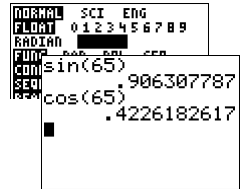
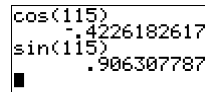


1a $\sin \alpha = \frac{\text{overstaande rechthoekszijde}}{\text{schuine zijde}} \Rightarrow$ (in figuur 6.1) $\sin 65^\circ = \frac{PQ}{OP} = \frac{PQ}{1} \Rightarrow PQ = 1 \cdot \sin 65^\circ \approx 0,91$.
 $\cos \alpha = \frac{\text{aanliggende rechthoekszijde}}{\text{schuine zijde}} \Rightarrow$ (in figuur 6.1) $\cos 65^\circ = \frac{OQ}{OP} = \frac{OQ}{1} \Rightarrow OQ = 1 \cdot \cos 65^\circ \approx 0,42$.



1b $P(0,42; 0,91)$.

1c $\angle POQ = 180^\circ - 115^\circ = 65^\circ$; $PQ \approx 0,91$ en $OQ \approx 0,42 \Rightarrow P(-0,42; 0,91)$.
 (P in figuur 6.2 is het spiegelbeeld van P in figuur 6.1 ten opzichte van de y -as)



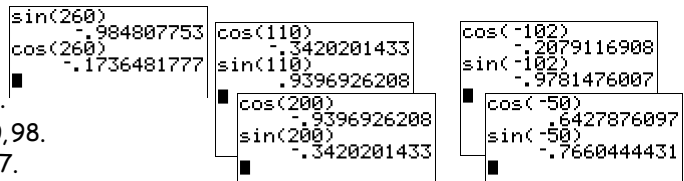
1d $\cos 115^\circ \approx -0,42$ en $\sin 115^\circ \approx 0,91$. Dus $\cos 115^\circ = x_p$ en $\sin 115^\circ = y_p$.

□

- | | |
|---|--|
| 2a □ $\alpha = 0^\circ \Rightarrow P = (1, 0) \Rightarrow \sin 0^\circ = y_p = 0$. | 2g □ $\alpha = 360^\circ \Rightarrow P = (1, 0) \Rightarrow \sin 360^\circ = y_p = 0$. |
| 2b □ $\alpha = 0^\circ \Rightarrow P = (1, 0) \Rightarrow \cos 0^\circ = x_p = 1$. | 2h □ $\alpha = 360^\circ \Rightarrow P = (1, 0) \Rightarrow \cos 360^\circ = x_p = 1$. |
| 2c □ $\alpha = 90^\circ \Rightarrow P = (0, 1) \Rightarrow \sin 90^\circ = y_p = 1$. | 2i □ $\alpha = 450^\circ \Rightarrow P = (0, 1) \Rightarrow \sin 450^\circ = y_p = 1$. |
| 2d □ $\alpha = 90^\circ \Rightarrow P = (0, 1) \Rightarrow \cos 90^\circ = x_p = 0$. | 2j □ $\alpha = -90^\circ \Rightarrow P = (0, -1) \Rightarrow \cos(-90^\circ) = x_p = 0$. |
| 2e □ $\alpha = 270^\circ \Rightarrow P = (0, -1) \Rightarrow \sin 270^\circ = y_p = -1$. | 2k □ $\alpha = -540^\circ \Rightarrow P = (-1, 0) \Rightarrow \sin(-540^\circ) = y_p = 0$. |
| 2f □ $\alpha = 270^\circ \Rightarrow P = (0, -1) \Rightarrow \cos 270^\circ = x_p = 0$. | 2l □ $\alpha = -180^\circ \Rightarrow P = (-1, 0) \Rightarrow \cos(-180^\circ) = x_p = -1$. |

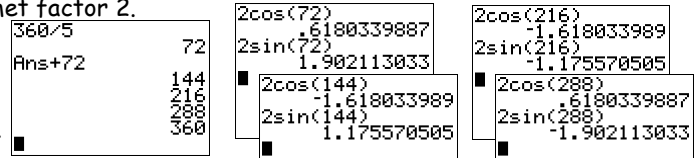
- | | |
|---|---|
| 3a □ $\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$. | 3d □ $\cos(-135^\circ) = \cos(-180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}$. |
| 3b □ $\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{1}{2}\sqrt{3}$. | 3e □ $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{1}{2}\sqrt{3}$. |
| 3c □ $\sin(-135^\circ) = \sin(-180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{1}{2}\sqrt{2}$. | 3f □ $\cos 300^\circ = \cos(360^\circ - 60^\circ) = \cos 60^\circ = \frac{1}{2}$. |

- 4a Zie het scherm hiernaast.
 4b $x_p = \cos 110^\circ \approx -0,34$ en $y_p = \sin 110^\circ \approx 0,94$.
 $x_q = \cos 200^\circ \approx -0,94$ en $y_q = \sin 200^\circ \approx -0,34$.
 $x_r = \cos(-102^\circ) \approx -0,21$ en $y_r = \sin(-102^\circ) \approx -0,98$.
 $x_s = \cos(-50^\circ) \approx 0,64$ en $y_s = \sin(-50^\circ) \approx -0,77$.



5 De cirkel is een vergroting van de eenheidscirkel met factor 2.

- $x_B = 2 \cdot \cos 72^\circ \approx 0,62$ en $y_B = 2 \cdot \sin 72^\circ \approx 1,90$.
 $x_C = 2 \cdot \cos 144^\circ \approx -1,62$ en $y_C = 2 \cdot \sin 144^\circ \approx 1,18$.
 $x_D = 2 \cdot \cos 216^\circ \approx -1,62$ en $y_D = 2 \cdot \sin 216^\circ \approx -1,18$.
 $x_E = 2 \cdot \cos 288^\circ \approx 0,62$ en $y_E = 2 \cdot \sin 288^\circ \approx -1,90$.

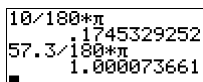


- 6a omtrek $= 2\pi r = 2\pi \cdot 1 = 2\pi$.
 6b $\alpha = 90^\circ \Rightarrow$ een kwart van de eenheidscirkel doorlopen \Rightarrow lengte van de doorlopen boog is $\frac{1}{4} \cdot 2\pi = \frac{2}{4}\pi = \frac{1}{2}\pi$.
 6c $\alpha = 180^\circ \Rightarrow$ een halve eenheidscirkel doorlopen \Rightarrow lengte van de doorlopen boog is $\frac{1}{2} \cdot 2\pi = \pi$.
 6d $1\frac{1}{2}\pi = \frac{3}{4} \cdot 2\pi \Rightarrow$ driekwart van de omtrek van de eenheidscirkel $\Rightarrow \alpha = \frac{3}{4} \cdot 360^\circ = 3 \cdot 90^\circ = 270^\circ$.

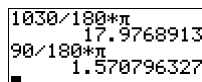
□ π radialen $= 180^\circ \Rightarrow 1$ radiaal $= \frac{180^\circ}{\pi}$ en $\frac{1}{180}\pi$ radialen $= 1^\circ$.

- | | |
|---|---|
| 7a □ $\frac{1}{6}\pi \text{ rad} = \frac{1}{6} \cdot 180^\circ = 30^\circ$. | 7e □ $\frac{5}{4}\pi \text{ rad} = \frac{5}{4} \cdot 180^\circ = 225^\circ$. |
| 7b □ $\frac{1}{4}\pi \text{ rad} = \frac{1}{4} \cdot 180^\circ = 45^\circ$. | 7f □ $\frac{5}{4} \text{ rad} = \frac{5}{4} \cdot \frac{180^\circ}{\pi} \approx 71,6^\circ$. |
| 7c □ $2\pi \text{ rad} = 2 \cdot 180^\circ = 360^\circ$. | 7g □ $-2\frac{1}{3}\pi \text{ rad} = -2\frac{1}{3} \cdot 180^\circ = -420^\circ$. |
| 7d □ $2 \text{ rad} = 2 \cdot \frac{180^\circ}{\pi} \approx 114,6^\circ$. | 7h □ $-2\frac{1}{3} \text{ rad} = -2\frac{1}{3} \cdot \frac{180^\circ}{\pi} \approx -133,7^\circ$. |
| 8a □ $360^\circ = \frac{360}{180} \cdot \pi \text{ rad} = 2\pi \text{ rad}$. | 8e □ $90^\circ = \frac{90}{180} \cdot \pi \text{ rad} = \frac{1}{2}\pi \text{ rad}$. |
| 8b □ $30^\circ = \frac{30}{180} \cdot \pi \text{ rad} = \frac{1}{6}\pi \text{ rad}$. | 8f □ $135^\circ = \frac{135}{180} \cdot \pi \text{ rad} = \frac{3}{4}\pi \text{ rad}$. |
| 8c □ $45^\circ = \frac{45}{180} \cdot \pi \text{ rad} = \frac{1}{4}\pi \text{ rad}$. | 8g □ $300^\circ = \frac{300}{180} \cdot \pi \text{ rad} = 1\frac{2}{3}\pi \text{ rad}$. |
| 8d □ $60^\circ = \frac{60}{180} \cdot \pi \text{ rad} = \frac{1}{3}\pi \text{ rad}$. | 8h □ $210^\circ = \frac{210}{180} \cdot \pi \text{ rad} = 1\frac{1}{6}\pi \text{ rad}$. |

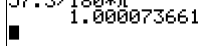
9a $10^\circ = \frac{10}{180} \cdot \pi \text{ rad} \approx 0,17 \text{ rad.}$



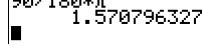
9c $1030^\circ = \frac{1030}{180} \cdot \pi \text{ rad} \approx 17,98 \text{ rad.}$



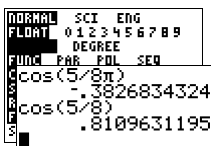
9b $57,3^\circ = \frac{57,3}{180} \cdot \pi \text{ rad} \approx 1,00 \text{ rad.}$



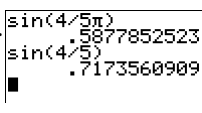
9d $90^\circ = \frac{90}{180} \cdot \pi \text{ rad} \approx 1,57 \text{ rad.}$



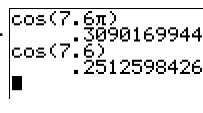
10a $\cos\left(\frac{5}{8}\pi\right) \approx -0,38.$



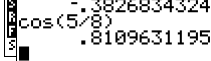
10c $\sin\left(\frac{4}{5}\pi\right) \approx 0,59.$



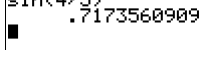
10e $\cos(7,6\pi) \approx 0,31.$



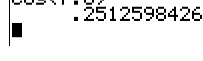
10b $\cos\left(\frac{5}{8}\right) \approx 0,81.$



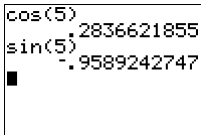
10d $\sin\left(\frac{4}{5}\right) \approx 0,72.$



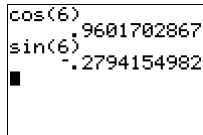
10f $\cos(7,6) \approx 0,25.$



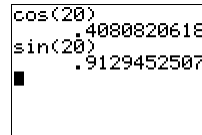
11a $x_\rho = \cos(5) \approx 0,28$ en $y_\rho = \sin(5) \approx -0,96.$



11b $x_\rho = \cos(6) \approx 0,96$ en $y_\rho = \sin(6) \approx -0,28.$



11c $x_\rho = \cos(20) \approx 0,41$ en $y_\rho = \sin(20) \approx 0,91.$



12a $\cos\left(\frac{1}{6}\pi\right) = \cos 30^\circ = \frac{1}{2}\sqrt{3}.$

hoek	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
sinus	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
cosinus	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0

Leer deze tabel uit het hoofd.

12b $\sin\left(\frac{1}{4}\pi\right) = \sin 45^\circ = \frac{1}{2}\sqrt{2}.$

□

13a $\sin\left(\frac{3}{4}\pi\right) = \sin\left(\pi - \frac{1}{4}\pi\right) = \sin\left(\frac{1}{4}\pi\right) = \frac{1}{2}\sqrt{2}.$

13d $\cos\left(\frac{5}{3}\pi\right) = \cos\left(-\frac{1}{3}\pi\right) = \cos\left(\frac{1}{3}\pi\right) = \frac{1}{2}.$

13b $\cos\left(\frac{7}{6}\pi\right) = \cos\left(\pi + \frac{1}{6}\pi\right) = -\cos\left(\frac{1}{6}\pi\right) = -\frac{1}{2}\sqrt{3}.$

13e $\cos\left(1\frac{1}{3}\pi\right) = \cos\left(\pi + \frac{1}{3}\pi\right) = -\cos\left(\frac{1}{3}\pi\right) = -\frac{1}{2}.$

13c $\sin\left(1\frac{1}{3}\pi\right) = \sin\left(\pi + \frac{1}{3}\pi\right) = -\sin\left(\frac{1}{3}\pi\right) = -\frac{1}{2}\sqrt{3}.$

13f $\sin\left(-\frac{1}{4}\pi\right) = -\sin\left(\frac{1}{4}\pi\right) = -\frac{1}{2}\sqrt{2}.$

14a $\sin(\alpha) = \frac{1}{2}\sqrt{3} = \sin\left(\frac{1}{3}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{3}\pi$ of $\alpha = \pi - \frac{1}{3}\pi = \frac{2}{3}\pi.$

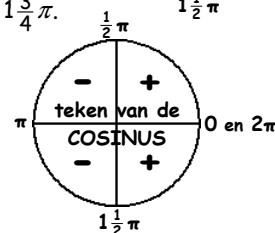
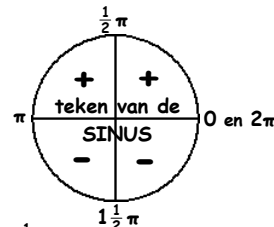
14b $\cos(\alpha) = -\frac{1}{2} = -\cos\left(\frac{1}{3}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi - \frac{1}{3}\pi = \frac{2}{3}\pi$ of $\alpha = \pi + \frac{1}{3}\pi = 1\frac{1}{3}\pi.$

14c $\sin(\alpha) = -\frac{1}{2}\sqrt{2} = -\sin\left(\frac{1}{4}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi + \frac{1}{4}\pi = 1\frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$

14d $\cos(\alpha) = 0 = \cos\left(\frac{1}{2}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{2}\pi$ of $\alpha = 2\pi - \frac{1}{2}\pi = 1\frac{1}{2}\pi.$

14e $\cos(\alpha) = \frac{1}{2}\sqrt{3} = \cos\left(\frac{1}{6}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = 2\pi - \frac{1}{6}\pi = 1\frac{5}{6}\pi.$

14f $\cos(\alpha) = \frac{1}{2}\sqrt{2} = \cos\left(\frac{1}{4}\pi\right)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$



15 Bij de snijpunten met de y -as horen de draaiingshoeken $\dots, -2\frac{1}{2}\pi, -1\frac{1}{2}\pi, -\frac{1}{2}\pi, \frac{1}{2}\pi, 1\frac{1}{2}\pi, 2\frac{1}{2}\pi, \dots$

16a $\sin(3x - \frac{1}{2}\pi) = 0 = \sin(0)$

$3x - \frac{1}{2}\pi = 0 + k \cdot \pi$

$3x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi.$

16c $\sin^2(x) - \sin(x) = 0$

$\sin(x) \cdot (\sin(x) - 1) = 0$

$\sin(x) = 0$ of $\sin(x) = 1$

$x = k \cdot \pi$ of $x = \frac{1}{2}\pi + k \cdot 2\pi.$

16b $\cos\left(\frac{1}{2}x - \frac{1}{6}\pi\right) = 0 = \cos\left(\frac{1}{2}\pi\right)$

$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$

$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$

$x = 1\frac{1}{3}\pi + k \cdot 2\pi.$

16d $\cos^2(2x) + \cos(2x) = 0$

$\cos(2x) \cdot (\cos(2x) + 1) = 0$

$\cos(2x) = 0$ of $\cos(2x) = -1$

$2x = \frac{1}{2}\pi + k \cdot \pi$ of $2x = \pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$ of $x = \frac{1}{2}\pi + k \cdot \pi.$

17a $\sin^2(2x) = 1$

$\sin(2x) = 1$ of $\sin(2x) = -1$

$2x = \frac{1}{2}\pi + k \cdot 2\pi$ of $2x = -\frac{1}{2}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \pi$ of $x = -\frac{1}{4}\pi + k \cdot \pi.$

of sneller: 17abc $\sin^2(2x) = 1$

$\sin(2x) = \pm 1$

$2x = \frac{1}{2}\pi + k \cdot \pi$

$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

17b $\dots, -2\frac{1}{4}\pi, -1\frac{1}{4}\pi, -\frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{3}{4}\pi, 2\frac{3}{4}\pi, \dots$

17c $\dots, -2\frac{1}{4}\pi, -1\frac{3}{4}\pi, -1\frac{1}{4}\pi, -\frac{3}{4}\pi, -\frac{1}{4}\pi, \frac{1}{4}\pi, \frac{3}{4}\pi, 1\frac{1}{4}\pi, 1\frac{3}{4}\pi, 2\frac{1}{4}\pi, 2\frac{3}{4}\pi, \dots$ is te schrijven als $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

18a $\cos^2(x - \frac{1}{5}\pi) = 1$
 $\cos(x - \frac{1}{5}\pi) = \pm 1$
 $x - \frac{1}{5}\pi = 0 + k \cdot \pi$
 $x = \frac{1}{5}\pi + k \cdot \pi.$

18c $\sin^3(x) - \sin(x) = 0$
 $\sin(x) \cdot (\sin^2(x) - 1) = 0$
 $\sin(x) = 0$ of $\sin^2(x) = 1$
 $\sin(x) = 0$ of $\sin(x) = \pm 1$
 $x = 0 + k \cdot \pi$ of $x = \frac{1}{2}\pi + k \cdot \pi$
 $x = k \cdot \frac{1}{2}\pi.$

18d $\cos^3(2x) - \cos(2x) = 0$
 $\cos(2x) \cdot (\cos^2(x) - 1) = 0$
 $\cos(2x) = 0$ of $\cos^2(2x) = 1$
 $\cos(2x) = 0$ of $\cos(2x) = \pm 1$
 $2x = \frac{1}{2}\pi + k \cdot \pi$ of $2x = 0 + k \cdot \pi$
 $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$ of $x = k \cdot \frac{1}{2}\pi$
 $x = k \cdot \frac{1}{4}\pi.$

18b $\sin^2(2x - \frac{1}{4}\pi) = 1$
 $\sin(2x - \frac{1}{4}\pi) = \pm 1$
 $2x - \frac{1}{4}\pi = \frac{1}{2}\pi + k \cdot \pi$
 $2x = \frac{3}{4}\pi + k \cdot \pi$
 $x = \frac{3}{8}\pi + k \cdot \frac{1}{2}\pi.$

19a $\sin(4x - \frac{1}{3}\pi) = 1$
 $4x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot 2\pi$
 $4x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{24}\pi + k \cdot \frac{1}{2}\pi.$

19c $\sin^2(\frac{1}{4}\pi x) = 1$
 $\sin(\frac{1}{4}\pi x) = \pm 1$
 $\frac{1}{4}\pi x = \frac{1}{2}\pi + k \cdot \pi$
 $x = 2 + k \cdot 4.$

19d $\sin(2x) \cdot \cos(2x) + \sin(2x) = 0$
 $\sin(2x) \cdot (\cos(2x) + 1) = 0$
 $\sin(2x) = 0$ of $\cos(2x) = -1$
 $2x = 0 + k \cdot \pi$ of $2x = \pi + k \cdot 2\pi$
 $x = k \cdot \frac{1}{2}\pi$ of $x = \frac{1}{2}\pi + k \cdot \pi$
 $x = k \cdot \frac{1}{2}\pi.$

19b $\cos(4\pi x) = -1$
 $4\pi x = \pi + k \cdot 2\pi$
 $x = \frac{1}{4} + k \cdot \frac{1}{2}.$

20a $\sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = \frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

20b $2\frac{1}{6}\pi = \frac{1}{6}\pi + 2\pi \Rightarrow \sin(2\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$ en $4\frac{1}{6}\pi = \frac{1}{6}\pi + 2 \cdot 2\pi \Rightarrow \sin(4\frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$.

20c $\sin(\frac{5}{6}\pi) = \sin(\pi - \frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$, dus $x = \frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}$.

20d $2\frac{5}{6}\pi = \frac{5}{6}\pi + 2\pi \Rightarrow \sin(2\frac{5}{6}\pi) = \sin(\frac{5}{6}\pi) = \frac{1}{2}$ en $-1\frac{1}{6}\pi = \frac{5}{6}\pi - 2\pi \Rightarrow \sin(-1\frac{1}{6}\pi) = \sin(\frac{5}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}$.

21a $2\sin(\frac{1}{2}x) = 1$
 $\sin(\frac{1}{2}x) = \frac{1}{2}$
 $\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot 4\pi$ of $x = \frac{5}{3}\pi + k \cdot 4\pi.$

21c $2\sin(2x - \frac{1}{4}\pi) = -\sqrt{3}$
 $\sin(2x - \frac{1}{4}\pi) = -\frac{1}{2}\sqrt{3}$
 $2x - \frac{1}{4}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$ of $2x - \frac{1}{4}\pi = 1\frac{1}{3}\pi + k \cdot 2\pi$
 $2x = -\frac{1}{12}\pi + k \cdot 2\pi$ of $2x = \frac{19}{12}\pi + k \cdot 2\pi$
 $x = -\frac{1}{24}\pi + k \cdot \pi$ of $x = \frac{19}{24}\pi + k \cdot \pi.$

21b $2\cos(x - \frac{1}{3}\pi) = 1$
 $\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$
 $x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi$ of $x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi$ of $x = k \cdot 2\pi.$

21d $2\cos(3x - \pi) = -1$
 $\cos(3x - \pi) = -\frac{1}{2}$
 $3x - \pi = \frac{2}{3}\pi + k \cdot 2\pi$ of $3x - \pi = -\frac{2}{3}\pi + k \cdot 2\pi$
 $3x = \frac{5}{3}\pi + k \cdot 2\pi$ of $3x = \frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{5}{9}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi.$

22a $2\sin(2x - \frac{1}{6}\pi) = \sqrt{2}$
 $\sin(2x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{2}$
 $2x - \frac{1}{6}\pi = \frac{1}{4}\pi + k \cdot 2\pi$ of $2x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$
 $2x = \frac{5}{12}\pi + k \cdot 2\pi$ of $2x = \frac{11}{12}\pi + k \cdot 2\pi$
 $x = \frac{5}{24}\pi + k \cdot \pi$ of $x = \frac{11}{24}\pi + k \cdot \pi.$
 x op $[0, 2\pi]$ geeft
 $x = \frac{5}{24}\pi$ of $x = 1\frac{5}{24}\pi$ of $x = \frac{11}{24}\pi$ of $x = 1\frac{11}{24}\pi.$

22b $2\cos(3x - \frac{1}{2}\pi) = \sqrt{3}$
 $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{3}$
 $3x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $3x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$
 $3x = \frac{2}{3}\pi + k \cdot 2\pi$ of $3x = \frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{9}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi.$
 x op $[0, 2\pi]$ geeft
 $x = \frac{2}{9}\pi$ of $x = \frac{8}{9}\pi$ of $x = \frac{14}{9}\pi$ of $x = \frac{1}{9}\pi$ of $x = \frac{7}{9}\pi$ of $x = \frac{13}{9}\pi.$

22c $\sin(\frac{2}{3}x) = -\frac{1}{2}\sqrt{2}$
 $\frac{2}{3}x = -\frac{1}{4}\pi + k \cdot 2\pi$ of $\frac{2}{3}x = \frac{5}{4}\pi + k \cdot 2\pi$
 $x = -\frac{3}{8}\pi + k \cdot 3\pi$ of $x = \frac{15}{8}\pi + k \cdot 3\pi.$
 x op $[0, 2\pi]$ geeft $x = \frac{15}{8}\pi.$

22d $\cos(\frac{1}{2}x) = -\frac{1}{2}\sqrt{3}$
 $\frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$ of $\frac{1}{2}x = -\frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{3}\pi + k \cdot 4\pi$ of $x = -\frac{5}{3}\pi + k \cdot 4\pi.$
 x op $[0, 2\pi]$ geeft $x = \frac{5}{3}\pi.$

- 23a $2\sin^2(x) = 1$
 $\sin^2(x) = \frac{1}{2}$
 $\sin(x) = \sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{4}} \cdot \sqrt{2} = \frac{1}{2}\sqrt{2}$ of $\sin(x) = -\sqrt{\frac{1}{2}} = -\frac{1}{2}\sqrt{2}$.
- 23b $\sin(x) = \frac{1}{2}\sqrt{2}$ geeft $x = \frac{1}{4}\pi + k \cdot 2\pi$ of $x = \frac{3}{4}\pi + k \cdot 2\pi$.
 $\sin(x) = -\frac{1}{2}\sqrt{2}$ geeft $x = -\frac{1}{4}\pi + k \cdot 2\pi$ of $x = \frac{5}{4}\pi + k \cdot 2\pi$.
- 23c Uitschrijven geeft: $\dots, -\frac{5}{4}\pi, -\frac{3}{4}\pi, -\frac{1}{4}\pi, \frac{1}{4}\pi, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi, \frac{9}{4}\pi, \frac{11}{4}\pi, \frac{13}{4}\pi, \frac{15}{4}\pi, \dots \Rightarrow x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.
 (de oplossingen in een schets op de rand van de eenheidscirkel uitzetten geeft ook snel de uiteindelijke oplossing)
- 23d of sneller: $2\sin^2(x) = 1$
 $\sin^2(x) = \frac{1}{2}$
 $\sin(x) = \pm\sqrt{\frac{1}{2}} = \pm\sqrt{\frac{2}{4}} = \pm\sqrt{\frac{1}{4}} \cdot \sqrt{2} = \pm\frac{1}{2}\sqrt{2}$
 Uit de exacte-waarden-cirkel lees je af:
 $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.
- 24a $2\cos^2(\frac{1}{2}x) = 1$
 $\cos^2(\frac{1}{2}x) = \frac{1}{2}$
 $\cos(\frac{1}{2}x) = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{2}\sqrt{2}$
 $\frac{1}{2}x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$
 $x = \frac{1}{2}\pi + k \cdot \pi$.
- 24b $4\sin^2(x - \frac{1}{6}\pi) = 1$
 $\sin^2(x - \frac{1}{6}\pi) = \frac{1}{4}$
 $\sin(x - \frac{1}{6}\pi) = \pm\frac{1}{2}$
 $x - \frac{1}{6}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $x - \frac{1}{6}\pi = \frac{5}{6}\pi + k \cdot 2\pi$ of $x - \frac{1}{6}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x - \frac{1}{6}\pi = 1\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot 2\pi$ of $x = \pi + k \cdot 2\pi$ of $x = k \cdot 2\pi$ of $x = 1\frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \pi$ of $x = \pi + k \cdot \pi$.
- 24c $4\cos^2(x + \frac{1}{4}\pi) = 3$
 $\cos^2(x + \frac{1}{4}\pi) = \frac{3}{4}$
 $\cos(x + \frac{1}{4}\pi) = \pm\sqrt{\frac{3}{4}} = \pm\frac{1}{2}\sqrt{3}$
 $x + \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x + \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi$ of $x + \frac{1}{4}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$
 $x = -\frac{1}{12}\pi + k \cdot 2\pi$ of $x = -\frac{5}{12}\pi + k \cdot 2\pi$ of $x = \frac{7}{12}\pi + k \cdot 2\pi$ of $x = -\frac{13}{12}\pi + k \cdot 2\pi$
 $x = -\frac{1}{12}\pi + k \cdot \pi$ of $x = -\frac{5}{12}\pi + k \cdot \pi$.
- 24d $4\sin^3(x) - \sin(x) = 0$
 $\sin(x) \cdot (4\sin^2(x) - 1) = 0$
 $\sin(x) = 0$ of $4\sin^2(x) = 1$
 $\sin(x) = 0$ of $\sin^2(x) = \frac{1}{4}$
 $\sin(x) = 0$ of $\sin(x) = \pm\frac{1}{2}$
 $x = k \cdot \pi$ of $x = \frac{1}{6}\pi + k \cdot 2\pi$ of $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $x = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x = \frac{7}{6}\pi + k \cdot 2\pi$
 $x = k \cdot \pi$ of $x = \frac{1}{6}\pi + k \cdot \pi$ of $x = \frac{5}{6}\pi + k \cdot \pi$.
- 24e $2\cos^2(x) = \cos(x) + 1$
 $2\cos^2(x) - \cos(x) - 1 = 0$
 $\cos^2(x) - \frac{1}{2}\cos(x) - \frac{1}{2} = 0$ even proberen geeft: $(\cos(x) - \dots) \cdot (\cos(x) + \dots) = 0$ (anders abc-formule)
 $(\cos(x) - 1) \cdot (\cos(x) + \frac{1}{2}) = 0$
 $\cos(x) = 1$ of $\cos(x) = -\frac{1}{2}$
 $x = k \cdot 2\pi$ of $x = \frac{2}{3}\pi + k \cdot 2\pi$ of $x = -\frac{2}{3}\pi + k \cdot 2\pi$
 $x = k \cdot \frac{2}{3}\pi$.
- 24f $\cos^2(x) - \cos(x) + \frac{1}{4} = 0$ even proberen geeft: $(\cos(x) - \dots) \cdot (\cos(x) - \dots) = 0$ (anders abc-formule)
 $(\cos(x) - \frac{1}{2}) \cdot (\cos(x) - \frac{1}{2}) = 0$
 $\cos(x) = \frac{1}{2}$
 $x = \frac{1}{3}\pi + k \cdot 2\pi$ of $x = -\frac{1}{3}\pi + k \cdot 2\pi$.

25a $\sin(\frac{1}{2}\pi x) = \frac{1}{2}\sqrt{3}$
 $\frac{1}{2}\pi x = \frac{1}{3}\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \frac{2}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{3} + k \cdot 4$ of $x = \frac{4}{3} + k \cdot 4$.
 x op $[0, 10]$ geeft
 $x = \frac{2}{3}$ of $x = 4\frac{2}{3}$ of $x = 8\frac{2}{3}$ of $x = 1\frac{1}{3}$ of $x = 5\frac{1}{3}$ of $x = 9\frac{1}{3}$.

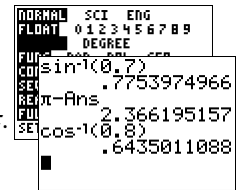
25b $\cos(\frac{1}{3}\pi x) = -\frac{1}{2}\sqrt{3}$
 $\frac{1}{3}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$ of $\frac{1}{3}\pi x = -\frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{15}{6} + k \cdot 6$ of $x = -\frac{15}{6} + k \cdot 6$.
 x op $[0, 10]$ geeft
 $x = 2\frac{1}{2}$ of $x = 8\frac{1}{2}$ of $x = 3\frac{1}{2}$ of $x = 9\frac{1}{2}$.

25c $4\sin^2(\frac{1}{5}\pi x) = 1$
 $\sin^2(\frac{1}{5}\pi x) = \frac{1}{4}$
 $\sin(\frac{1}{5}\pi x) = \pm\frac{1}{2}$
 $\frac{1}{5}\pi x = \frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = \frac{5}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = -\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{5}\pi x = \frac{7}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{6} + k \cdot 10$ of $x = \frac{25}{6} + k \cdot 10$ of $x = -\frac{5}{6} + k \cdot 10$ of $x = \frac{35}{6} + k \cdot 10$.
 x op $[0, 10]$ geeft $x = \frac{5}{6}$ of $x = 4\frac{1}{6}$ of $x = 9\frac{1}{6}$ of $x = 5\frac{5}{6}$.

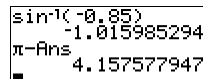
25d $2\cos^2(0,1\pi x) + \cos(0,1\pi x) = 1$
 $2\cos^2(0,1\pi x) + \cos(0,1\pi x) - 1 = 0$
 $\cos^2(0,1\pi x) + \frac{1}{2}\cos(0,1\pi x) - \frac{1}{2} = 0$ proberen geeft: $(\cos(0,1\pi x) + \dots) \cdot (\cos(0,1\pi x) - \dots) = 0$ (anders abc-formule)
 $(\cos(0,1\pi x) + 1) \cdot (\cos(0,1\pi x) - \frac{1}{2}) = 0$
 $\cos(0,1\pi x) = -1$ of $\cos(0,1\pi x) = \frac{1}{2}$
 $0,1\pi x = \pi + k \cdot 2\pi$ of $0,1\pi x = \frac{1}{3}\pi + k \cdot 2\pi$ of $0,1\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = 10 + k \cdot 20$ of $x = \frac{10}{3} + k \cdot 20$ of $x = -\frac{10}{3} + k \cdot 20$.
 x op $[0, 10]$ geeft $x = 10$ of $x = 3\frac{1}{3}$.

26a $\sin(x) = 0,7$
 $x \approx 0,775 + k \cdot 2\pi$ of $x \approx \pi - 0,775 + k \cdot 2\pi$
 $x \approx 0,775 + k \cdot 2\pi$ of $x \approx 2,366 + k \cdot 2\pi$.

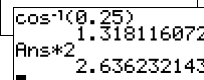
26b $\cos(x) = 0,8$
 $x \approx 0,644 + k \cdot 2\pi$ of $x \approx -0,644 + k \cdot 2\pi$.



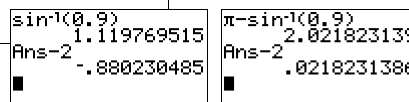
27a $\sin(x) = -0,85$
 $x \approx -1,016 + k \cdot 2\pi$ of $x \approx \pi - 1,016 + k \cdot 2\pi$
 $x \approx -1,016 + k \cdot 2\pi$ of $x \approx 4,158 + k \cdot 2\pi$.



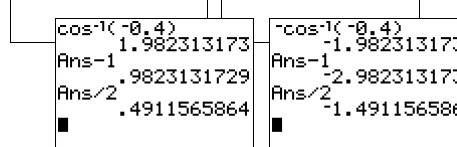
27b $\cos(\frac{1}{2}x) = 0,25$
 $\frac{1}{2}x \approx 1,318 + k \cdot 2\pi$ of $\frac{1}{2}x \approx -1,318 + k \cdot 2\pi$
 $x \approx 2,636 + k \cdot 4\pi$ of $x \approx -2,636 + k \cdot 4\pi$.



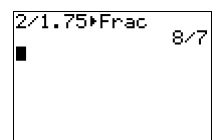
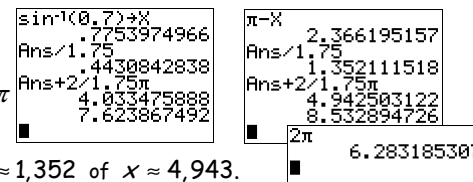
27c $\sin(x+2) = 0,9$
 $x+2 \approx 1,120 + k \cdot 2\pi$ of $x+2 \approx \pi - 1,120 + k \cdot 2\pi$
 $x \approx -0,880 + k \cdot 2\pi$ of $x \approx 0,022 + k \cdot 2\pi$.



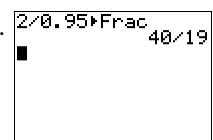
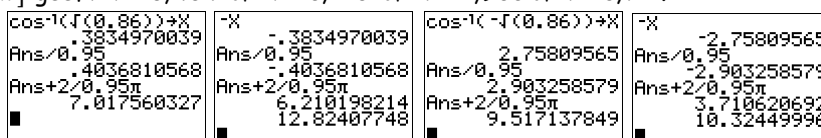
27d $\cos(2x+1) = -0,4$
 $2x+1 \approx 1,982 + k \cdot 2\pi$ of $2x+1 \approx -1,982 + k \cdot 2\pi$
 $2x \approx 0,982 + k \cdot 2\pi$ of $2x \approx -2,982 + k \cdot 2\pi$
 $x \approx 0,491 + k \cdot \pi$ of $x \approx -1,491 + k \cdot \pi$.



28a $2\sin(1,75x) = 1,4$
 $\sin(1,75x) = 0,7$
 $1,75x \approx 0,775 + k \cdot 2\pi$ of $1,75x \approx \pi - 0,775 + k \cdot 2\pi$
 $x \approx 0,443 + k \cdot \frac{2}{1,75}\pi$ of $x \approx 1,352 + k \cdot \frac{2}{1,75}\pi$.
 x op $[0, 2\pi]$ geeft $x \approx 0,443$ of $x \approx 4,033$ of $x \approx 1,352$ of $x \approx 4,943$.



28b $\cos^2(0,95x) = 0,86$
 $\cos(0,95x) = \pm\sqrt{0,86}$
 $0,95x \approx 0,383 + k \cdot 2\pi$ of $0,95x \approx -0,383 + k \cdot 2\pi$ of $0,95x \approx 2,758 + k \cdot 2\pi$ of $0,95x \approx -2,758 + k \cdot 2\pi$
 $x \approx 0,404 + k \cdot \frac{2}{0,95}\pi$ of $x \approx -0,404 + k \cdot \frac{2}{0,95}\pi$ of $x \approx 2,903 + k \cdot \frac{2}{0,95}\pi$ of $x \approx -2,903 + k \cdot \frac{2}{0,95}\pi$.
 x op $[0, 2\pi]$ geeft $x \approx 0,404$ of $x \approx 6,210$ of $x \approx 2,903$ of $x \approx 3,711$.



29a $\sin(3x) = \sin(\frac{1}{6}\pi)$
 $3x = \frac{1}{6}\pi + k \cdot 2\pi$ of $3x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$.

29b $\cos(3x) = \cos(\frac{1}{6}\pi)$
 $3x = \frac{1}{6}\pi + k \cdot 2\pi$ of $3x = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$ of $x = -\frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$.

30a $\sin(x+1) = \sin(2x+3)$
 $x+1 = 2x+3 + k \cdot 2\pi$ of $x+1 = \pi - (2x+3) + k \cdot 2\pi$
 $x+1 = 2x+3 + k \cdot 2\pi$ of $x+1 = \pi - 2x - 3 + k \cdot 2\pi$
 $-x = 2 + k \cdot 2\pi$ of $3x = \pi - 4 + k \cdot 2\pi$
 $x = -2 + k \cdot 2\pi$ of $x = \frac{1}{3}\pi - \frac{4}{3} + k \cdot \frac{2}{3}\pi$.

30b $\cos(2x-1) = \cos(x+1)$
 $2x-1 = x+1 + k \cdot 2\pi$ of $2x-1 = -(x+1) + k \cdot 2\pi$
 $2x-1 = x+1 + k \cdot 2\pi$ of $2x-1 = -x-1 + k \cdot 2\pi$
 $x = 2 + k \cdot 2\pi$ of $3x = k \cdot 2\pi$
 $x = 2 + k \cdot 2\pi$ of $x = k \cdot \frac{2}{3}\pi$.

30c $\sin(2x - \frac{1}{2}\pi) = \sin(x + \frac{1}{3}\pi)$
 $2x - \frac{1}{2}\pi = x + \frac{1}{3}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \pi - (x + \frac{1}{3}\pi) + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \pi - x - \frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = \frac{2}{3}\pi - x + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $3x = \frac{7}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$.

30d $\cos(x - \frac{1}{3}\pi) = \cos(2x)$
 $x - \frac{1}{3}\pi = 2x + k \cdot 2\pi$ of $x - \frac{1}{3}\pi = -2x + k \cdot 2\pi$
 $-x = \frac{1}{3}\pi + k \cdot 2\pi$ of $3x = \frac{1}{3}\pi + k \cdot 2\pi$
 $x = -\frac{1}{3}\pi + k \cdot 2\pi$ of $x = \frac{1}{9}\pi + k \cdot \frac{2}{3}\pi$.

30e $\sin(2\pi x) = \sin(\pi(x-1))$
 $2\pi x = \pi(x-1) + k \cdot 2\pi$ of $2\pi x = \pi - \pi(x-1) + k \cdot 2\pi$
 $2\pi x = \pi x - \pi + k \cdot 2\pi$ of $2\pi x = \pi - \pi x + \pi + k \cdot 2\pi$
 $\pi x = -\pi + k \cdot 2\pi$ of $3\pi x = 2\pi + k \cdot 2\pi$
 $x = -1 + k \cdot 2$ of $x = \frac{2}{3} + k \cdot \frac{2}{3}$
 $x = -1 + k \cdot 2$ of $x = k \cdot \frac{2}{3}$.

30f $\cos(\frac{1}{2}\pi x) = \cos(\pi(x-2))$
 $\frac{1}{2}\pi x = \pi(x-2) + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi(x-2) + k \cdot 2\pi$
 $\frac{1}{2}\pi x = \pi x - 2\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
 $-\frac{1}{2}\pi x = -2\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = -\pi x + 2\pi + k \cdot 2\pi$
 $x = 4 + k \cdot 4$ of $\frac{3}{2}\pi x = 2\pi + k \cdot 2\pi$
 $x = k \cdot 4$ of $x = \frac{4}{3} + k \cdot \frac{4}{3}$
 $x = k \cdot 4$ of $x = k \cdot \frac{4}{3}$
 $x = k \cdot \frac{4}{3}$.

31a $\sin(2x - \frac{1}{3}\pi) = \sin(x + \frac{1}{4}\pi)$
 $2x - \frac{1}{3}\pi = x + \frac{1}{4}\pi + k \cdot 2\pi$ of $2x - \frac{1}{3}\pi = \pi - (x + \frac{1}{4}\pi) + k \cdot 2\pi$
 $x = \frac{7}{12}\pi + k \cdot 2\pi$ of $2x - \frac{1}{3}\pi = \pi - x - \frac{1}{4}\pi + k \cdot 2\pi$
 $x = \frac{7}{12}\pi + k \cdot 2\pi$ of $3x = \frac{13}{12}\pi + k \cdot 2\pi$
 $x = \frac{7}{12}\pi + k \cdot 2\pi$ of $x = \frac{13}{36}\pi + k \cdot \frac{2}{3}\pi$
 x op $[0, 2\pi]$ geeft $x = \frac{7}{12}\pi$ of $x = \frac{13}{36}\pi$ of $x = \frac{37}{36}\pi$ of $x = \frac{61}{36}\pi$.

31b $\cos(3x + \frac{1}{2}\pi) = \cos(2x - \frac{1}{4}\pi)$
 $3x + \frac{1}{2}\pi = 2x - \frac{1}{4}\pi + k \cdot 2\pi$ of $3x + \frac{1}{2}\pi = -(2x - \frac{1}{4}\pi) + k \cdot 2\pi$
 $x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $3x + \frac{1}{2}\pi = -2x + \frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $5x = -\frac{1}{4}\pi + k \cdot 2\pi$
 $x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $x = -\frac{1}{20}\pi + k \cdot \frac{2}{5}\pi$
 x op $[0, 2\pi]$ geeft $x = \frac{5}{4}\pi$ of $x = \frac{7}{20}\pi$ of $x = \frac{15}{20}\pi$ of $x = \frac{23}{20}\pi$ of $x = \frac{31}{20}\pi$ of $x = \frac{39}{20}\pi$.

32a Zie de plot hiernaast. (denk ook aan de eenheidscirkel)

32b De toppen zijn: $(-\frac{1}{2}\pi, 1)$, $(-\frac{1}{2}\pi, -1)$, $(\frac{1}{2}\pi, 1)$ en $(\frac{1}{2}\pi, -1)$.

32c Snijpunten met de x -as: $(-2\pi, 0)$, $(-\pi, 0)$, $(0, 0)$, $(\pi, 0)$ en $(2\pi, 0)$.

33a Zie de plot hiernaast. (denk ook aan de eenheidscirkel)

33b De toppen zijn: $(-2\pi, 1)$, $(-\pi, -1)$, $(0, 1)$, $(\pi, -1)$ en $(2\pi, 1)$.

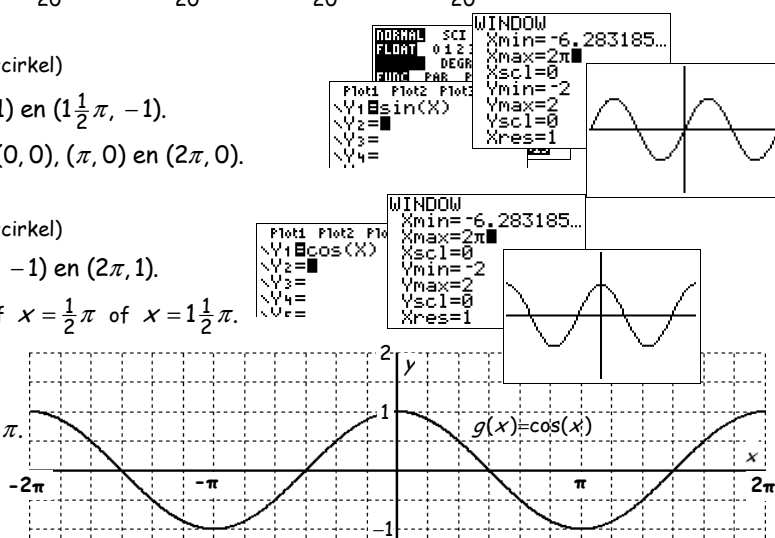
33c $g(x) = \cos x = 0 \Rightarrow x = -\frac{1}{2}\pi$ of $x = -\frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$.

OF: $\cos x = 0 \Rightarrow x = \frac{1}{2}\pi + k \cdot \pi$

Dus op domein $[-2\pi, 2\pi]$ $\cos x = 0$ geeft

$x = -\frac{1}{2}\pi$ of $x = -\frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$.

33d Zie de grafiek hiernaast.



34a $f(x) = \sin(x) \xrightarrow{\text{translatie } (0,2)} g(x) = \sin(x) + 2.$
evenwichtswaarde 0 \Rightarrow evenwichtswaarde 2

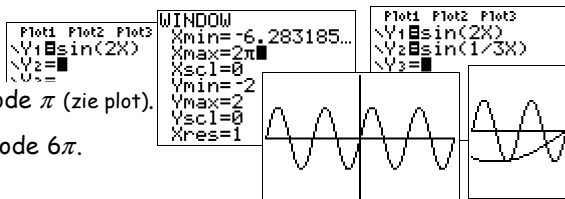
34b $f(x) = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} h(x) = \sin(x - \frac{1}{3}\pi).$ Nulpunten: $h(x) = \sin(x - \frac{1}{3}\pi) = 0$

34c $f(x) = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 4} k(x) = 4 \sin(x).$ $x - \frac{1}{3}\pi = 0 + k \cdot \pi$
amplitude 1 \Rightarrow amplitude 4 $x = \frac{1}{3}\pi + k \cdot \pi$

35a Zie de plot hiernaast.

35b $f(x) = \sin(2x)$ heeft periode π (zie plot).

35c $g(x) = \sin(\frac{1}{3}x)$ heeft periode 6π .



$\left. \begin{aligned} &\sin(2x) = 1 \\ &2x = \frac{1}{2}\pi + k \cdot 2\pi \\ &x = \frac{1}{4}\pi + k \cdot \pi \Rightarrow \text{maxima om de } \pi \end{aligned} \right\}$
Dus $\sin(2x)$ heeft periode π

36 \square Zie het schema voor de cosinus hieronder. (de oorspronkelijke grafiek van $y = \cos(x)$ is steeds iets dunner getekend)

translatie $(0, a)$	vermenigvuldiging x -as, b	vermenigvuldiging y -as, $\frac{1}{c}$	translatie $(d, 0)$
tel a op bij de functiewaarde	verm. de functiewaarde met b	vervang x door cx	vervang x door $x - d$
$y = a + \cos(x)$ evenwichtsstand is a	$y = b \cos(x)$ amplitude is b	$y = \cos(cx)$ periode is $\frac{2\pi}{c}$	$y = \cos(x - d)$ punt op de y -as komt bij $x = d$

37a \square $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2 \sin(x) \xrightarrow{\text{translatie } (-3, 0)} f(x) = 2 \sin(x + 3).$ (of in omgekeerde volgorde)

37b \square $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } \frac{1}{3}} y = \frac{1}{3} \sin(x) \xrightarrow{\text{translatie } (0, \frac{1}{5})} g(x) = \frac{1}{3} \sin(x) + \frac{1}{5}.$ (in deze volgorde)

37c \square $y = \cos(x) \xrightarrow{\text{translatie } (12, 0)} y = \cos(x - 12) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} h(x) = \cos(3x - 12).$ (in deze volgorde)

37d \square $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } \frac{1}{2}} y = \frac{1}{2} \cos(x) \xrightarrow{\text{verm. } y\text{-as, } 4} j(x) = 2 \cos(\frac{1}{4}x).$ (of in omgekeerde volgorde)

38a \square $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 1,2} y = 1,2 \cos(x) \xrightarrow{\text{translatie } (\frac{1}{6}\pi, 5)} f(x) = 1,2 \cos(x - \frac{1}{6}\pi) + 5.$ (in deze volgorde)

38b \square $y = \sin(x) \xrightarrow{\text{verm. } y\text{-as, } 5} y = 5 \sin(\frac{1}{5}x) \xrightarrow{\text{translatie } (-\frac{1}{3}\pi, 0,4)} g(x) = 5 \sin(\frac{1}{5}(x + \frac{1}{3}\pi)) + 0,4.$ (in deze volgorde)

38c \square $y = \cos(x) \xrightarrow{\text{translatie } (-4, 2; 0)} y = \cos(x + 4, 2) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = \cos(3x + 4, 2)$ (in deze volgorde)
 $y = \cos(3x + 4, 2) \xrightarrow{\text{verm. } x\text{-as, } 0,29} h(x) = 0,29 \cos(3x + 4, 2)$ (deze laatste vermenigvuldiging mag ook eerder staan).

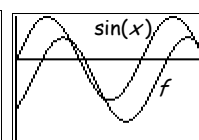
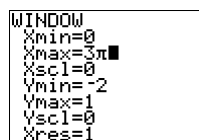
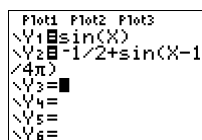
38d \square $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2 \sin(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = 2 \sin(3x)$ (of in omgekeerde volgorde en daarna)
 $y = 2 \sin(3x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi, -0,8)} j(x) = 2 \sin(3(x - \frac{1}{2}\pi)) - 0,8.$ (deze translatie moet als laatste staan)

39 \square $y = \sin(x) \xrightarrow{\text{verm. } y\text{-as, } 3} y = 3 \sin(\frac{1}{3}x) \xrightarrow{\text{translatie } (4; -1,5)} f(x) = 3 \sin(\frac{1}{3}(x - 4)) - 1,5.$

40a $y = \cos(x) \xrightarrow{\text{transl. } (\frac{1}{4}\pi, 4)} y = \cos(x - \frac{1}{4}\pi) + 4 \xrightarrow{\text{verm. } x\text{-as, } 3} f(x) = 3(\cos(x - \frac{1}{4}\pi) + 4) = 3 \cos(x - \frac{1}{4}\pi) + 12.$

40b $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3 \cos(x) \xrightarrow{\text{translatie } (\frac{1}{4}\pi; 4)} g(x) = 3 \cos(x - \frac{1}{4}\pi) + 4.$

41a $y = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{4}\pi, -\frac{1}{2})} f(x) = \sin(x - \frac{1}{4}\pi) - \frac{1}{2}.$
Maak een schets van de plot van f hiernaast.



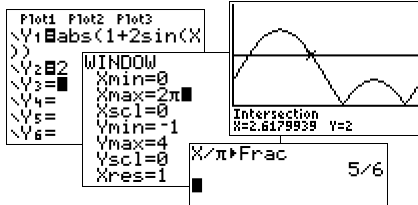
41b Beginpunt (0, 0) (waar $y = \sin(x)$ stijgend door de evenwichtswaarde gaat) ligt door de translatie in $(\frac{1}{4}\pi, -\frac{1}{2})$; periode blijft 2π , dus $f(x) = -\frac{1}{2} + \sin(x - \frac{1}{4}\pi)$ snijdt evenwichtswaarde in $(\frac{1}{4}\pi, -\frac{1}{2})$, $(\frac{5}{4}\pi, -\frac{1}{2})$ en $(\frac{9}{4}\pi, -\frac{1}{2})$.

41c De toppen (een kwart periode rechts van de snijpunten met de evenwichtswaarde) zijn: $(\frac{3}{4}\pi, \frac{1}{2})$, $(\frac{7}{4}\pi, \frac{1}{2})$ en $(\frac{11}{4}\pi, \frac{1}{2})$.

41d $f(x) = -\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = 0$
 $\sin(x - \frac{1}{4}\pi) = \frac{1}{2}$
 $x - \frac{1}{4}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $x - \frac{1}{4}\pi = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{12}\pi + k \cdot 2\pi$ of $x = \frac{13}{12}\pi + k \cdot 2\pi$.
 Op $[0, 3\pi]$ geeft: $x = \frac{5}{12}\pi$ of $x = \frac{13}{12}\pi$ of $x = \frac{25}{12}\pi$ of $x = \frac{37}{12}\pi$.
 Dus $x_A = \frac{5}{12}\pi$, $x_B = \frac{13}{12}\pi$ en $x_C = \frac{25}{12}\pi$.
 $AB = x_B - x_A = \frac{13}{12}\pi - \frac{5}{12}\pi = \frac{8}{12}\pi$.

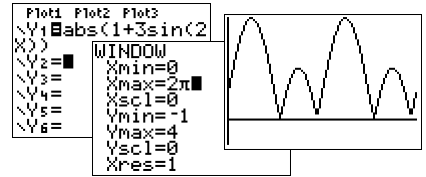
41e $f(x) = -\frac{1}{2} + \sin(x - \frac{1}{4}\pi) = -1$
 $\sin(x - \frac{1}{4}\pi) = -\frac{1}{2}$
 $x - \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x - \frac{1}{4}\pi = \frac{7}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{12}\pi + k \cdot 2\pi$ of $x = \frac{17}{12}\pi + k \cdot 2\pi$.
 Op $[0, 3\pi]$ geeft: $x = \frac{1}{12}\pi$ of $x = \frac{17}{12}\pi$ of $x = \frac{37}{12}\pi$.
 Gebruik vervolgens de plot om daarin af te lezen:
 $f(x) \geq -1$ voor $\frac{1}{12}\pi \leq x \leq \frac{5}{12}\pi$ of $\frac{17}{12}\pi \leq x \leq \frac{37}{12}\pi$.

42 $f(x) = |1 + 2\sin(x)| = 2$
 $1 + 2\sin(x) = 2$ of $1 + 2\sin(x) = -2$
 $2\sin(x) = 1$ of $2\sin(x) = -3$
 $\sin(x) = \frac{1}{2}$ of $\sin(x) = -\frac{3}{2}$ (kan niet)
 $x = \frac{1}{6}\pi + k \cdot 2\pi$ of $x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{6}\pi + k \cdot 2\pi$ of $x = \frac{5}{6}\pi + k \cdot 2\pi$.
 Op $[0, 2\pi]$ geeft: $x = \frac{1}{6}\pi$ of $x = \frac{5}{6}\pi$.
 Gebruik nu een plot om af te lezen:
 $f(x) \geq 2$ voor $\frac{1}{6}\pi \leq x \leq \frac{5}{6}\pi$.

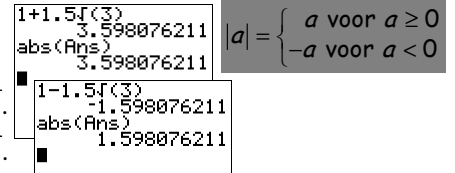


43a Maak een schets van de plot hiernaast. (de grafiek komt 4 keer aan de x-as)

43b $y = \sin(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = \sin(2x)$.
 De toppen van $y = \sin(2x)$ zijn $(\frac{1}{4}\pi, 1)$, $(\frac{3}{4}\pi, 1)$, $(\frac{5}{4}\pi, 1)$ en $(\frac{7}{4}\pi, 1)$.
 De toppen van $f(x)$ zijn $(\frac{1}{4}\pi, 4)$, $(\frac{3}{4}\pi, 4)$, $(\frac{5}{4}\pi, 4)$ en $(\frac{7}{4}\pi, 4)$.

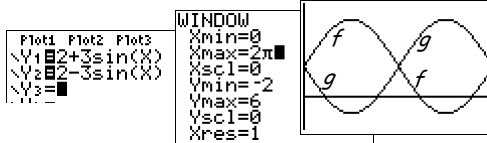


43c $f(\frac{1}{6}\pi) = |1 + 3\sin(2 \cdot \frac{1}{6}\pi)| = |1 + 3\sin(\frac{1}{3}\pi)| = |1 + 3 \cdot \frac{1}{2}\sqrt{3}| = |1 + \frac{3}{2}\sqrt{3}| = 1 + \frac{3}{2}\sqrt{3}$.
 $f(\frac{1}{3}\pi) = |1 + 3\sin(2 \cdot \frac{1}{3}\pi)| = |1 + 3\sin(\frac{2}{3}\pi)| = |1 + 3 \cdot \frac{1}{2}\sqrt{3}| = |1 + \frac{3}{2}\sqrt{3}| = 1 + \frac{3}{2}\sqrt{3}$.
 $f(\frac{2}{3}\pi) = |1 + 3\sin(2 \cdot \frac{2}{3}\pi)| = |1 + 3\sin(\frac{4}{3}\pi)| = |1 + 3 \cdot (-\frac{1}{2}\sqrt{3})| = |1 - \frac{3}{2}\sqrt{3}| = -1 + \frac{3}{2}\sqrt{3}$.
 $f(\frac{5}{6}\pi) = |1 + 3\sin(2 \cdot \frac{5}{6}\pi)| = |1 + 3\sin(\frac{5}{3}\pi)| = |1 + 3 \cdot (-\frac{1}{2}\sqrt{3})| = |1 - \frac{3}{2}\sqrt{3}| = -1 + \frac{3}{2}\sqrt{3}$.



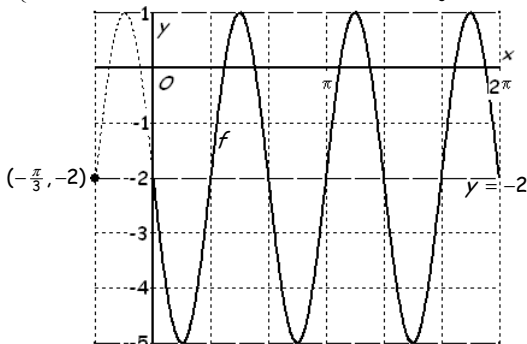
44a Zie de plot hiernaast.

44b De amplitude is bij beide grafieken 3.



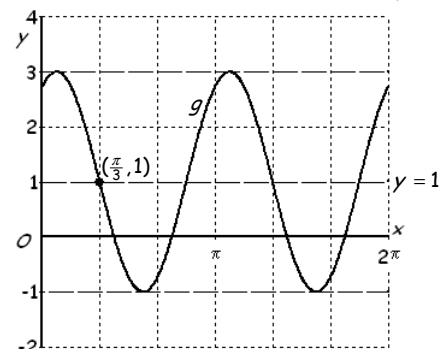
45a $f(x) = -2 + 3\sin(3x + \pi) = -2 + 3\sin(3(x + \frac{1}{3}\pi))$.

evenwichtsstand -2
 amplitude 3
 periode $\frac{2\pi}{3} = \frac{2}{3}\pi$
 $3 > 0 \Rightarrow$ stijgend door evenwichtsstand in $(-\frac{1}{3}\pi, -2)$



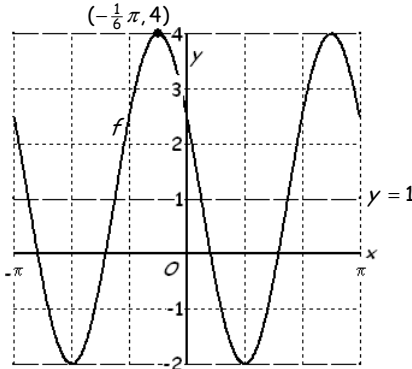
45b $g(x) = 1 - 2\sin(2x - \frac{2}{3}\pi) = 1 - 2\sin(2(x - \frac{1}{3}\pi))$.

evenwichtsstand 1
 amplitude 2
 periode $\frac{2\pi}{2} = \pi$
 $-2 < 0 \Rightarrow$ dalend door evenwichtsstand in $(\frac{1}{3}\pi, 1)$



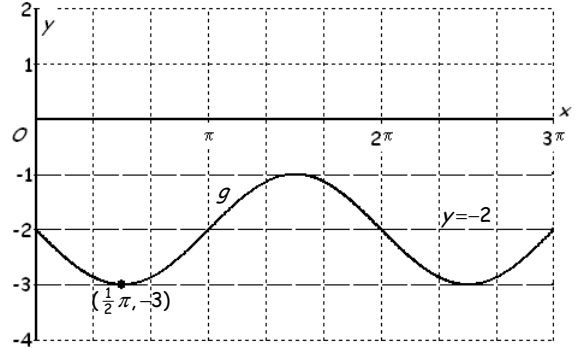
46a $f(x) = 1 + 3 \cos(2x + \frac{1}{3}\pi) = 1 + 3 \cos(2(x + \frac{1}{6}\pi))$.

evenwichtsstand 1
amplitude 3
periode $\frac{2\pi}{2} = \pi$
 $3 > 0 \Rightarrow$ beginpunt $(-\frac{1}{6}\pi, 4)$ is hoogste punt



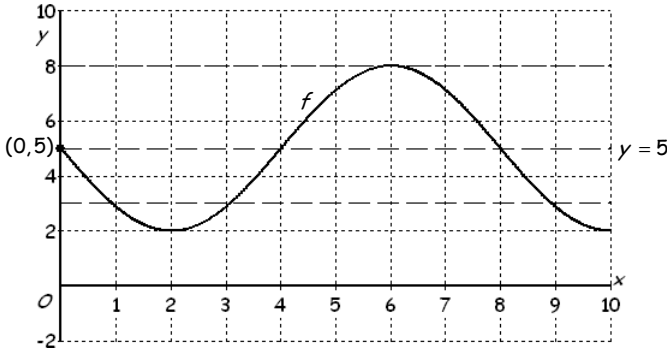
46b $g(x) = -2 - \cos(x - \frac{1}{2}\pi)$.

evenwichtsstand -2
amplitude 1
periode $\frac{2\pi}{1} = 2\pi$
 $-1 < 0 \Rightarrow$ beginpunt $(\frac{1}{2}\pi, -3)$ is laagste punt



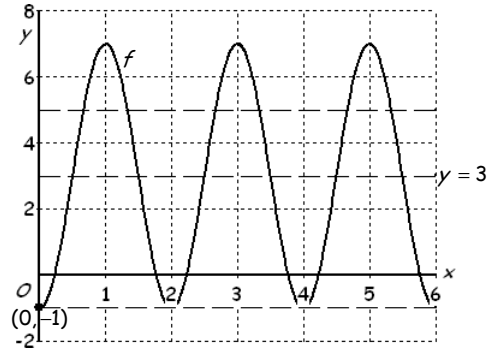
47 $f(x) = 5 - 3 \sin(\frac{1}{4}\pi x)$.

evenwichtsstand 5
amplitude 3
periode $\frac{2\pi}{\frac{1}{4}\pi} = 8$
 $-3 < 0 \Rightarrow$ dalend door evenwichtsstand in $(0, 5)$



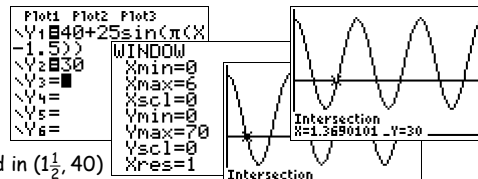
48 $f(x) = 3 - 4 \cos(\pi x)$.

evenwichtsstand 3
amplitude 4
periode $\frac{2\pi}{\pi} = 2$
 $-4 < 0 \Rightarrow$ beginpunt $(0, -1)$ is laagste punt



49a $A = 40 + 25 \sin(\pi(t - 1\frac{1}{2}))$.

evenwichtsstand 40
amplitude 25
periode $\frac{2\pi}{\pi} = 2$
 $25 > 0 \Rightarrow$ stijgend door evenwichtsstand in $(1\frac{1}{2}, 40)$



49b $A = 30$ (intersect) \Rightarrow (bedenk dat de periode 2 is anders alle snijpunten met de GR zoeken)
 $t \approx 0,63$ of $t \approx 1,37$ of $t \approx 2,63$ of $t \approx 3,37$ of $t \approx 4,63$ of $t \approx 5,37$.

Met gebruik van de plot (of de grafiek) vind je:

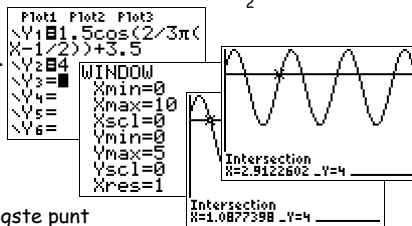
$A < 30$ op $[0, 6]$ voor $0,63 < t < 1,37$ of $2,63 < t < 3,37$ of $4,63 < t < 5,37$.

49c Maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat.

Dus in $(1\frac{1}{2}, 40) \Rightarrow$ de grootste helling is $\left[\frac{dA}{dt}\right]_{t=1\frac{1}{2}} \approx 78,5$.

50a $N = 1\frac{1}{2} \cos(\frac{2}{3}\pi(t - \frac{1}{2})) + 3\frac{1}{2}$.

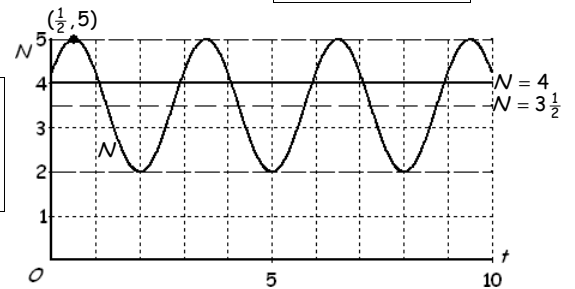
evenwichtsstand $3\frac{1}{2}$
amplitude $1\frac{1}{2}$
periode $\frac{2\pi}{\frac{2}{3}\pi} = 3$
 $1\frac{1}{2} > 0 \Rightarrow$ beginpunt $(\frac{1}{2}, 5)$ is hoogste punt



50b $A = 4$ (intersect) \Rightarrow (bedenk dat de periode 3 is)

$t \approx 1,09$ of $t \approx 2,91$ of $t \approx 4,09$ of $t \approx 5,91$ of $t \approx 7,09$ of $t \approx 8,91$.

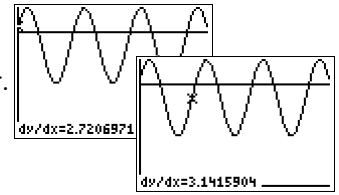
Met behulp van de grafiek vind je: $N > 4$ voor $0 \leq t < 1,09$ of $2,91 < t < 4,09$ of $5,91 < t < 7,09$ of $8,91 < t \leq 10$.



50c De helling in het snijpunt met de verticale as ($t = 0$) is $\left[\frac{dN}{dt}\right]_{t=0} \approx 2,72$.

50d Maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat. Dit is een kwart periode voor het hoogste punt \Rightarrow in $(\frac{1}{2} + \frac{3}{4} \cdot 3, 3\frac{1}{2})$ of in $(\frac{11}{4}, 3\frac{1}{2})$.

De grootste helling is $\left[\frac{dN}{dt}\right]_{t=2\frac{3}{4}} \approx 3,1$.



51a $j(x) = \sin(2x) - \frac{1}{2}$. (de enige met amplitude 1 of evenwichtsstand $-\frac{1}{2}$)

51b $f(x) = 1\frac{1}{2}\sin(2x)$. (de enige met amplitude $1\frac{1}{2}$ én evenwichtsstand 0)

51c $g(x) = 1\frac{1}{2}\sin(x) + 1$. (de enige met evenwichtsstand 1)

51d $h(x) = 2\sin(1\frac{1}{2}x)$. (de enige met amplitude 2)

52a $y = a + b\sin(c(x-d))$ met a (= evenwichtsstand = $\frac{\max+\min}{2}$) = $\frac{50+(-10)}{2} = \frac{40}{2} = 20$; b (= amplitude) = $50 - 20 = 30$;
 c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{50} = \frac{1}{25}\pi$ en $d = 0$ (de sinus gaat stijgend door de evenwichtsstand voor $x = 0$) $\Rightarrow y = 20 + 30\sin(\frac{1}{25}\pi x)$.

52b $y = a + b\sin(c(x-d))$ met $d = 25$ (sinus gaat dalend door de evenwichtsstand voor $x = 25$) $\Rightarrow y = 20 - 30\sin(\frac{1}{25}\pi(x-25))$.

52c $y = a + b\cos(c(x-d))$ met $d = 12\frac{1}{2}$ (de cosinus heeft hoogste punt voor $x = 12\frac{1}{2}$) $\Rightarrow y = 20 + 30\cos(\frac{1}{25}\pi(x-12\frac{1}{2}))$.

52d $y = a + b\cos(c(x-d))$ met $d = 37\frac{1}{2}$ (de cosinus heeft laagste punt voor $x = 37\frac{1}{2}$) $\Rightarrow y = 20 - 30\cos(\frac{1}{25}\pi(x-37\frac{1}{2}))$.

53a $N = a + b\sin(c(t-d))$ met a (= evenwichtsstand = $\frac{\max+\min}{2}$) = $\frac{100+(-220)}{2} = \frac{-120}{2} = -60$; b (= amplitude) = $100 + 60 = 160$;
 c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{6,8} = \frac{5}{17}\pi$ en $d = 4$ (sinus gaat stijgend door de evenwichtsstand voor $t = 4$) $\Rightarrow N = -60 + 160\sin(\frac{5}{17}\pi(t-4))$.

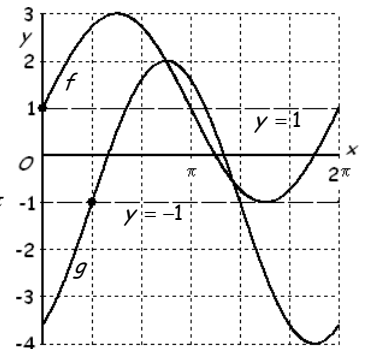
53b $N = a + b\cos(c(t-d))$ met $d = 5,7$ (de cosinus heeft hoogste punt voor $t = 5,7$) $\Rightarrow N = -60 + 160\cos(\frac{5}{17}\pi(t-5,7))$.

54a $f(x) = 1 + 2\sin(x)$.

evenwichtsstand 1
amplitude 2
periode $\frac{2\pi}{1} = 2\pi$
 $2 > 0 \Rightarrow$ stijgend door evenw. stand voor $x = 0$

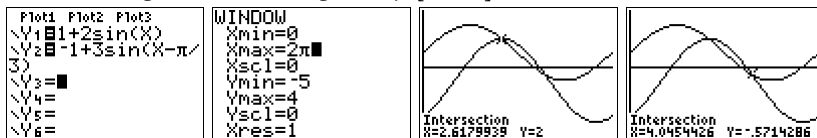
$g(x) = -1 + 3\sin(x - \frac{1}{3}\pi)$

evenwichtsstand -1
amplitude 3
periode $\frac{2\pi}{1} = 2\pi$
 $3 > 0 \Rightarrow$ stijgend door evenw. stand voor $x = \frac{1}{3}\pi$



54b $f(x) = g(x)$ (intersect) $\Rightarrow x \approx 2,62$ of $x \approx 4,05$.

Dan met de grafiek: $f(x) > g(x)$ op $[0, 2\pi]$ voor $0 \leq x < 2,62$ of $4,05 < x \leq 2\pi$.



54c De evenwichtsstand van f is 1 en die van g is $-1 \Rightarrow$ evenwichtsstand van s is $1 + (-1) = 0 \Rightarrow a = 0$.

De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van s is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1$.

54d $s(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).

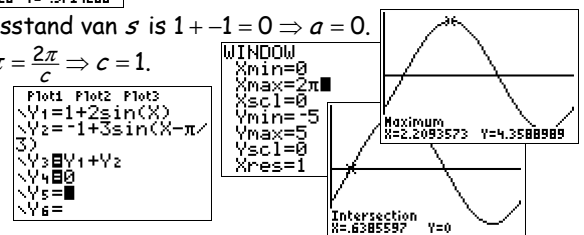
De evenwichtsstand van s is 0 en de periode is 2π .

Dus $s(x) = b\sin(x - d)$ met $b \approx 4,36$ (optie maximum).

(b = amplitude = maximum - evenwichtsstand = $4,36 - 0 = 4,36$)

$s(x) = 0$ (intersect) $\Rightarrow x \approx 0,64 = d$.

(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand) Dus $s(x) = 4,36\sin(x - 0,64)$.



55a De evenwichtsstand van f is -3 en die van g is $-2 \Rightarrow$ evenwichtsstand van s is $-3 + (-2) = -5 \Rightarrow a = -5$.

De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van s is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1 \Rightarrow s(x) = -5 + b\cos(x - d)$.

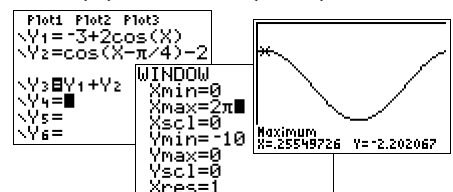
$s(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).

Optie maximum geeft maximum $s(0,26) \approx -2,20 \Rightarrow b \approx 2,80$ en $d \approx 0,26$.

(b = amplitude = maximum - evenwichtsstand = $-2,20 - (-5) = -2,20 + 5 = 2,80$)

(de cosinus heeft, als $b > 0$, maximum bij $x = d$)

Dus $s(x) = -5 + 2,80\cos(x - 0,26)$.



55b De evenwichtsstand van f is -3 en die van g is $-2 \Rightarrow$ evenwichtsstand van v is $-3 - (-2) = -3 + 2 = -1 \Rightarrow a = -1$.
De periode van f en van g zijn beide $2\pi \Rightarrow$ periode van v is $2\pi = \frac{2\pi}{c} \Rightarrow c = 1 \Rightarrow v(x) = -1 + b \sin(x - d)$.

$v(x) = f(x) - g(x)$ (voer deze formule in op de GR; zet f en g uit).

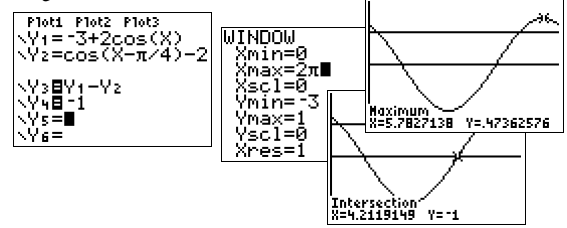
Optie maximum geeft maximum $v(5,78) \approx 0,47 \Rightarrow b \approx 1,47$.

($b =$ amplitude $=$ maximum $-$ evenwichtsstand $= 0,47 - (-1) = 1,47$)

$v(x) = -1$ (intersect) $\Rightarrow x \approx 4,21 = d$.

(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand)

Dus $v(x) = -1 + 1,47 \sin(x - 4,21)$.



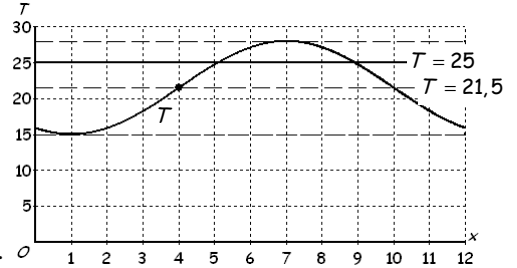
56a $T = 21,5 + 6,5 \sin(\frac{1}{6} \pi(t - 4))$.

evenwichtsstand 21,5

amplitude 6,5

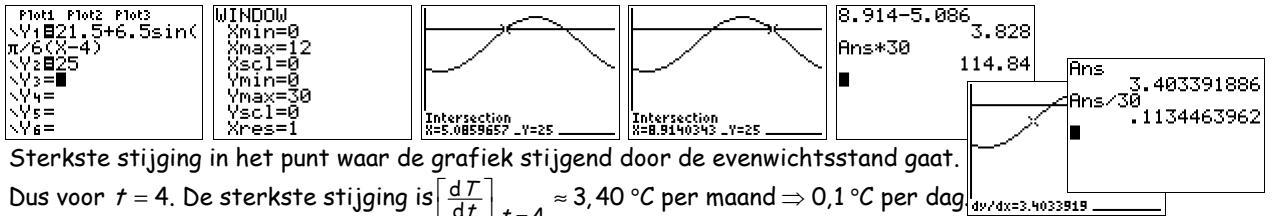
periode $\frac{2\pi}{\frac{1}{6}\pi} = 12$

$6,5 > 0 \Rightarrow$ stijgend door evenwichtsstand voor $t = 4$



56b $T = 21,5 + 6,5 \sin(\frac{1}{6} \pi(t - 4)) = 25$ (intersect) $\Rightarrow x \approx 5,086$ of $x \approx 8,914$.

Dus gedurende $8,914 - 5,086 = 3,828$ maanden $\Rightarrow 3,828 \cdot 30 \approx 115$ dagen.



56c Sterkste stijging in het punt waar de grafiek stijgend door de evenwichtsstand gaat.

Dus voor $t = 4$. De sterkste stijging is $\left[\frac{dT}{dt}\right]_{t=4} \approx 3,40 \text{ }^\circ\text{C per maand} \Rightarrow 0,1 \text{ }^\circ\text{C per dag}$.

56d $a = 17,5$; $b = 17,5 - 15 = 2,5$; $c = \frac{2\pi}{12} = \frac{1}{6}\pi$ en $d = 2 + \frac{1}{4} \cdot 12 = 2 + 3 = 5$

(stijgend door de evenwichtsstand voor $t = d$, een kwart periode na de laagste temperatuur op 1 maart met $t = 2 \Rightarrow d = 2 + 3 = 5$).

Diagnostische toets

D1a $\sin(-270^\circ) = \sin(-270^\circ + 360^\circ) = \sin(90^\circ) = 1.$

D1d $\cos(-120^\circ) = \cos(-180^\circ + 60^\circ) = -\cos 60^\circ = -\frac{1}{2}.$

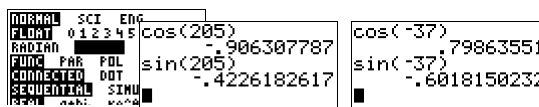
D1b $\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{2}\sqrt{2}.$

D1e $\sin 240^\circ = \sin(180^\circ + 60^\circ) = -\sin 60^\circ = -\frac{1}{2}\sqrt{3}.$

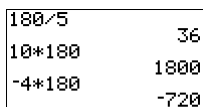
D1c $\cos 225^\circ = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}.$

D1f $\cos 330^\circ = \cos(-30^\circ) = \cos 30^\circ = \frac{1}{2}\sqrt{3}.$

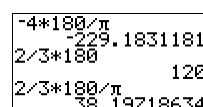
D2 $x_P = \cos 205^\circ \approx -0,91$ en $y_P = \sin 205^\circ \approx -0,42.$
 $x_Q = \cos(-37^\circ) \approx 0,80$ en $y_Q = \sin(-37^\circ) \approx -0,60.$



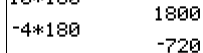
D3a $\frac{1}{5}\pi \text{ rad} = \frac{1}{5} \cdot 180^\circ = 36^\circ.$



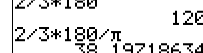
D3d $-4 \text{ rad} = -4 \cdot \frac{180^\circ}{\pi} \approx -229,2^\circ.$



D3b $10\pi \text{ rad} = 10 \cdot 180^\circ = 1800^\circ.$



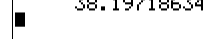
D3e $\frac{2}{3}\pi \text{ rad} = \frac{2}{3} \cdot 180^\circ = 120^\circ.$



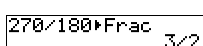
D3c $-4\pi \text{ rad} = -4 \cdot 180^\circ = -720^\circ.$



D3f $\frac{2}{3} \text{ rad} = \frac{2}{3} \cdot \frac{180^\circ}{\pi} \approx 38,2^\circ.$



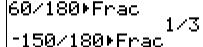
D4a $270^\circ = \frac{270}{180} \cdot \pi \text{ rad} = 1\frac{1}{2}\pi \text{ rad}.$



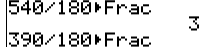
D4d $-135^\circ = \frac{-135}{180} \cdot \pi \text{ rad} = -\frac{3}{4}\pi \text{ rad}.$



D4b $60^\circ = \frac{60}{180} \cdot \pi \text{ rad} = \frac{1}{3}\pi \text{ rad}.$



D4e $540^\circ = \frac{540}{180} \cdot \pi \text{ rad} = 3\pi \text{ rad}.$



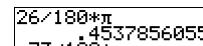
D4c $-150^\circ = \frac{-150}{180} \cdot \pi \text{ rad} = -\frac{5}{6}\pi \text{ rad}.$



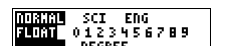
D4f $390^\circ = \frac{390}{180} \cdot \pi \text{ rad} = 2\frac{1}{6}\pi \text{ rad}.$



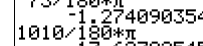
D5a $26^\circ = \frac{26}{180} \cdot \pi \text{ rad} \approx 0,45 \text{ rad}.$



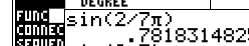
D6a $\sin(\frac{2}{7}\pi) \approx 0,78.$



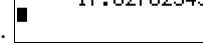
D5b $-73^\circ = \frac{-73}{180} \cdot \pi \text{ rad} \approx -1,27 \text{ rad}.$



D6b $\sin(\frac{2}{7}) \approx 0,28.$



D5c $1010^\circ = \frac{1010}{180} \cdot \pi \text{ rad} \approx 17,63 \text{ rad}.$



D6c $\cos(1\frac{3}{5}\pi) \approx 0,31.$



D7a $\sin(\frac{5}{6}\pi) = \sin(\pi - \frac{1}{6}\pi) = \sin(\frac{1}{6}\pi) = \frac{1}{2}.$

D7b $\cos(\frac{3}{4}\pi) = \cos(\pi - \frac{1}{4}\pi) = -\cos(\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}.$

D7c $\cos(1\frac{1}{3}\pi) = \cos(\pi + \frac{1}{3}\pi) = -\cos(\frac{1}{3}\pi) = -\frac{1}{2}.$

D8a $\sin(\alpha) = \frac{1}{2} = \sin(\frac{1}{6}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = \pi - \frac{1}{6}\pi = \frac{5}{6}\pi.$

D8b $\sin(\alpha) = -\frac{1}{2}\sqrt{2} = -\sin(\frac{1}{4}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \pi + \frac{1}{4}\pi = 1\frac{1}{4}\pi$ of $\alpha = 2\pi - \frac{1}{4}\pi = 1\frac{3}{4}\pi.$

D8c $\cos(\alpha) = \frac{1}{2}\sqrt{3} = \cos(\frac{1}{6}\pi)$ (uit het hoofd weten) $\Rightarrow \alpha = \frac{1}{6}\pi$ of $\alpha = 2\pi - \frac{1}{6}\pi = 1\frac{5}{6}\pi.$

D9a $\sin(2x + \frac{1}{2}\pi) = 0$

$2x + \frac{1}{2}\pi = k \cdot \pi$

$2x = -\frac{1}{2}\pi + k \cdot \pi$

$x = -\frac{1}{4}\pi + k \cdot \frac{1}{2}\pi.$

D9b $\cos(2x + \frac{1}{6}\pi) = 1$

$2x + \frac{1}{6}\pi = k \cdot 2\pi$

$2x = -\frac{1}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{12}\pi + k \cdot \pi.$

D9c $\sin^2(\frac{1}{2}x) - \sin(\frac{1}{2}x) = 0$

$\sin(\frac{1}{2}x) \cdot (\sin(\frac{1}{2}x) - 1) = 0$

$\sin(\frac{1}{2}x) = 0$ of $\sin(\frac{1}{2}x) = 1$

$\frac{1}{2}x = k \cdot \pi$ of $\frac{1}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$

$x = k \cdot 2\pi$ of $x = \pi + k \cdot 4\pi.$

D10a $\sin(\frac{1}{2}x + \pi) = \frac{1}{2}\sqrt{2}$

$\frac{1}{2}x + \pi = \frac{1}{4}\pi + k \cdot 2\pi$ of $\frac{1}{2}x + \pi = \pi - \frac{1}{4}\pi + k \cdot 2\pi$

$\frac{1}{2}x = -\frac{3}{4}\pi + k \cdot 2\pi$ of $\frac{1}{2}x = -\frac{1}{4}\pi + k \cdot 2\pi$

$x = -\frac{3}{2}\pi + k \cdot 4\pi$ of $x = -\frac{1}{2}\pi + k \cdot 4\pi.$

D10c $4\cos^2(\frac{1}{2}\pi x) = 3$

$\cos^2(\frac{1}{2}\pi x) = \frac{3}{4}$

$\cos(\frac{1}{2}\pi x) = \pm\sqrt{\frac{3}{4}} = \pm\sqrt{\frac{1}{4} \cdot 3} = \pm\frac{1}{2}\sqrt{3}$

$\frac{1}{2}\pi x = \pm\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \pm\frac{5}{6}\pi + k \cdot 2\pi$

$x = \pm\frac{1}{3} + k \cdot 4$ of $x = \pm\frac{5}{3} + k \cdot 4$

$x = -\frac{1}{3} + k \cdot 2$ of $x = \frac{1}{3} + k \cdot 2.$

D10b $\cos(-\frac{1}{3}x + \frac{1}{2}\pi) = -\frac{1}{2}.$

$-\frac{1}{3}x + \frac{1}{2}\pi = \frac{2}{3}\pi + k \cdot 2\pi$ of $-\frac{1}{3}x + \frac{1}{2}\pi = -\frac{2}{3}\pi + k \cdot 2\pi$

$-\frac{1}{3}x = \frac{1}{6}\pi + k \cdot 2\pi$ of $-\frac{1}{3}x = -\frac{7}{6}\pi + k \cdot 2\pi$

$x = -\frac{1}{2}\pi + k \cdot 6\pi$ of $x = \frac{7}{2}\pi + k \cdot 6\pi.$

D11a \square $2\sin(2x) = -\sqrt{3}$
 $\sin(2x) = -\frac{1}{2}\sqrt{3}$
 $2x = -\frac{1}{3}\pi + k \cdot 2\pi$ of $2x = \pi - \frac{1}{3}\pi + k \cdot 2\pi$
 $x = -\frac{1}{6}\pi + k \cdot \pi$ of $2x = \frac{4}{3}\pi + k \cdot 2\pi$
 $x = -\frac{1}{6}\pi + k \cdot \pi$ of $x = \frac{2}{3}\pi + k \cdot \pi$
 x op $[0, 2\pi]$ geeft
 $x = \frac{5}{6}\pi$ of $x = 1\frac{5}{6}\pi$ of $x = \frac{2}{3}\pi$ of $x = 1\frac{2}{3}\pi$.

D11c \square $\sin^2(x) - \frac{1}{2}\sin(x) - \frac{1}{2} = 0$
 proberen met: $(\sin(x) - \dots) \cdot (\sin(x) + \dots) = 0$ geeft
 $(\sin(x) - 1) \cdot (\sin(x) + \frac{1}{2}) = 0$
 $\sin(x) = 1$ of $\sin(x) = -\frac{1}{2}$
 $x = \frac{1}{2}\pi + k \cdot 2\pi$ of $x = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x = \pi - \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi$ of $x = -\frac{1}{6}\pi + k \cdot 2\pi$ of $x = 1\frac{1}{6}\pi + k \cdot 2\pi$
 x op $[0, 2\pi]$ geeft $x = \frac{1}{2}\pi$ of $x = 1\frac{5}{6}\pi$ of $x = 1\frac{1}{6}\pi$.

D11b \square $2\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\sqrt{2}$
 $\cos(1\frac{1}{2}x - \frac{1}{6}\pi) = -\frac{1}{2}\sqrt{2}$
 $1\frac{1}{2}x - \frac{1}{6}\pi = \frac{3}{4}\pi + k \cdot 2\pi$ of $1\frac{1}{2}x - \frac{1}{6}\pi = -\frac{3}{4}\pi + k \cdot 2\pi$
 $\frac{3}{2}x = \frac{11}{12}\pi + k \cdot 2\pi$ of $\frac{3}{2}x = -\frac{7}{12}\pi + k \cdot 2\pi$
 $x = \frac{11}{18}\pi + k \cdot \frac{4}{3}\pi$ of $x = -\frac{7}{18}\pi + k \cdot \frac{4}{3}\pi$
 x op $[0, 2\pi]$ geeft $x = \frac{11}{18}\pi$ of $x = \frac{35}{18}\pi$ of $x = \frac{17}{18}\pi$.

D12a \square $\sin(2x - 1) = \sin(x + 2)$
 $2x - 1 = x + 2 + k \cdot 2\pi$ of $2x - 1 = \pi - (x + 2) + k \cdot 2\pi$
 $x = 3 + k \cdot 2\pi$ of $2x - 1 = \pi - x - 2 + k \cdot 2\pi$
 $x = 3 + k \cdot 2\pi$ of $3x = \pi - 1 + k \cdot 2\pi$
 $x = 3 + k \cdot 2\pi$ of $x = \frac{1}{3}\pi - \frac{1}{3} + k \cdot \frac{2}{3}\pi$.

D12c \square $\sin(\frac{1}{2}\pi x) = \sin(\pi(x + 1))$
 $\frac{1}{2}\pi x = \pi(x + 1) + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \pi - \pi(x + 1) + k \cdot 2\pi$
 $\frac{1}{2}\pi x = \pi x + \pi + k \cdot 2\pi$ of $\frac{1}{2}\pi x = \pi - \pi x - \pi + k \cdot 2\pi$
 $-\frac{1}{2}\pi x = \pi + k \cdot 2\pi$ of $1\frac{1}{2}\pi x = k \cdot 2\pi$
 $x = -2 + k \cdot 4$ of $x = k \cdot \frac{4}{3}$.

D12b \square $\cos(x + \frac{1}{3}\pi) = \cos(2x - \frac{1}{2}\pi)$
 $x + \frac{1}{3}\pi = 2x - \frac{1}{2}\pi + k \cdot 2\pi$ of $x + \frac{1}{3}\pi = -(2x - \frac{1}{2}\pi) + k \cdot 2\pi$
 $-x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $x + \frac{1}{3}\pi = -2x + \frac{1}{2}\pi + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $3x = \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{5}{6}\pi + k \cdot 2\pi$ of $x = \frac{1}{18}\pi + k \cdot \frac{2}{3}\pi$.

D13a \square $y = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi, 0)} y = \sin(x - \frac{1}{2}\pi) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} y = \sin(3x - \frac{1}{2}\pi)$ (moet in deze volgorde)
 $y = \sin(3x - \frac{1}{2}\pi) \xrightarrow{\text{verm. } x\text{-as, } 2} f(x) = 2\sin(3x - \frac{1}{2}\pi)$. (deze vermenigvuldiging mag ook eerder staan)

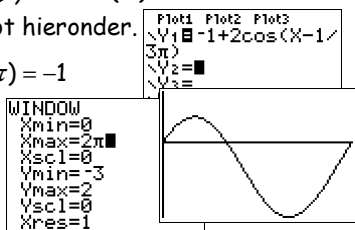
D13b \square $y = \cos(x) \xrightarrow{\text{verm. } y\text{-as, } 3} y = \cos(\frac{1}{3}x) \xrightarrow{\text{translatie } (-2, 5)} g(x) = \cos(\frac{1}{3}(x + 2)) + 5$. (moet in deze volgorde)

D13c \square $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2\sin(x) \xrightarrow{\text{translatie } (\frac{1}{4}\pi, 1)} y = 2\sin(x - \frac{1}{4}\pi) + 1$
 $y = 2\sin(x - \frac{1}{4}\pi) + 1 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{3}} h(x) = 2\sin(3x - \frac{1}{4}\pi) + 1$ (neem je beide translaties samen dan in deze volgorde)

D14 \square $y = \sin(x) \xrightarrow{\text{translatie } (\frac{1}{2}\pi, 3)} y = \sin(x - \frac{1}{2}\pi) + 3 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{5}} f(x) = \sin(5x - \frac{1}{2}\pi) + 3$.

D15a \square $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 2} y = 2\cos(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, -1)} f(x) = 2\cos(x - \frac{1}{3}\pi) - 1$. (in deze volgorde)
 Maak een schets van de plot hieronder.

D15b \square $f(x) = y = -1 + 2\cos(x - \frac{1}{3}\pi) = -1$
 $2\cos(x - \frac{1}{3}\pi) = 0$
 $\cos(x - \frac{1}{3}\pi) = 0$
 $x - \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$
 $x = \frac{5}{6}\pi + k \cdot \pi$
 x op $[0, 2\pi]$ geeft $x = \frac{5}{6}\pi$ of $x = 1\frac{5}{6}\pi$.

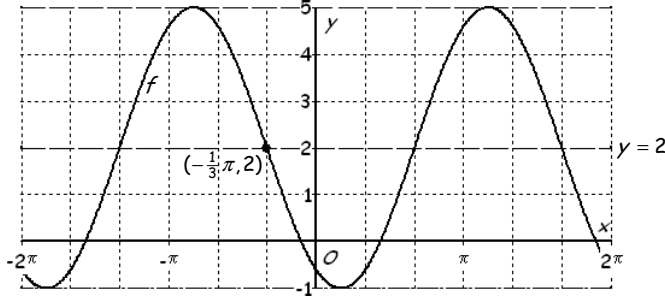


D15d \square $f(x) = y = -1 + 2\cos(x - \frac{1}{3}\pi) = 0$
 $2\cos(x - \frac{1}{3}\pi) = 1$
 $\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$
 $x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi$ of $x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$
 $x = \frac{2}{3}\pi + k \cdot 2\pi$ of $x = k \cdot 2\pi$
 x op $[0, 2\pi]$ geeft $x = \frac{2}{3}\pi$ of $x = 0$ of $x = 2\pi$.

D15c \square $y = 2\cos x$ heeft toppen $(0, 2)$, $(\pi, -2)$ en $(2\pi, 2)$.
 Dus de toppen van f zijn: $(\frac{1}{3}\pi, 1)$ en $(1\frac{1}{3}\pi, -3)$.

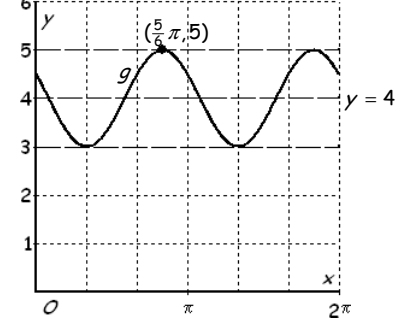
D16a $f(x) = 2 - 3 \sin(x + \frac{1}{3}\pi)$.

evenwichtsstand 2
amplitude 3
periode $\frac{2\pi}{1} = 2\pi$
 $-3 < 0 \Rightarrow$ dalend door evenwichtsstand in $(-\frac{1}{3}\pi, 2)$



D16b $g(x) = 4 + \cos(2x - 1\frac{2}{3}\pi) = 4 + \cos(2(x - \frac{5}{6}\pi))$

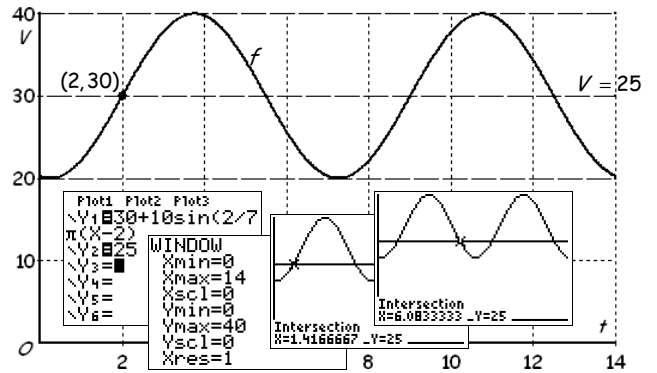
evenwichtsstand 4
amplitude 1
periode $\frac{2\pi}{2} = \pi$
 $1 > 0 \Rightarrow$ beginpunt $(\frac{5}{6}\pi, 5)$ is hoogste punt



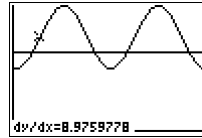
D17a $f(x) = 30 + 10 \sin(\frac{2}{7}\pi(t - 2))$.

evenwichtsstand 30
amplitude 10
periode $\frac{2\pi}{\frac{2}{7}\pi} = 7$
 $10 > 0 \Rightarrow$ stijgend door evenwichtsstand in $(2, 30)$

D17b $V = 25$ (intersect) \Rightarrow (bedenk dat de periode 7)
 $t \approx 1,42$ of $t \approx 6,08$ of $t \approx 8,42$ of $t \approx 13,08$.
Met gebruik van de plot (of de grafiek) vind je:
 $V > 25$ op $[0, 14]$ voor $1,42 < t < 6,08$ of $8,42 < t < 13,08$.



D17c De grootste helling is $[\frac{dV}{dt}]_{t=2} \approx 8,98$.
(maximale helling in een punt waar de grafiek stijgend door de evenwichtsstand gaat)

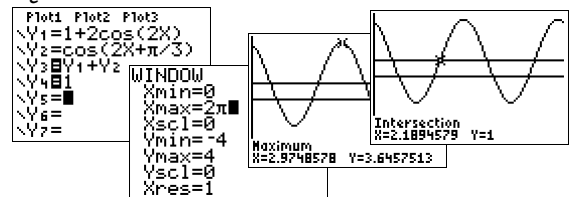


D18a $N = a + b \sin(c(t - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{65 + (-35)}{2} = \frac{30}{2} = 15$; b (= -amplitude) = $-(65 - 15) = -50$;
 c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{30} = \frac{1}{15}\pi$ en $d = 25$ (sinus gaat dalend door de evenwichtsstand voor $t = 25$) $\Rightarrow N = 15 - 50 \sin(\frac{1}{15}\pi(t - 25))$.

D18b $N = a + b \cos(c(t - d))$ met $a = 15$; b (= amplitude) = 50 ;
 $c = \frac{1}{15}\pi$ en $d = 17,5$ (cosinus heeft maximum voor $t = \frac{10 + 25}{2} = \frac{35}{2} = 17,5$) $\Rightarrow N = 15 + 50 \sin(\frac{1}{15}\pi(t - 17,5))$.

D19 De evenwichtsstand van f is 1 en die van g is 0 \Rightarrow evenwichtsstand van h is $1 + 0 = 1 \Rightarrow a = 1$.
De periode van f en van g zijn beide $\pi \Rightarrow$ periode van h is $\pi = \frac{2\pi}{c} \Rightarrow c = 2 \Rightarrow h(x) = 1 + b \sin(2(x - d))$.

$h(x) = f(x) + g(x)$ (voer deze formule in op de GR; zet f en g uit).
Optie maximum geeft maximum $h(2,97) \approx 3,65 \Rightarrow b \approx 2,65$.
(b = amplitude = maximum - evenwichtsstand = $3,65 - 1 = 2,65$)
 $h(x) = 1$ (intersect) $\Rightarrow x \approx 2,19 = d$.
(de sinus gaat, als $b > 0$, voor $x = d$ stijgend door de evenwichtsstand)
Dus $h(x) = 1 + 2,65 \sin(2(x - 2,19))$.



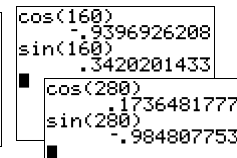
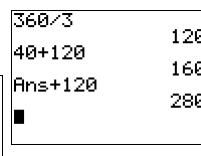
Gemengde opgaven 6. Goniometrische formules

G13 \square $\angle AOB = \angle BOC = \angle COA = 120^\circ$ (alle drie even groot en samen een volle hoek).

$x_A = \cos 40^\circ \approx 0,766$ en $y_A = \sin 40^\circ \approx 0,643$.

$x_B = \cos 160^\circ \approx -0,940$ en $y_B = \sin 160^\circ \approx 0,342$.

$x_C = \cos 280^\circ \approx 0,174$ en $y_C = \sin 280^\circ \approx -0,985$.



G14a \square $x_A = \cos(\alpha) = \cos(\frac{2}{3}\pi) = \cos(\pi - \frac{1}{3}\pi) = -\cos(\frac{1}{3}\pi) = -\frac{1}{2}$ en $y_A = \sin(\alpha) = \sin(\frac{2}{3}\pi) = \sin(\pi - \frac{1}{3}\pi) = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$.

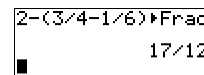
G14b \square $x_C = \cos(\gamma) = \cos(-\frac{1}{6}\pi) = \cos(\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$ en $y_C = \sin(\gamma) = \sin(-\frac{1}{6}\pi) = -\sin(\frac{1}{6}\pi) = -\frac{1}{2}$.

G14c \square $x_B = \cos(\beta) = -\frac{1}{2}\sqrt{2} = -\cos(\frac{1}{4}\pi)$ (en ligt in het 3^e kwart = kwadrant III) $\Rightarrow \beta = -\pi + \frac{1}{4}\pi = -\frac{3}{4}\pi$.

G14d \square De langste cirkelboog BC is de boog van C via A naar B.

De lengte van deze boog = de omtrek van de cirkel - de lengte van de kortste boog BC

$= 2\pi - (\frac{3}{4}\pi - \frac{1}{6}\pi) = 2\pi - (\frac{9}{12}\pi - \frac{2}{12}\pi) = 2\pi - \frac{7}{12}\pi = 1\frac{5}{12}\pi$.



G15a \square In 12 seconden één rondgang \Rightarrow in 12 seconden een hoek van 2π rad.

In 2 seconden een hoek van $\frac{2}{12} \cdot 2\pi \text{ rad} = \frac{1}{3}\pi \text{ rad} \Rightarrow x_p = \cos(\frac{1}{3}\pi) = \frac{1}{2}$ en $y_p = \sin(\frac{1}{3}\pi) = \frac{1}{2}\sqrt{3}$.

In 7,5 seconden een hoek van $\frac{7,5}{12} \cdot 2\pi \text{ rad} = 1\frac{1}{4}\pi \text{ rad} \Rightarrow x_p = \cos(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$ en $y_p = \sin(1\frac{1}{4}\pi) = -\frac{1}{2}\sqrt{2}$.

In 11 seconden een hoek van $\frac{11}{12} \cdot 2\pi \text{ rad} = \frac{11}{6}\pi \text{ rad} \Rightarrow x_p = \cos(\frac{11}{6}\pi) = \cos(-\frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$ en $y_p = \sin(-\frac{1}{6}\pi) = -\frac{1}{2}$.

G15b \square $x_p = -\frac{1}{2} = \cos(\frac{2}{3}\pi) \Rightarrow$ draaiingshoek $\frac{2}{3}\pi$ (in kwadrant II) of $2\pi - \frac{2}{3}\pi = \frac{4}{3}\pi$ (in kwadrant III).

Dit is na $\frac{\frac{2}{3}\pi}{2\pi} = \frac{1}{3}$ rondgang of na $\frac{\frac{4}{3}\pi}{2\pi} = \frac{2}{3}$ rondgang. Dus na $\frac{1}{3} \cdot 12 = 4$ seconden of na $\frac{2}{3} \cdot 12 = 8$ seconden.

G16a \square $\cos(3x - \frac{1}{2}\pi) = \frac{1}{2}\sqrt{2}$

$3x - \frac{1}{2}\pi = \frac{1}{4}\pi + k \cdot 2\pi$ of $3x - \frac{1}{2}\pi = -\frac{1}{4}\pi + k \cdot 2\pi$

$3x = \frac{3}{4}\pi + k \cdot 2\pi$ of $3x = \frac{1}{4}\pi + k \cdot 2\pi$

$x = \frac{1}{4}\pi + k \cdot \frac{2}{3}\pi$ of $x = \frac{1}{12}\pi + k \cdot \frac{2}{3}\pi$.

G16d \square $4\cos^2(2\pi x - \frac{1}{2}\pi) = 3$

$\cos^2(2\pi x - \frac{1}{2}\pi) = \frac{3}{4}$

$\cos(2\pi x - \frac{1}{2}\pi) = \pm\sqrt{\frac{3}{4}} = \pm\sqrt{\frac{1}{4} \cdot 3} = \pm\frac{1}{2}\sqrt{3}$

$2\pi x - \frac{1}{2}\pi = \frac{1}{6}\pi + k \cdot 2\pi$ of $2\pi x - \frac{1}{2}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$

of $2\pi x - \frac{1}{2}\pi = \frac{5}{6}\pi + k \cdot 2\pi$ of $2\pi x - \frac{1}{2}\pi = -\frac{5}{6}\pi + k \cdot 2\pi$

$2\pi x = \frac{2}{3}\pi + k \cdot 2\pi$ of $2\pi x = \frac{1}{3}\pi + k \cdot 2\pi$

of $2\pi x = \frac{4}{3}\pi + k \cdot 2\pi$ of $2\pi x = -\frac{1}{3}\pi + k \cdot 2\pi$

$x = \frac{1}{3} + k \cdot 1$ of $x = \frac{1}{6} + k \cdot 1$ of $x = \frac{2}{3} + k \cdot 1$ of $x = -\frac{1}{6} + k \cdot 1$

$x = \frac{1}{3} + k \cdot \frac{1}{2}$ of $x = \frac{1}{6} + k \cdot \frac{1}{2}$.

G16b \square $\sin(\frac{1}{3}x + \frac{1}{4}\pi) = -\frac{1}{2}$

$\frac{1}{3}x + \frac{1}{4}\pi = -\frac{1}{6}\pi + k \cdot 2\pi$ of $\frac{1}{3}x + \frac{1}{4}\pi = \pi - \frac{1}{6}\pi + k \cdot 2\pi$

$\frac{1}{3}x = -\frac{5}{12}\pi + k \cdot 2\pi$ of $\frac{1}{3}x = \frac{11}{12}\pi + k \cdot 2\pi$

$x = -\frac{5}{4}\pi + k \cdot 6\pi$ of $x = \frac{11}{4}\pi + k \cdot 6\pi$.

G16c \square $\sin(\frac{1}{2}x - \frac{1}{3}\pi) \cdot \cos(2x) = 0$

$\sin(\frac{1}{2}x - \frac{1}{3}\pi) = 0$ of $\cos(2x) = 0$

$\frac{1}{2}x - \frac{1}{3}\pi = k \cdot \pi$ of $2x = \frac{1}{2}\pi + k \cdot \pi$

$\frac{1}{2}x = \frac{1}{3}\pi + k \cdot \pi$ of $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$

$x = \frac{2}{3}\pi + k \cdot 2\pi$ of $x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi$.

G17a \square $\cos(2x - \frac{1}{2}\pi) = \cos(\pi - x)$

$2x - \frac{1}{2}\pi = \pi - x + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = -(\pi - x) + k \cdot 2\pi$

$3x = 1\frac{1}{2}\pi + k \cdot 2\pi$ of $2x - \frac{1}{2}\pi = -\pi + x + k \cdot 2\pi$

$x = \frac{1}{2}\pi + k \cdot \frac{2}{3}\pi$ of $x = -\frac{1}{2}\pi + k \cdot 2\pi$.

G17c \square $\sin(\pi x) = \sin(2\pi x)$

$\pi x = 2\pi x + k \cdot 2\pi$ of $\pi x = \pi - 2\pi x + k \cdot 2\pi$

$-\pi x = k \cdot 2\pi$ of $3\pi x = \pi + k \cdot 2\pi$

$x = k \cdot 2$ of $x = \frac{1}{3} + k \cdot \frac{2}{3}$.

G17b \square $\sin(2x + \frac{1}{3}\pi) = \sin(x - \frac{1}{2}\pi)$

$2x + \frac{1}{3}\pi = x - \frac{1}{2}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = \pi - (x - \frac{1}{2}\pi) + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = \pi - x + \frac{1}{2}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $3x = \frac{7}{6}\pi + k \cdot 2\pi$

$x = -\frac{5}{6}\pi + k \cdot 2\pi$ of $x = \frac{7}{18}\pi + k \cdot \frac{2}{3}\pi$.

G17d \square $\cos(10\pi x) = \cos(5\pi x - 6\pi)$

$10\pi x = 5\pi x - 6\pi + k \cdot 2\pi$ of $10\pi x = -(5\pi x - 6\pi) + k \cdot 2\pi$

$5\pi x = -6\pi + k \cdot 2\pi$ of $10\pi x = -5\pi x + 6\pi + k \cdot 2\pi$

$x = -\frac{6}{5} + k \cdot \frac{2}{5}$ of $15\pi x = 6\pi + k \cdot 2\pi$

$x = -\frac{6}{5} + k \cdot \frac{2}{5}$ of $x = \frac{2}{5} + k \cdot \frac{2}{15}$

$x = k \cdot \frac{2}{5}$ of $x = k \cdot \frac{2}{15}$

$x = k \cdot \frac{2}{15}$.

G18a \square $\sin(1\frac{1}{2}x - \frac{1}{6}\pi) = \frac{1}{2}\sqrt{3}$
 $1\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{3}\pi + k \cdot 2\pi$ of $1\frac{1}{2}x - \frac{1}{6}\pi = \pi - \frac{1}{3}\pi + k \cdot 2\pi$
 $\frac{3}{2}x = \frac{1}{2}\pi + k \cdot 2\pi$ of $\frac{3}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{3}\pi + k \cdot \frac{4}{3}\pi$ of $x = \frac{5}{9}\pi + k \cdot \frac{4}{3}\pi$
 x op $[0, 2\pi]$ geeft
 $x = \frac{1}{3}\pi$ of $x = \frac{5}{3}\pi$ of $x = \frac{5}{9}\pi$ of $x = \frac{17}{9}\pi$.

G18c \square $\sin^2(1,5x) = \sin(1,5x) + 2$
 $\sin^2(1,5x) - \sin(1,5x) - 2 = 0$
 proberen $(\sin(1,5x) - \dots) \cdot (\sin(1,5x) + \dots) = 0$ geeft
 $(\sin(1,5x) - 2) \cdot (\sin(1,5x) + 1) = 0$
 $\sin(1,5x) = 1$ (kan niet) of $\sin(1,5x) = -1$
 $\frac{3}{2}x = -\frac{1}{2}\pi + k \cdot 2\pi$
 $x = -\frac{1}{3}\pi + k \cdot \frac{4}{3}\pi$
 x op $[0, 2\pi]$ geeft $x = \pi$.

G18b \square $\cos^3(2\frac{1}{2}x) + \cos(2\frac{1}{2}x) = 0$
 $\cos(2\frac{1}{2}x) \cdot (\cos^2(2\frac{1}{2}x) + 1) = 0$
 $\cos(2\frac{1}{2}x) = 0$ of $\cos^2(2\frac{1}{2}x) = -1$ (kan niet)
 $\frac{5}{2}x = \frac{1}{2}\pi + k \cdot \pi$
 $x = \frac{1}{5}\pi + k \cdot \frac{2}{5}\pi$
 x op $[0, 2\pi]$ geeft:
 $x = \frac{1}{5}\pi$ of $x = \frac{3}{5}\pi$ of $x = \pi$ of $x = \frac{7}{5}\pi$ of $x = \frac{9}{5}\pi$.

G18d \square $\cos(2x + \frac{1}{3}\pi) = \cos(3x - \frac{1}{6}\pi)$
 $2x + \frac{1}{3}\pi = 3x - \frac{1}{6}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = -(3x - \frac{1}{6}\pi) + k \cdot 2\pi$
 $-x = -\frac{1}{2}\pi + k \cdot 2\pi$ of $2x + \frac{1}{3}\pi = -3x + \frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi$ of $5x = -\frac{1}{6}\pi + k \cdot 2\pi$
 $x = \frac{1}{2}\pi + k \cdot 2\pi$ of $x = -\frac{1}{30}\pi + k \cdot \frac{2}{5}\pi$
 x op $[0, 2\pi]$ geeft: $x = \frac{1}{2}\pi$ of $x = \frac{11}{30}\pi$ of $x = \frac{23}{30}\pi$ of
 $x = \frac{35}{30}\pi$ of $x = \frac{47}{30}\pi$ of $x = \frac{59}{30}\pi$.

G19a \square $y = \cos(x) \xrightarrow{\text{translatie } (0, -2)} y = \cos(x) - 2 \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3 \cdot (\cos(x) - 2) = 3\cos(x) - 6$.

G19b \square $y = \cos(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} y = \cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = \cos(2x - \frac{1}{3}\pi)$.

G19c \square $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{translatie } (0, -2)} y = 3\cos(x) - 2$
 $y = 3\cos(x) - 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x) - 2$.

G19d \square $y = \cos(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = \cos(2x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(2x)$
 $y = 3\cos(2x) \xrightarrow{\text{translatie } (0, -2)} y = 3\cos(2x) - 2$.

G19e \square $y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x)$
 $y = 3\cos(2x) \xrightarrow{\text{translatie } (0, -2)} y = 3\cos(2x) - 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2 \cdot 2x) - 2 = 3\cos(4x) - 2$.

G19f \square $y = \cos(x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 0)} y = \cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } x\text{-as, } 3} y = 3\cos(x - \frac{1}{3}\pi)$
 $y = 3\cos(x - \frac{1}{3}\pi) \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} y = 3\cos(2x - \frac{1}{3}\pi) \xrightarrow{\text{translatie } (0, -2)} y = 3\cos(2x - \frac{1}{3}\pi) - 2$.

G20a \square $y = \sin(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{3}{2}]{\text{verm. } x\text{-as, } 3} y = 3\sin(\frac{2}{3}x) \xrightarrow{\text{translatie } (0, 5)} f(x) = 3\sin(\frac{2}{3}x) + 5$.

G20b \square Evenwichtsstand 5, amplitude 3, periode $\frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2} = 3\pi$ en $3 > 0$ dus grafiek stijgend door beginpunt $(0, 5)$.

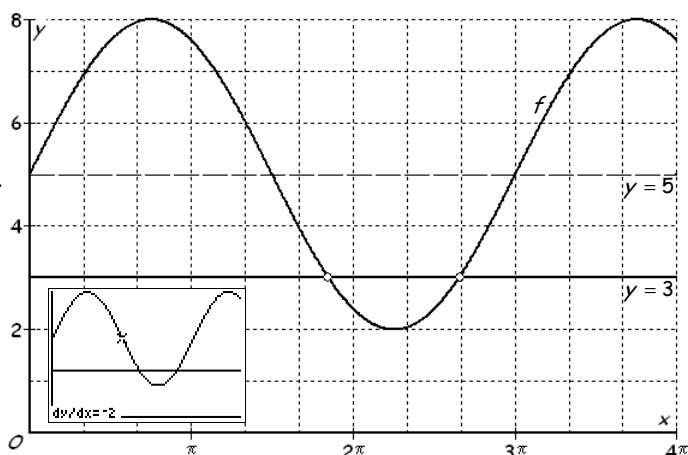
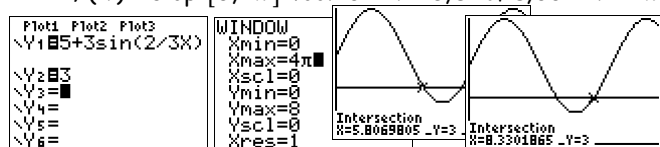
Zie de grafiek hiernaast.

G20c \square $B_f = [3 \cdot -1 + 5, 3 \cdot 1 + 5] = [2, 8]$.

G20d \square $5 + 3\sin(\frac{2}{3}x) = 3$ (intersect) $\Rightarrow x \approx 5,81$ of $x \approx 8,33$.

Met behulp van de grafiek lees je dan af:

$f(x) > 3$ op $[0, 2\pi]$ voor $0 \leq x < 5,81$ of $8,33 < x \leq 2\pi$.

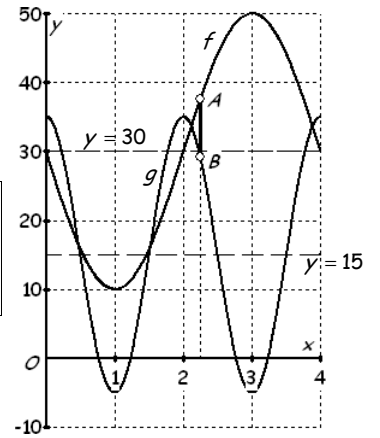
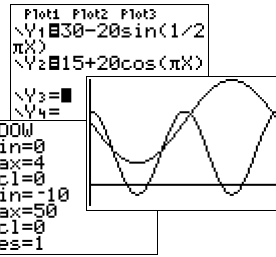


G20e \square Minimale helling (grootste daling) in een punt van f door de evenwichtsstand \Rightarrow in $(1\frac{1}{2}\pi, 5)$.

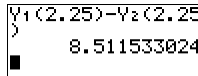
$\left. \frac{dy}{dx} \right|_{x=1\frac{1}{2}\pi} = -2 \Rightarrow$ de kleinste helling is -2 .

G21a \square $y = \sin(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{2}{\pi}]{\text{verm. } x\text{-as, } -20} y = -20\sin(\frac{\pi}{2}x) \xrightarrow{\text{translatie } (0, 30)} f(x) = -20\sin(\frac{\pi}{2}x) + 30$
 $y = \cos(x) \xrightarrow[\text{verm. } y\text{-as, } \frac{1}{\pi}]{\text{verm. } x\text{-as, } 20} y = 20\cos(\pi x) \xrightarrow{\text{translatie } (0, 15)} g(x) = 20\cos(\pi x) + 15$.

- G21b \square f : evenwichtsstand 30, amplitude 20, periode $\frac{2\pi}{\frac{1}{2}\pi} = 4$ en $-20 < 0$ dus dalend door evenwichtsstand in $(0, 30)$.
 g : evenwichtsstand 15, amplitude 20, periode $\frac{2\pi}{\pi} = 2$ en $20 > 0$ dus beginpunt $(0, 35)$ is hoogste punt.
 Zie de grafieken hiernaast.



G21c \square $AB = f(2,25) - g(2,25) \approx 8,51$.



- G21d \square Voor $-5 \leq p < 10$ heeft $g(x) = p$ wel oplossingen en $f(x) = p$ geen oplossingen.

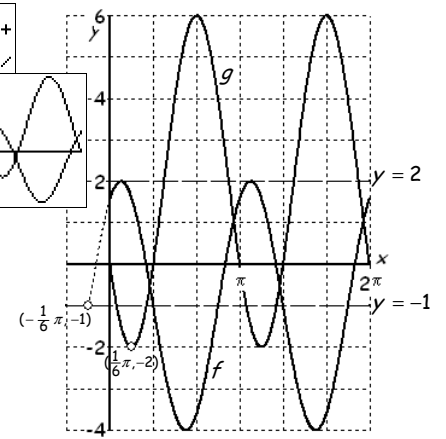
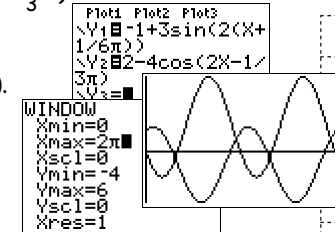
G22a \square $y = \sin(x) \xrightarrow{\text{verm. } x\text{-as, } 3 \text{ verm. } y\text{-as, } \frac{1}{2}} y = 3 \sin(2x) \xrightarrow{\text{translatie } (-\frac{1}{6}\pi, -1)} f(x) = 3 \sin(2(x + \frac{1}{6}\pi)) - 1$.

$y = \cos(x) \xrightarrow{\text{verm. } x\text{-as, } -4} y = -4 \cos(\pi x) \xrightarrow{\text{translatie } (\frac{1}{3}\pi, 2)} y = -4 \cos(x - \frac{1}{3}\pi) + 2$

$y = -4 \cos(x - \frac{1}{3}\pi) + 2 \xrightarrow{\text{verm. } y\text{-as, } \frac{1}{2}} g(x) = -4 \cos(2x - \frac{1}{3}\pi) + 2$.

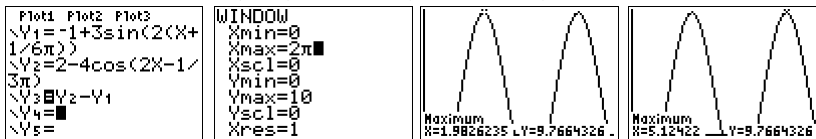
- G22b \square f : evenwichtsstand -1 , amplitude 3, periode $\frac{2\pi}{2} = \pi$ en $3 > 0$ dus stijgend door evenwichtsstand in $(-\frac{1}{6}\pi, -1)$.

- g : evenwichtsstand 2, amplitude 4, periode $\frac{2\pi}{2} = \pi$ en $-4 < 0$ dus beginpunt $(\frac{1}{6}\pi, -2)$ is laagste punt.
 Zie de grafieken hiernaast.



G22c \square $AB = g(a) - f(a) = 2 - 4 \cos(2a - \frac{1}{3}\pi) - (-1 + 3 \sin(2(a + \frac{1}{6}\pi)))$. (met $AB > 0$)

Voer deze formule in op de GR (schakel de andere formules uit door ENTER op =).
 Optie maximum geeft $x \approx 1,98$ met $y \approx 9,77$ en $x \approx 5,12$ met $y \approx 9,77$.
 Dus AB is maximaal (9,77) voor $a \approx 1,98$ of $a \approx 5,12$.



- G22d \square Voer de formule $h(x) = f(x) + g(x)$ in op de GR.

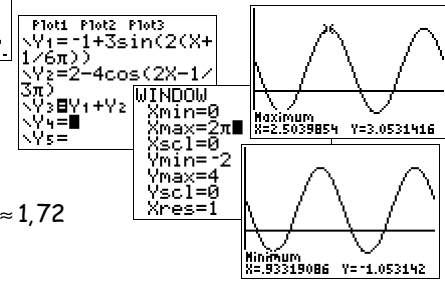
Optie maximum geeft $x \approx 2,504$ met $y \approx 3,053$.
 Optie minimum geeft $x \approx 0,933$ met $y \approx -1,053$.
 De evenwichtsstand is $-1 + 2 = 1$ (of $\frac{3,053 + (-1,053)}{2}$).

De grafiek gaat stijgend door de evenwichtsstand voor $x \approx \frac{2,504 + 0,933}{2} \approx 1,72$

De amplitude is $\frac{3,052 - (-1,053)}{2} \approx 2,05$.

De periode is π (f en g hebben beide periode π) $\Rightarrow c = 2$.

Dus $h(x) = 1 + 2,05 \sin(2(x - 1,72))$.



- G23a \square Zie de grafieken hiernaast.

G23b \square $|1 + 2 \sin(x)| = 1$

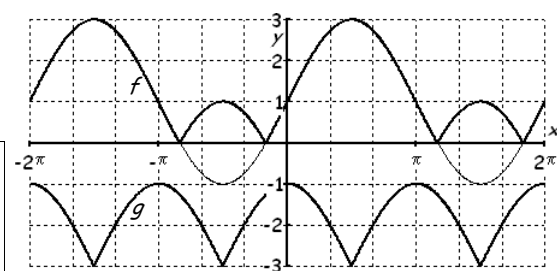
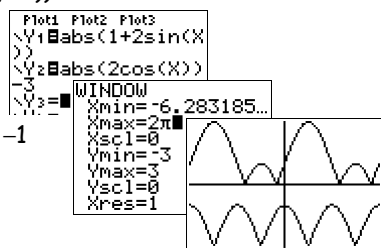
$1 + 2 \sin(x) = 1$ of $1 + 2 \sin(x) = -1$

$2 \sin(x) = 0$ of $2 \sin(x) = -2$

$\sin(x) = 0$ of $\sin(x) = -1$

$x = k \cdot \pi$ of $x = -\frac{1}{2}\pi + k \cdot 2\pi$

x op $[-2\pi, 2\pi]$ geeft: $x = -2\pi$ of $x = -\pi$ of $x = 0$ of $x = \pi$ of $x = 2\pi$ of $x = -\frac{1}{2}\pi$ of $x = \frac{1}{2}\pi$.

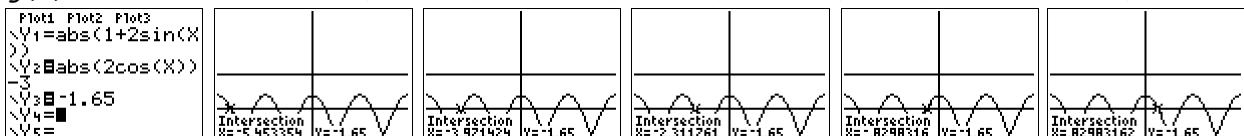


G23c \square $g(x) = -1,65 \Rightarrow |2 \cos(x)| - 3 = -1,65$ (intersect) \Rightarrow

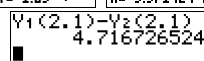
$x \approx -5,45$ of $x \approx -3,97$ of $x \approx -2,31$ of $x \approx -0,83$ of $x \approx 0,83$ of $x \approx 2,31$ of $x \approx 3,97$ of $x \approx 5,45$.

Nu aflezen in de plot (of de grafiek):

$g(x) > -1,65$ voor $-2\pi \leq x < -5,45$ of $-3,97 < x < -2,31$ of $-0,83 < x < 0,83$ of $2,31 < x < 3,97$ of $5,45 < x \leq 2\pi$.



G23d \square $AB = f(2,1) - g(2,1) \approx 4,72$.



G24a \square In de tweede figuur is dat $6500 - 1500 = 5000 \text{ (cm}^3\text{)}$.

G24b \square Lees in de tweede figuur af: $V = 5500$ voor $t = 1\frac{1}{2}$ of $t = 6 \text{ (sec.)}$.

Dus per periode (van 15 sec.) is $6 - 1\frac{1}{2} = 4\frac{1}{2}$ seconde meer dan 5500 cm^3 lucht in de longen.

Dus per minuut (= 60 sec.) is $4 \cdot 4\frac{1}{2} = 18$ seconden meer dan 5500 cm^3 lucht in de longen.

G24c \square In de eerste figuur is de periode 6 seconden, dus er zijn 10 ademhalingen per minuut.

Het *minuutvolume* is $10 \cdot 500 = 5000 \text{ (cm}^3\text{, dus 5 liter)}$.

In de tweede figuur is de periode 15 seconden, dus er zijn 4 ademhalingen per minuut.

Het *minuutvolume* is $4 \cdot 5000 = 20000 \text{ (cm}^3\text{, dus 20 liter)}$.

De verhouding van het *minuutvolume* bij de ademritmes van de eerste en tweede figuur is $5000 : 20000 = 1 : 4$.

G24d \square $V = a + b \sin(c(t - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{4000 + 3500}{2} = 3750$; b (= amplitude) = $4000 - 3750 = 250$;

c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{6} = \frac{1}{3}\pi$ en $d = 0$ (sinus gaat stijgend door de evenwichtsstand voor $t = 0$) $\Rightarrow V = 3750 + 250 \sin(\frac{1}{3}\pi t)$.

G24e \square $V = a + b \cos(c(t - d))$ met a (= evenwichtsstand) = $\frac{6500 + 1500}{2} = 4000$; b (= amplitude) = $6500 - 4000 = 2500$;

c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{15} = \frac{2}{15}\pi$ en $d = \frac{15}{4}$ (cosinus heeft maximum voor $t = \frac{15}{4}$) $\Rightarrow V = 4000 + 2500 \cos(\frac{2}{15}\pi(t - \frac{15}{4}))$.

G24f \square $V = a - b \cos(c(t - d))$ met a (= evenwichtsstand) = 4200 (gegeven); b (= -amplitude) = -2500 ;

(de periode van één ademhaling is $1\frac{1}{2}$ seconde, want er zijn 40 ademhalingen per minuut \Rightarrow)

c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{1,5} = \frac{20}{15}\pi = \frac{4}{3}\pi$ en $d = 0$ (gegeven: cosinus heeft minimum voor $t = 0$) $\Rightarrow V = 4200 - 2500 \cos(\frac{4}{3}\pi t)$.

G25a \square $f(x) = a + b \sin(c(x - d))$ met a (= evenwichtsstand = $\frac{\max + \min}{2}$) = $\frac{3\frac{1}{2} + -1\frac{1}{2}}{2} = \frac{2}{2} = 1$; b (= amplitude) = $3\frac{1}{2} - 1 = 2\frac{1}{2}$;

c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{\frac{3}{4}} = 2 \cdot \frac{3}{4} = 1\frac{1}{2}$ en $d = \frac{1}{3}\pi$ (sinus stijgend door evenwichtsstand voor $x = \frac{1}{3}\pi$) $\Rightarrow f(x) = 1 + 2\frac{1}{2} \sin(1\frac{1}{2}(x - \frac{1}{3}\pi))$.

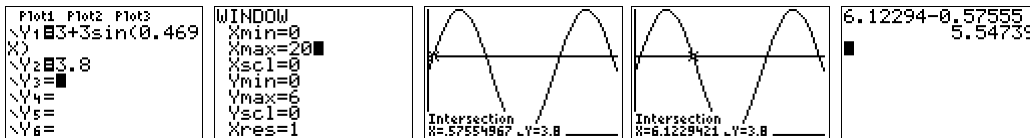
G25b \square $N = a + b \cos(c(t - d))$ met a (= evenwichtsstand) = $\frac{40 + 0}{2} = 20$; b (= amplitude) = $40 - 20 = 20$;

(tussen het maximum en het minimum ligt een halve periode \Rightarrow de periode is $2 \times (7 - 1) = 2 \times 6 = 12 \Rightarrow$)

c (= $\frac{2\pi}{\text{periode}}$) = $\frac{2\pi}{12} = \frac{1}{6}\pi$ en $d = 1$ (cosinus heeft maximum voor $t = 1$) $\Rightarrow N = 20 + 20 \cos(\frac{1}{6}\pi(t - 1))$.

G26a \square $y = 3 + 3 \sin(0,469x) = 3,8$ (met intersect twee snijpunten naast één top) $\Rightarrow x \approx 0,57555$ of $x \approx 6,12294$.

Dus de breedte van het blokje is $6,12294 - 0,57555 \approx 5,5 \text{ (cm)}$.



G26b \square $SQ = \sqrt{55^2 + 67^2} \approx 86,68$.

S ligt even hoog als P ; hetzelfde aantal golven; even hoge toppen; enz.

De nieuwe grafiek is een horizontale uitrekking van G.11 met factor $\frac{86,68}{67}$.

$y = 3 + 3 \sin(0,469x) \xrightarrow{\text{verm. } y\text{-as, } \frac{86,68}{67}} y = 3 + 3 \sin(0,469 \cdot \frac{67}{86,68} x)$

Dus $y = 3 + 3 \sin(0,363x)$.

