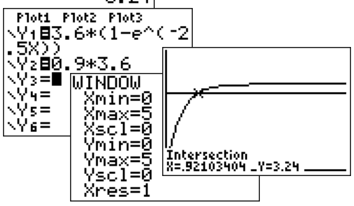
$1-e^{-2{,}5t}=0{,}9$

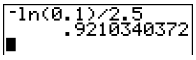
$-e^{-2{,}5t}=-0{,}1$

$e^{-2{,}5t}=0{,}1$

$-2{,}5t=\ln(0{,}1)$

$t=-\frac{\ln(0{,}1)}{2{,}5}\approx 0{,}9$ (seconden).

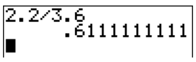




G12b De maximale hoeveelheid verse lucht is 2,2 liter (zie figuur G.2 in boek).

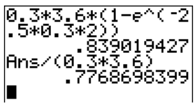
$\alpha\cdot 3{,}6=2{,}2$

$\alpha=\frac{2{,}2}{3{,}6}\approx 0{,}6.$



G12c $L_{0,3}(2)=0{,}3\cdot 3{,}6\cdot(1-e^{-2{,}5\cdot 0{,}3\cdot 2})\approx 0{,}84$ (liter).

Dus $\frac{0{,}84}{0{,}3\cdot 3{,}6}\times 100\% \approx 78\%$.



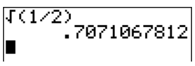
G12d $L_\alpha(t)=\alpha\cdot 3{,}6(1-e^{-2{,}5\alpha t})=3{,}6\alpha-3{,}6\alpha\cdot e^{-2{,}5\alpha t} \Rightarrow L_\alpha'(t)=-3{,}6\alpha\cdot e^{\boxed{-2{,}5\alpha t}}\cdot -2{,}5\alpha=9\alpha^2\cdot e^{-2{,}5\alpha t}$.

De maxmale vulsnelheid is $L_\alpha'(0)=9\alpha^2\cdot e^0=9\alpha^2$.

$9\alpha^2=4{,}5$

$\alpha^2=\frac{1}{2}$

$\alpha=\sqrt{\frac{1}{2}}\approx 0{,}71.$



Voor $g > 0$, $g \neq 1$, $a > 0$ en $b > 0$ geldt:

- ☐ ${}^{g}\log(a) + {}^{g}\log(b) = {}^{g}\log(a \cdot b)$
- ☐ ${}^{g}\log(a) - {}^{g}\log(b) = {}^{g}\log(\frac{a}{b})$
- ☐ $c \cdot {}^{g}\log(a) = {}^{g}\log(a^{c})$
- ☐ ${}^{g}\log(a) = \frac{{}^{p}\log(a)}{{}^{p}\log(b)}$
- ☐ $g^{{}^{g}\log(a)} = a$
- ☐ ${}^{g}\log(g^{c}) = c$

Dus: ${}^{g}\log(1) = {}^{g}\log(g^{0}) = 0$
${}^{g}\log(g) = {}^{g}\log(g^{1}) = 1$

$f(x) = e^{x} \Rightarrow f'(x) = e^{x}$
$f(x) = g^{x} \Rightarrow f'(x) = g^{x} \cdot \ln(g)$
$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(x^{a}) \Rightarrow f'(x) = \frac{a}{x}$
$f(x) = {}^{g}\log(x) \Rightarrow f'(x) = \frac{1}{x \cdot \ln(g)}$