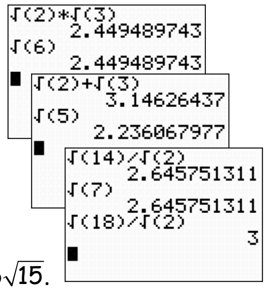
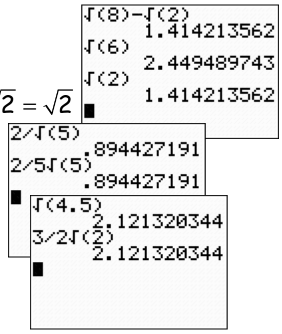
1 I $\sqrt{2}+\sqrt{3}\neq\sqrt{2+3}=\sqrt{5}$
II $\sqrt{2}\cdot\sqrt{3}=\sqrt{2\cdot 3}=\sqrt{6}$
III $\frac{\sqrt{14}}{\sqrt{2}}=\sqrt{\frac{14}{2}}=\sqrt{7}$
IV $\frac{\sqrt{18}}{\sqrt{2}}=\sqrt{\frac{18}{2}}=\sqrt{9}=3$



V $\sqrt{8}-\sqrt{2}\neq\sqrt{8-2}=\sqrt{6}$
VI $\sqrt{8}-\sqrt{2}=\sqrt{4}\cdot\sqrt{2}-\sqrt{2}=2\sqrt{2}-\sqrt{2}=\sqrt{2}$
VII $\frac{2}{\sqrt{5}}=\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{5}=\frac{2}{5}\sqrt{5}$
VIII $\sqrt{4\frac{1}{2}}=\sqrt{\frac{9}{2}}=\frac{\sqrt{9}}{\sqrt{2}}=\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{3}{2}\sqrt{2}.$



2a $2\sqrt{3}\cdot 3\sqrt{5}=2\cdot 3\cdot\sqrt{3}\cdot\sqrt{5}=6\sqrt{15}.$

2b $\frac{5\sqrt{10}}{\sqrt{5}}=\frac{5}{1}\cdot\frac{\sqrt{10}}{\sqrt{5}}=5\sqrt{2}.$

2c $3a\sqrt{2}\cdot a\sqrt{7}=3a\cdot a\cdot\sqrt{2}\cdot\sqrt{7}=3a^2\cdot\sqrt{14}.$

2d $\frac{2\sqrt{14}}{3\sqrt{7}}=\frac{2}{3}\cdot\frac{\sqrt{14}}{\sqrt{7}}=\frac{2}{3}\sqrt{2}.$

2e $\frac{1}{2}a\sqrt{2}\cdot\frac{1}{2}a\sqrt{3}=\frac{1}{2}\cdot\frac{1}{2}\cdot a\cdot a\cdot\sqrt{2}\cdot\sqrt{3}=\frac{1}{4}a^2\cdot\sqrt{6}.$

2f $\frac{6}{5\sqrt{2}}=\frac{6}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{{}^{3}\cancel{6}\sqrt{2}}{5\cdot\cancel{2}}=\frac{3}{5}\sqrt{2}.$

3a $\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}=\frac{1}{3}\sqrt{3}.$

3b $\sqrt{\frac{1}{2}}=\frac{\sqrt{1}}{\sqrt{2}}=\frac{\sqrt{1}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\frac{1}{2}\sqrt{2}.$

3c $\sqrt{4\frac{1}{2}}=\sqrt{\frac{9}{2}}=\frac{\sqrt{9}}{\sqrt{2}}=\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2}=1\frac{1}{2}\sqrt{2}.$

3d $(\frac{1}{2}\sqrt{5})^2=(\frac{1}{2})^2\cdot(\sqrt{5})^2=\frac{1}{4}\cdot 5=\frac{5}{4}=1\frac{1}{4}.$

3e $(\frac{1}{2}a\sqrt{2})^2=(\frac{1}{2}a)^2\cdot(\sqrt{2})^2=\frac{1}{4}a^2\cdot 2=\frac{1}{2}a^2.$

3f $(\frac{2}{3}a\sqrt{3})^2=(\frac{2}{3}a)^2\cdot(\sqrt{3})^2=\frac{4}{9}a^2\cdot 3=\frac{4}{3}a^2=1\frac{1}{3}a^2.$

4a $\sqrt{24}+\sqrt{6}=\sqrt{4\cdot 6}+\sqrt{6}=2\sqrt{6}+\sqrt{6}=3\sqrt{6}.$

4b $\sqrt{80}-\frac{10}{\sqrt{5}}=\sqrt{16\cdot 5}-\frac{10}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=4\sqrt{5}-\frac{10\sqrt{5}}{5}=4\sqrt{5}-2\sqrt{5}=2\sqrt{5}.$

4c $\sqrt{18a}-\sqrt{8a}=\sqrt{9\cdot 2a}-\sqrt{4\cdot 2a}=3\sqrt{2a}-2\sqrt{2a}=\sqrt{2a}.$ (voorwaarde: $a\geq 0$)

4d $\sqrt{3a^2}+\sqrt{12a^2}=\sqrt{a^2\cdot 3}+\sqrt{4\cdot a^2\cdot 3}=|a|\sqrt{3}+2|a|\sqrt{3}=3|a|\sqrt{3}.$

4e $\sqrt{\frac{3}{4}a^2}=\sqrt{\frac{1}{4}\cdot a^2\cdot 3}=\frac{1}{2}|a|\sqrt{3}.$

4f $\sqrt{\frac{7}{9}a^2}=\sqrt{\frac{1}{9}\cdot a^2\cdot 7}=\frac{1}{3}|a|\sqrt{7}.$

5a $a\sqrt{8}-a\sqrt{2}=a\sqrt{4\cdot 2}-a\sqrt{2}=2a\sqrt{2}-a\sqrt{2}=a\sqrt{2}.$

5b $\sqrt{2a^2}+\sqrt{\frac{1}{2}a^2}=\sqrt{a^2\cdot 2}+\frac{\sqrt{a^2}}{\sqrt{2}}=|a|\sqrt{2}+\frac{|a|}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=|a|\sqrt{2}+\frac{|a|\sqrt{2}}{2}=1\frac{1}{2}|a|\sqrt{2}.$

5c $\sqrt{24\frac{1}{2}a^2}-\sqrt{2a^2}=\sqrt{\frac{49a^2}{2}}-\sqrt{a^2\cdot 2}=\frac{7|a|}{\sqrt{2}}-|a|\sqrt{2}=\frac{7|a|}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}-|a|\sqrt{2}=3\frac{1}{2}|a|\sqrt{2}-|a|\sqrt{2}=2\frac{1}{2}|a|\sqrt{2}.$

5d $a^2\cdot\sqrt{50}-a^2\cdot\sqrt{32}=a^2\cdot\sqrt{25\cdot 2}-a^2\cdot\sqrt{16\cdot 2}=5a^2\cdot\sqrt{2}-4a^2\cdot\sqrt{2}=a^2\cdot\sqrt{2}.$

5e $(\frac{1}{4}a\sqrt{2})^2+(\frac{3}{4}a\sqrt{2})^2=\frac{1}{4}a\sqrt{2}\cdot\frac{1}{4}a\sqrt{2}+\frac{3}{4}a\sqrt{2}\cdot\frac{3}{4}a\sqrt{2}=\frac{1}{16}a^2\cdot 2+\frac{9}{16}a^2\cdot 2=\frac{2}{16}a^2+\frac{18}{16}a^2=\frac{20}{16}a^2=\frac{5}{4}a^2=1\frac{1}{4}a^2.$

5f $\frac{a}{2\sqrt{3}}+\frac{a}{\sqrt{3}}=\frac{a}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{a\sqrt{3}}{6}+\frac{a\sqrt{3}}{3}=\frac{1}{6}a\sqrt{3}+\frac{1}{3}a\sqrt{3}=\frac{1}{6}a\sqrt{3}+\frac{2}{6}a\sqrt{3}=\frac{1}{2}a\sqrt{3}.$

6 I $(\sqrt{2}+\sqrt{3})^2=(\sqrt{2}+\sqrt{3})\cdot(\sqrt{2}+\sqrt{3})=2+\sqrt{6}+\sqrt{6}+3=5+2\sqrt{6}.$ ($\neq 2+3$)
II $(\sqrt{3}+\sqrt{2})\cdot(\sqrt{3}-\sqrt{2})=3-\sqrt{6}+\sqrt{6}-2=3-2.$ Dus waar.
III $(1+\sqrt{2})^2=(1+\sqrt{2})\cdot(1+\sqrt{2})=1+\sqrt{2}+\sqrt{2}+2=3+2\sqrt{2}.$ Dus waar.
IV $(10-\sqrt{3})^2=(10-\sqrt{3})\cdot(10-\sqrt{3})=100-10\sqrt{3}-10\sqrt{3}+3=103-20\sqrt{3}.$ ($\neq 100-3$)

Merkwaardige producten:

$(\square+\bigcirc)^2=\square^2+2\cdot\square\cdot\bigcirc+\bigcirc^2$; $(\square-\bigcirc)^2=\square^2-2\cdot\square\cdot\bigcirc+\bigcirc^2$ en $(\square+\bigcirc)\cdot(\square-\bigcirc)=\square^2-\bigcirc^2$.

7a $(3\sqrt{2}-\sqrt{5})^2=(3\sqrt{2})^2-2\cdot 3\sqrt{2}\cdot\sqrt{5}+\sqrt{5}^2=9\cdot 2-6\sqrt{10}+5=23-6\sqrt{10}.$

7b $(2\sqrt{2}+3\sqrt{3})^2=(2\sqrt{2})^2+2\cdot 2\sqrt{2}\cdot 3\sqrt{3}+(3\sqrt{3})^2=4\cdot 2+12\sqrt{6}+9\cdot 3=35+12\sqrt{6}.$

7c $(5\sqrt{3}+2)\cdot(5\sqrt{3}-2)=(5\sqrt{3})^2-2^2=25\cdot 3-4=71.$

7d $(a-\sqrt{3})^2=a^2-2\cdot a\cdot\sqrt{3}+\sqrt{3}^2=a^2-2a\sqrt{3}+3.$

7e $(a-a\sqrt{2})^2=a^2-2\cdot a\cdot a\sqrt{2}+(a\sqrt{2})^2=a^2-2a^2\cdot\sqrt{2}+a^2\cdot 2=3a^2-2a^2\cdot\sqrt{2}.$

7f $(4-\frac{1}{2}a\sqrt{2})^2=4^2-2\cdot 4\cdot\frac{1}{2}a\sqrt{2}+(\frac{1}{2}a\sqrt{2})^2=16-4a\cdot\sqrt{2}+\frac{1}{4}\cdot a^2\cdot 2=16-4a\sqrt{2}+\frac{1}{2}a^2.$

8a $\frac{2}{\sqrt{5}-1}=\frac{2}{\sqrt{5}-1}\cdot\frac{\sqrt{5}+1}{\sqrt{5}+1}=\frac{2\cdot(\sqrt{5}+1)}{5-1}=\frac{2\sqrt{5}+2}{4}=\frac{1}{2}\sqrt{5}+\frac{1}{2}.$

8b $\frac{10}{\sqrt{2}+\sqrt{3}}=\frac{10}{\sqrt{2}+\sqrt{3}}\cdot\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}=\frac{10\cdot(\sqrt{2}-\sqrt{3})}{2-3}=\frac{10\sqrt{2}-10\sqrt{3}}{-1}=-10\sqrt{2}+10\sqrt{3}.$

8c $\frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}}=\frac{12\sqrt{2}}{\sqrt{10}-\sqrt{2}}\cdot\frac{\sqrt{10}+\sqrt{2}}{\sqrt{10}+\sqrt{2}}=\frac{12\sqrt{2}\cdot(\sqrt{10}+\sqrt{2})}{10-2}=\frac{12\sqrt{20}+12\cdot 2}{8}=\frac{12\sqrt{4\cdot 5}+24}{8}=\frac{24\sqrt{5}+24}{8}=3\sqrt{5}+3.$

9a $(2a\sqrt{2}-a\sqrt{3})^2=(2a\sqrt{2})^2-2\cdot 2a\sqrt{2}\cdot a\sqrt{3}+(a\sqrt{3})^2=8a^2-4a^2\cdot\sqrt{6}+3a^2=11a^2-4a^2\cdot\sqrt{6}.$

9b $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}\cdot\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}+\sqrt{2})^2}{5-2}=\frac{\sqrt{5}^2+2\cdot\sqrt{5}\cdot\sqrt{2}+\sqrt{2}^2}{3}=\frac{5+2\cdot\sqrt{10}+2}{3}=\frac{7+2\cdot\sqrt{10}}{3}=2\frac{1}{3}+\frac{2}{3}\sqrt{10}.$

9c $(\frac{1}{2}\sqrt{2}+\frac{3}{4}\sqrt{3})^2=(\frac{1}{2}\sqrt{2})^2+2\cdot\frac{1}{2}\sqrt{2}\cdot\frac{3}{4}\sqrt{3}+(\frac{3}{4}\sqrt{3})^2=\frac{1}{4}\cdot 2+\frac{3}{4}\cdot\sqrt{6}+\frac{9}{16}\cdot 3=\frac{8}{16}+\frac{3}{4}\sqrt{6}+\frac{27}{16}=\frac{35}{16}+\frac{3}{4}\sqrt{6}=2\frac{3}{16}+\frac{3}{4}\sqrt{6}.$

9d $\frac{\sqrt{72}}{3-\sqrt{3}}=\frac{\sqrt{36\cdot 2}}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}=\frac{6\sqrt{2}\cdot(3+\sqrt{3})}{9-3}=\frac{18\sqrt{2}+6\sqrt{6}}{6}=3\sqrt{2}+\sqrt{6}.$

9e $\left(\frac{1}{\sqrt{2}-1}\right)^2=\frac{1}{(\sqrt{2}-1)^2}=\frac{1}{\sqrt{2}^2-2\cdot\sqrt{2}\cdot 1+1^2}=\frac{1}{3-2\sqrt{2}}=\frac{1}{3-2\sqrt{2}}\cdot\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{3+2\sqrt{2}}{3^2-(2\sqrt{2})^2}=\frac{3+2\sqrt{2}}{9-4\cdot 2}=\frac{3+2\sqrt{2}}{1}=3+2\sqrt{2}.$

9f $\left(\frac{a}{2\sqrt{5}}+\frac{a}{\sqrt{5}}\right)^2=\left(\frac{a}{2\sqrt{5}}+\frac{2a}{2\sqrt{5}}\right)^2=\left(\frac{3a}{2\sqrt{5}}\right)^2=\frac{9a^2}{4\cdot 5}=\frac{9a^2}{20}=\frac{9}{20}a^2.$

10 I $\frac{1}{x}+\frac{1}{y}=\frac{1}{x}\cdot\frac{y}{y}+\frac{1}{y}\cdot\frac{x}{x}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}\neq\frac{2}{x+y}.$

II $\frac{1}{x}-\frac{1}{y}=\frac{1}{x}\cdot\frac{y}{y}-\frac{1}{y}\cdot\frac{x}{x}=\frac{y}{xy}-\frac{x}{xy}=\frac{y-x}{xy}$. Dus waar.

III $\frac{3}{2x}+\frac{2}{3x}=\frac{3}{2x}\cdot\frac{3}{3}+\frac{2}{3x}\cdot\frac{2}{2}=\frac{9}{6x}+\frac{4}{6x}=\frac{9+4}{6x}=\frac{13}{6x}\neq\frac{1}{x}.$

IV $\frac{3}{2x}-\frac{2}{3x}=\frac{3}{2x}\cdot\frac{3}{3}-\frac{2}{3x}\cdot\frac{2}{2}=\frac{9}{6x}-\frac{4}{6x}=\frac{9-4}{6x}=\frac{5}{6x}$. Dus waar.

11a $\frac{1}{2x}+\frac{2}{x}=\frac{1}{2x}+\frac{2}{x}\cdot\frac{2}{2}=\frac{1}{2x}+\frac{4}{2x}=\frac{5}{2x}.$

11b $\frac{3}{2a}-\frac{2}{3a}=\frac{3}{2a}\cdot\frac{3}{3}-\frac{2}{3a}\cdot\frac{2}{2}=\frac{9}{6a}-\frac{4}{6a}=\frac{5}{6a}.$

11c $\frac{1}{ab}-\frac{1}{b}=\frac{1}{ab}-\frac{1}{b}\cdot\frac{a}{a}=\frac{1}{ab}-\frac{a}{ab}=\frac{1-a}{ab}.$

11d $\frac{a}{b}-\frac{1}{a}=\frac{a}{b}\cdot\frac{a}{a}-\frac{1}{a}\cdot\frac{b}{b}=\frac{a^2}{ab}-\frac{b}{ab}=\frac{a^2-b}{ab}.$

11e $2+\frac{1}{x}=\frac{2}{1}\cdot\frac{x}{x}+\frac{1}{x}=\frac{2x}{x}+\frac{1}{x}=\frac{2x+1}{x}.$

11f $3a-\frac{2}{a}=\frac{3a}{1}\cdot\frac{a}{a}-\frac{2}{a}=\frac{3a^2}{a}-\frac{2}{a}=\frac{3a^2-2}{a}.$

12a $\frac{1}{x}+\frac{1}{x+2}=\frac{1}{x}\cdot\frac{x+2}{x+2}+\frac{1}{x+2}\cdot\frac{x}{x}=\frac{x+2}{x(x+2)}+\frac{x}{x(x+2)}=\frac{2x+2}{x(x+2)}.$

12b $\frac{1}{x+3}+\frac{1}{x+4}=\frac{1}{x+3}\cdot\frac{x+4}{x+4}+\frac{1}{x+4}\cdot\frac{x+3}{x+3}=\frac{x+4}{(x+3)(x+4)}+\frac{x+3}{(x+3)(x+4)}=\frac{2x+7}{(x+3)(x+4)}.$

12c $\frac{x}{x-2}-\frac{1}{x+2}=\frac{x}{x-2}\cdot\frac{x+2}{x+2}-\frac{1}{x+2}\cdot\frac{x-2}{x-2}=\frac{x^2+2x}{(x-2)(x+2)}-\frac{x-2}{(x-2)(x+2)}=\frac{x^2+2x-x+2}{(x-2)(x+2)}=\frac{x^2+x+2}{(x-2)(x+2)}.$

12d $\frac{x+2}{x+3}-\frac{x}{x-2}=\frac{x+2}{x+3}\cdot\frac{x-2}{x-2}-\frac{x}{x-2}\cdot\frac{x+3}{x+3}=\frac{x^2-4}{(x-2)(x+3)}-\frac{x^2+3x}{(x-2)(x+3)}=\frac{x^2-4-x^2-3x}{(x-2)(x+3)}=\frac{-4-3x}{(x-2)(x+3)}.$

12e $\frac{2x}{x+2}-\frac{3x}{x+3}=\frac{2x}{x+2}\cdot\frac{x+3}{x+3}-\frac{3x}{x+3}\cdot\frac{x+2}{x+2}=\frac{2x^2+6x}{(x+2)(x+3)}-\frac{3x^2+6x}{(x+2)(x+3)}=\frac{2x^2+6x-3x^2-6x}{(x+2)(x+3)}=\frac{-x^2}{(x+2)(x+3)}.$

12f $\frac{x+2}{x+3}-\frac{x+3}{x+2}=\frac{x+2}{x+3}\cdot\frac{x+2}{x+2}-\frac{x+3}{x+2}\cdot\frac{x+3}{x+3}=\frac{x^2+4x+4}{(x+2)(x+3)}-\frac{x^2+6x+9}{(x+2)(x+3)}=\frac{x^2+4x+4-x^2-6x-9}{(x+2)(x+3)}=\frac{-2x-5}{(x+2)(x+3)}.$

13a
$\frac{1}{a}=b+\frac{1}{c}$
$\frac{1}{a}=\frac{b}{1}\cdot\frac{c}{c}+\frac{1}{c}$
$\frac{1}{a}=\frac{bc}{c}+\frac{1}{c}$
$\frac{1}{a}=\frac{bc+1}{c}$ (kruiselings vermenigvuldigen)
$a\cdot(bc+1)=c\cdot 1$
$a=\frac{c}{bc+1}.$

13b
$\frac{1}{p}=2q-\frac{1}{q}$
$\frac{1}{p}=\frac{2q}{1}\cdot\frac{q}{q}-\frac{1}{q}$
$\frac{1}{p}=\frac{2q^2}{q}-\frac{1}{q}$
$\frac{1}{p}=\frac{2q^2-1}{q}$
$p\cdot(2q^2-1)=q\cdot 1$
$p=\frac{q}{2q^2-1}.$

13c
$\frac{3}{y}=x-\frac{x}{x-1}$
$\frac{3}{y}=\frac{x}{1}\cdot\frac{x-1}{x-1}-\frac{x}{x-1}$
$\frac{3}{y}=\frac{x^2-x}{x-1}-\frac{x}{x-1}$
$\frac{3}{y}=\frac{x^2-x-x}{x-1}$
$\frac{3}{y}=\frac{x^2-2x}{x-1}$
$y\cdot(x^2-2x)=3\cdot(x-1)$
$y=\frac{3(x-1)}{x^2-2x}.$

14a $\frac{3}{x^2y}-\frac{2}{xy^2}=\frac{3y}{x^2y^2}-\frac{2x}{x^2y^2}=\frac{3y-2x}{x^2y^2}.$

14b $2x-\frac{x^2}{x+1}=\frac{2x(x+1)}{x+1}-\frac{x^2}{x+1}=\frac{2x^2+2x-x^2}{x+1}=\frac{x^2+2x}{x+1}.$

14c $\frac{5a}{3b}-\frac{a}{b+1}=\frac{5a(b+1)}{3b(b+1)}-\frac{a\cdot 3b}{3b(b+1)}=\frac{5ab+5a-3ab}{3b(b+1)}=\frac{2ab+5a}{3b(b+1)}.$

14d $\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}=\frac{1}{x}\cdot\frac{x^2}{x^2}+\frac{1}{x^2}\cdot\frac{x}{x}+\frac{1}{x^3}=\frac{x^2}{x^3}+\frac{x}{x^3}+\frac{1}{x^3}=\frac{x^2+x+1}{x^3}.$

14e $\frac{2x+1}{x+1}-\frac{x-1}{x+2}=\frac{(2x+1)(x+2)}{(x+1)(x+2)}-\frac{(x-1)(x+1)}{(x+1)(x+2)}=\frac{2x^2+4x+x+2}{(x+1)(x+2)}-\frac{x^2-1}{(x+1)(x+2)}=\frac{2x^2+5x+2-x^2+1}{(x+1)(x+2)}=\frac{x^2+5x+3}{(x+1)(x+2)}.$

14f $\frac{p^2}{p+1}-\frac{p^3}{p+2}=\frac{p^2(p+2)}{(p+1)(p+2)}-\frac{p^3(p+1)}{(p+1)(p+2)}=\frac{p^3+2p^2}{(p+1)(p+2)}-\frac{p^4+p^3}{(p+1)(p+2)}=\frac{p^3+2p^2-p^4-p^3}{(p+1)(p+2)}=\frac{-p^4+2p^2}{(p+1)(p+2)}.$

15a $\frac{1}{2}+\frac{1}{x+1}=\frac{x+1}{x+4}$

$\frac{x+1}{2(x+1)}+\frac{2}{2(x+1)}=\frac{x+1}{x+4}$

$\frac{x+3}{2(x+1)}=\frac{x+1}{x+4}$

$\frac{x+3}{2x+2}=\frac{x+1}{x+4}$ (kruiselings vermenigvuldigen)

$(2x+2)(x+1)=(x+3)(x+4)$ (hiernaast verder)

$2x^2+2x+2x+2=x^2+4x+3x+12$

$2x^2+4x+2=x^2+7x+12$

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5 \vee x=-2$

voldoet voldoet (noemers worden niet nul)

15b $x+1+\frac{1}{x-1}=\frac{x}{x+3}$

$\frac{(x+1)(x-1)}{x-1}+\frac{1}{x-1}=\frac{x}{x+3}$

$\frac{x^2-1}{x-1}+\frac{1}{x-1}=\frac{x}{x+3}$

$\frac{x^2}{x-1}=\frac{x}{x+3}$ (kruiselings vermenigvuldigen)

$x^2(x+3)=x(x-1)$ (hiernaast verder)

$x^3+3x^2=x^2-x$

$x^3+2x^2+x=0$

$x(x^2+2x+1)=0$

$x(x+1)(x+1)=0$

$x=0 \vee x=-1$

voldoet voldoet

15c $\frac{1}{x}+\frac{1}{x-3}=\frac{3}{x+1}$

$\frac{x-3}{x(x-3)}+\frac{x}{x(x-3)}=\frac{3}{x+1}$

$\frac{2x-3}{x(x-3)}=\frac{3}{x+1}$ (kruiselings vermenigvuldigen)

$3x(x-3)=(2x-3)(x+1)$

$3x^2-9x=2x^2+2x-3x-3$ (hiernaast verder)

$x^2-8x+3=0$ ($a=1;\ b=-8$ en $c=3$)

$D=b^2-4ac=(-8)^2-4\cdot 1\cdot 3=64-12=52$

$x=\frac{-b\pm\sqrt{D}}{2a}=\frac{8\pm\sqrt{52}}{2}=\frac{8\pm\sqrt{4\cdot 13}}{2}=\frac{8\pm 2\sqrt{13}}{2}=4\pm\sqrt{13}$

$x=4+\sqrt{13} \vee x=4-\sqrt{13}.$

voldoet voldoet

16 $\frac{2x^2+1}{x}=\frac{2x^2}{x}+\frac{1}{x}=2x+\frac{1}{x}.$ $\frac{x^2-1}{x}=\frac{x^2}{x}-\frac{1}{x}=x-\frac{1}{x}\neq x-1.$ $\frac{x^2-1}{x^2+2x+1}=\frac{\cancel{(x+1)}(x-1)}{(x+1)\cancel{(x+1)}}=\frac{x-1}{x+1}.$

17a $\frac{x^2-9}{x^2+6x+9}=\frac{\cancel{(x+3)}(x-3)}{(x+3)\cancel{(x+3)}}=\frac{x-3}{x+3}.$

17b $\frac{x^2-5x}{x^2-x-20}=\frac{x\cancel{(x-5)}}{\cancel{(x-5)}(x+4)}=\frac{x}{x+4}.$

17c $\frac{a^2-4a}{a^2+a}=\frac{\cancel{a}(a-4)}{\cancel{a}(a+1)}=\frac{a-4}{a+1}.$

17d $\frac{a^2-4a-5}{a^3+a^2}=\frac{(a-5)\cancel{(a+1)}}{a^2\cancel{(a+1)}}=\frac{a-5}{a^2}.$

17e $\frac{x^3-11x^2+30x}{x^2-10x+25}=\frac{x(x^2-11x+30)}{(x-5)(x-5)}=\frac{x(x-6)\cancel{(x-5)}}{\cancel{(x-5)}(x-5)}=\frac{x(x-6)}{(x-5)}.$

17f $\frac{x^2+6x+5}{2x+2}=\frac{(x+5)\cancel{(x+1)}}{2\cancel{(x+1)}}=\frac{x+5}{2}=\frac{1}{2}x+2\frac{1}{2}.$

18a $A=\frac{p^2+p}{p^2-1}=\frac{p\cancel{(p+1)}}{\cancel{(p+1)}(p-1)}=\frac{p}{p-1}.$

18b $T=\frac{t^3+4t^2}{t^2-16}=\frac{t^2\cancel{(t+4)}}{\cancel{(t+4)}(t-4)}=\frac{t^2}{t-4}.$

18c $N=\frac{a^4+a^2-2}{a^4+3a^2+2}=\frac{\cancel{(a^2+2)}(a^2-1)}{\cancel{(a^2+2)}(a^2+1)}=\frac{a^2-1}{a^2+1}$ (eventueel nog) $=\frac{(a+1)(a-1)}{a^2+1}.$

19a $\frac{4x^2+7}{x}=\frac{4x^2}{x}+\frac{7}{x}=4x+\frac{7}{x}.$

19b $\frac{a^2-2a+6}{2a}=\frac{a^2}{2a}-\frac{2a}{2a}+\frac{6}{2a}=\frac{1}{2}a-1+\frac{3}{a}.$

19c $\frac{p^3-3p^2+2}{2p}=\frac{p^3}{2p}-\frac{3p^2}{2p}+\frac{2}{2p}=\frac{1}{2}p^2-1\frac{1}{2}p+\frac{1}{p}.$

20a $F=\frac{a^2+2a-3}{a-1}+\frac{a^2+1}{a}=\frac{(a+3)\cancel{(a-1)}}{\cancel{a-1}}+\frac{a^2}{a}+\frac{1}{a}=a+3+a+\frac{1}{a}=2a+3+\frac{1}{a}.$

20b $R=\frac{m^4-4}{m^4+2m^2}+\frac{m^2+6}{2m^2}=\frac{\cancel{(m^2+2)}(m^2-2)}{m^2\cancel{(m^2+2)}}+\frac{m^2}{2m^2}+\frac{6}{2m^2}=\frac{m^2-2}{m^2}+\frac{1}{2}+\frac{3}{m^2}=\frac{m^2}{m^2}-\frac{2}{m^2}+\frac{1}{2}+\frac{3}{m^2}=1-\frac{2}{m^2}+\frac{1}{2}+\frac{3}{m^2}=1\frac{1}{2}+\frac{1}{m^2}.$

20c $H=\frac{c^3+4c^2+1}{2c^2}-\frac{c^2-5c+6}{2c-6}=\frac{c^3}{2c^2}+\frac{4c^2}{2c^2}+\frac{1}{2c^2}-\frac{\cancel{(c-3)}(c-2)}{2\cancel{(c-3)}}=\frac{1}{2}c+2+\frac{1}{2c^2}-\frac{1}{2}c+1=3+\frac{1}{2c^2}.$

21a $\frac{x^2+4x+4}{x^2-4}=\frac{10}{x-2}$
$\frac{(x+2)(x+2)}{(x-2)(x+2)}=\frac{10}{x-2}$
$x+2=10$
$x=8.$
(voldoet)

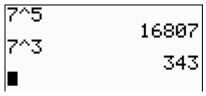
21b $\frac{x^2-9x+14}{x^2+x-6}=\frac{3-x}{2x-6}$
$\frac{(x-7)(x-2)}{(x+3)(x-2)}=\frac{-1(-3+x)}{2(x-3)}$
$\frac{x-7}{x+3}=-\frac{1}{2}$ (kruiselings verm.)
$2\cdot(x-7)=-1\cdot(x+3)$
$2x-14=-x-3$
$3x=11$
$x=\frac{11}{3}=3\frac{2}{3}.$
(voldoet)

21c $\frac{x^2-6}{x-3}=\frac{x^2-4}{x^2-x-2}$
$\frac{x^2-6}{x-3}=\frac{(x+2)(x-2)}{(x-2)(x+1)}$ (kruiselings verm.)
$(x-3)\cdot(x+2)=(x+1)\cdot(x^2-6)$
$x^2+2x-3x-6=x^3-6x+x^2-6$
$-x^3+5x=0$
$-x(x^2-5)=0$
$x=0 \vee x^2=5.$
$x=0 \vee x\pm\sqrt{5}$ (maken noemers niet nul)

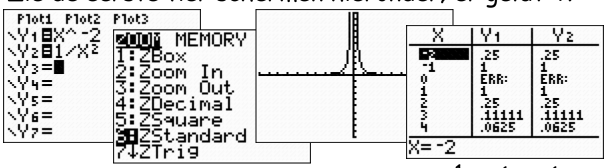
22 I $x^2\cdot x^3=x^{2+3}=x^5\neq x^6.$ III $(2x)^3=2x\cdot 2x\cdot 2x=2^3\cdot x^3=8x^3\neq 6x^3.$
II $\frac{x^6}{x^2}=x^{6-2}=x^4.$ IV $(x^3)^2=x^3\cdot x^3=x^{3+3}=x^{2\cdot 3}=x^6.$ Dus II en IV zijn waar.

23a $2a^3\cdot 4a^7=8a^{10}.$ 23d $(3ab^2)^4=81a^4b^8.$ 23g $(-2a)^3\cdot 3a^3=-8a^3\cdot 3a^3=-24a^6.$
23b $2a^3+4a^7-a^3=4a^7+a^3.$ 23e $(5a^3)^3\cdot 2b^7=125a^9\cdot 2b^7=250a^9b^7.$ 23h $(-2a)^2+3a^2=4a^2+3a^2=7a^2.$
23c $a^6\cdot\frac{1}{a^4}=\frac{a^6}{a^4}=a^2.$ 23f $\frac{15a^{18}}{3a^6}=5a^{12}.$ 23i $\frac{1}{a^8}\cdot(a^3)^4=\frac{a^{12}}{a^8}=a^4.$

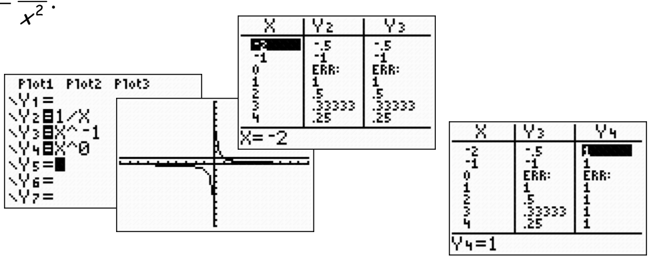
24a $7a^3+5a^3=12a^3.$ 24d $7a^3\cdot 5a^3=35a^6.$ 24g $(2a)^2+(\frac{1}{2}a)^2=4a^2+\frac{1}{4}a^2=4\frac{1}{4}a^2.$
24b $7a^3-a^3=6a^3.$ 24e $(7a^3)^5=16\,807a^{15}.$ 24h $(3a)^2-8a^2=9a^2-8a^2=a^2.$
24c $7a^5:a^3=7a^2.$ 24f $(7a)^3+5a^3=343a^3+5a^3=348a^3.$ 24i $(\frac{1}{3}a)^3-a^3=\frac{1}{27}a^3-a^3=-\frac{26}{27}a^3.$



25a Zie de eerste vier schermen hieronder; er geldt: $x^{-2}=\frac{1}{x^2}.$



25b Zie de schermen hiernaast; er geldt: $x^{-1}=\frac{1}{x^1}=\frac{1}{x}.$
25c De grafiek is de horizontale lijn $y=1$ voor $x\neq 0.$



26a $a^2:\frac{1}{a^4}=a^2:a^{-4}=a^{2--4}=a^6.$ 26c $(a^3)^{-2}=a^{3\cdot -2}=a^{-6}.$ 26e $\frac{1}{a^5}:a=a^{-5}:a=a^{-5-1}=a^{-6}.$
26b $a^8:a^0=a^{8-0}=a^8.$ 26d $\frac{a}{a^{12}}=a^{1-12}=a^{-11}.$ 26f $1=a^0.$

27a $7^{-2}=\frac{1}{7^2}=\frac{1}{49}.$ 27c $3\cdot 5^{-2}=3\cdot\frac{1}{25}=\frac{3}{25}.$ 27e $4\cdot 10^{-3}=4\cdot\frac{1}{10^3}=\frac{4}{1000}=\frac{1}{250}.$
27b $(\frac{1}{3})^{-2}=(3^{-1})^{-2}=3^2=9.$ 27d $(\frac{2}{5})^{-1}=\frac{1}{(\frac{2}{5})^1}=\frac{1}{\frac{2}{5}}=\frac{5}{2}=2\frac{1}{2}.$ 27f $\frac{1}{2}:6^{-2}=\frac{1}{2}:\frac{1}{6^2}=\frac{1}{2}:\frac{1}{36}=\frac{1}{2}\times 36=18.$

28a $6a^{-5}\cdot b^3=6\cdot\frac{1}{a^5}\cdot b^3=\frac{6b^3}{a^5}.$ 28d $(\frac{1}{2}a)^{-3}=\frac{1}{(\frac{1}{2}a)^3}=\frac{1}{\frac{1}{8}a^3}=\frac{1}{\frac{1}{8}a^3}\cdot\frac{8}{8}=\frac{8}{a^3}.$
28b $\frac{1}{3}a^{-3}=\frac{1}{3}\cdot\frac{1}{a^3}=\frac{1}{3a^3}.$ 28e $-4\cdot(\frac{2}{3}a)^{-2}=-4\cdot\frac{1}{(\frac{2}{3}a)^2}=\frac{-4}{\frac{4}{9}a^2}=\frac{-4}{\frac{4}{9}a^2}\cdot\frac{9}{9}=\frac{-9}{a^2}.$
28c $3a^{-4}=3\cdot\frac{1}{a^4}=\frac{3}{a^4}.$ 28f $(3a)^{-2}\cdot b^{-3}=\frac{1}{(3a)^2}\cdot\frac{1}{b^3}=\frac{1}{9a^2}\cdot\frac{1}{b^3}=\frac{1}{9a^2b^3}.$

29 De formules $y_1=x^{\frac{1}{5}}$ en $y_3=\sqrt[5]{x}$ komen op hetzelfde neer. (plot de grafieken en bekijk hun tabellen)

30a $5a^{\frac13} = 5\cdot\sqrt[3]{a}$.

30b $2a^{-\frac14}\cdot b = 2b\cdot\frac{1}{a^{\frac14}} = \frac{2b}{\sqrt[4]{a}}$.

30c $3a^{-\frac23} = 3\cdot\frac{1}{a^{\frac23}} = \frac{3}{\sqrt[3]{a^2}}$.

30d $a^{-3}\cdot b^{\frac13} = \frac{1}{a^3}\cdot\sqrt[3]{b} = \frac{\sqrt[3]{b}}{a^3}$.

30e $\frac15 a^{-\frac12}\cdot b^{\frac15} = \frac15\cdot\frac{1}{a^{\frac12}}\cdot\sqrt[5]{b} = \frac{\sqrt[5]{b}}{5\cdot\sqrt{a}}$.

30f $(5a)^{-\frac12} = \frac{1}{(5a)^{\frac12}} = \frac{1}{\sqrt{5a}}$.

31a $a\cdot\sqrt[3]{a} = a^1\cdot a^{\frac13} = a^{1\frac13}$.

31b $\frac{1}{\sqrt{a}} = \frac{1}{a^{\frac12}} = a^{-\frac12}$.

31c $\frac{1}{a\sqrt{a}} = \frac{1}{a^1\cdot a^{\frac12}} = \frac{1}{a^{1\frac12}} = a^{-1\frac12}$.

31d $\frac{1}{\sqrt[4]{a^3}} = \frac{1}{a^{\frac34}} = a^{-\frac34}$.

31e $a^2\cdot\sqrt{a} = a^2\cdot a^{\frac12} = a^{2\frac12}$.

31f $\sqrt[3]{\frac{1}{a^2}} = \sqrt[3]{a^{-2}} = a^{-\frac23}$.

31g $\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$.

31h $a^4\cdot\sqrt[3]{a} = a^4\cdot a^{\frac13} = a^{4\frac13}$.

31i $\frac{a^3}{\sqrt[3]{a}} = \frac{a^3}{a^{\frac13}} = a^{2\frac23}$.

32a $\frac{x^6}{x^2\cdot\sqrt{x}} = \frac{x^6}{x^2\cdot x^{\frac12}} = \frac{x^6}{x^{2\frac12}} = x^{3\frac12}$.

32b $x\cdot\sqrt[7]{x^3} = x^1\cdot x^{\frac37} = x^{1\frac37}$.

32c $\frac{x}{\sqrt[5]{x}} = \frac{x^1}{x^{\frac15}} = x^{\frac45}$.

32d $x^4\cdot\sqrt{x} = x^4\cdot x^{\frac12} = x^{4\frac12}$.

32e $\frac{\sqrt[3]{x}}{\sqrt{x}} = \frac{x^{\frac13}}{x^{\frac12}} = x^{\frac13-\frac12} = x^{-\frac16}$.

32f $\frac{1}{x^2} : \sqrt{x} = x^{-2} : x^{\frac12} = x^{-2\frac12}$.

32g $\sqrt[3]{x^2}\cdot\frac{1}{x^3} = x^{\frac23}\cdot x^{-3} = x^{-2\frac13}$.

32h $x^5\cdot\sqrt[3]{x^6} = x^5\cdot x^{\frac63} = x^7$.

32i $\frac{x^4\cdot\sqrt[5]{x}}{x^5\cdot\sqrt[4]{x}} = \frac{x^4\cdot x^{\frac15}}{x^5\cdot x^{\frac14}} = \frac{x^{4\frac15}}{x^{5\frac14}} = x^{-1\frac{1}{20}}$.

```
(4+1/5)-(5+1/4)▸
Frac
            -21/20
```

33a $x^{1,6} = 50$

```
50^(1/1.6)
       11.53071539
```

$x = 50^{\frac{1}{1,6}} \approx 11{,}531$.

33b $x^{-4,1} = 5$

```
5^(1/-4.1)
       .6753353935
```

$x = 5^{\frac{1}{-4,1}} \approx 0{,}675$.

33c $x^{-1,3} = 11$

```
11^(1/-1.3)
       .1580988474
```

$x = 11^{\frac{1}{-1,3}} \approx 0{,}158$.

33d $x^{-1} = 21$

```
21^(1/-1)
       .0476190476
```

$x = 21^{\frac{1}{-1}} \approx 0{,}048$.

33e $x^{0,55} = 18$

$x = 18^{\frac{1}{0,55}} \approx 191{,}564$.

33f $\sqrt[3]{x^2} = 28$

$x^{\frac23} = 28$

```
18^(1/0.55)
       191.5638892
28^(1/(2/3))
       148.1620734
```

$x = 28^{\frac{1}{\left(\frac23\right)}} \approx 148{,}162$.

34a $3x^{2,25} + 1 = 27$

$3x^{2,25} = 26$

$x^{2,25} = \frac{26}{3}$

$x = \left(\frac{26}{3}\right)^{\frac{1}{2,25}} \approx 2{,}611$.

```
27-1
                26
Ans/3
       8.666666667
Ans^(1/2.25)
       2.611097882
```

34b $5x^{-1,3} + 8 = 21$

$5x^{-1,3} = 13$

$x^{-1,3} = \frac{13}{5}$

$x = \left(\frac{13}{5}\right)^{\frac{1}{-1,3}} \approx 0{,}480$.

```
21-8
                13
Ans/5
               2.6
Ans^(1/-1.3)
       .4795012375
```

34c $4x^{-1,8} + 16 = 5\,000$

$4x^{-1,8} = 4\,984$

$x^{-1,8} = 1246$

$x = 1246^{\frac{1}{-1,8}} \approx 0{,}019$.

```
5000-16
              4984
Ans/4
              1246
Ans^(1/-1.8)
       .0190663585
```

34d $8 - 3x^{1,16} = 1$

$-3x^{1,16} = -7$

$x^{1,16} = \frac{-7}{-3} = \frac73$

$x = \left(\frac73\right)^{\frac{1}{1,16}} \approx 2{,}076$.

```
1-8
                -7
Ans/-3
       2.333333333
Ans^(1/1.16)
       2.075971403
```

34e $5\cdot\sqrt[3]{x} = 8$

$\sqrt[3]{x} = \frac85$

$x^{\frac13} = 1{,}6$

$x = (1{,}6)^3 = 4{,}096$.

```
8/5
               1.6
Ans^3
             4.096
```

34f $3\cdot\sqrt[4]{x^3} - 1 = 36$

$3\cdot\sqrt[4]{x^3} = 37$

$x^{\frac34} = \frac{37}{3}$

$x = \left(\frac{37}{3}\right)^{\frac43} \approx 28{,}495$.

```
36+1
                37
Ans/3
       12.33333333
Ans^(4/3)
        28.4953476
```

35a $P = 800\cdot l^{-2,25} = 800\cdot\frac{1}{l^{2,25}} = \frac{800}{l^{2,25}}$.

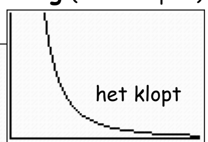
Als l groter wordt, dan wordt de noemer van de breuk groter en dan wordt de breuk zelf, dus P, kleiner.

Dat wil zeggen dat er minder organismen per km^2 leven ofwel de organismen leven gemiddeld verder van elkaar.

Alternatieve uitwerking (met een plot)

```
Plot1 Plot2 Plot3
\Y1=800*X^-2.25
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=0
 Ymin=0
 Ymax=1000
 Yscl=0
 Xres=1
```



35b $l = 0{,}9$ (m) $\Rightarrow P = 800\cdot 0{,}9^{-2,25} \approx 1014$ (ringslangen/km^2).

De populatiedichtheid is ongeveer 1000 ringslangen per km^2.

```
800*0.9^-2.25
       1014.01491
```

35c $P = 800\cdot l^{-2,25} = 1350$ (intersect of)

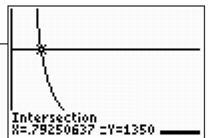
$l^{-2,25} = \frac{1350}{800} = 1{,}6875$

$l = 1{,}6875^{\frac{1}{-2,25}} \approx 0{,}79$ (m). Dus gemiddeld ongeveer 80 cm lang.

```
1350/800
            1.6875
Ans^(1/-2.25)
       .7925063738
```

```
Plot1 Plot2 Plot3
\Y1=800*X^-2.25
\Y2=1350
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=0
 Ymin=0
 Ymax=2000
 Yscl=0
```

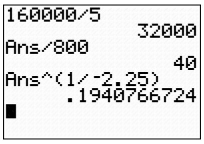


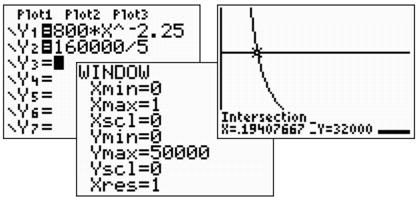
35d $l = 2{,}15$ (m) $\Rightarrow P = 800\cdot 2{,}15^{-2,25} \approx 143$ (kariboes/km^2).

In een gebied van 250 km^2 geeft dit ongeveer 36000 kariboes.

```
800*2.15^-2.25
       142.9234372
Ans*250
       35730.85931
143*250
             35750
```

35e $P = \frac{160000}{5} = 32\,000$
$800 \cdot l^{-2,25} = 32\,000$ (intersect of)
$l^{-2,25} = \frac{32000}{800} = 40$
$l = 40^{\frac{1}{-2,25}} \approx 0,194$ (m). Dus gemiddeld ongeveer 20 cm lang.

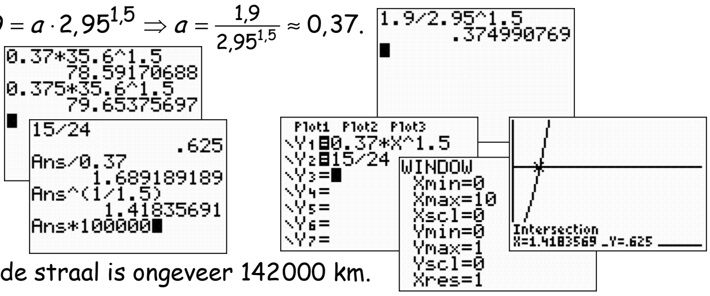




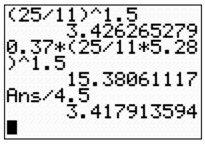
36a $T = a \cdot R^{1,5}$ door (2,95; 1,9) $\Rightarrow 1,9 = a \cdot 2,95^{1,5} \Rightarrow a = \frac{1,9}{2,95^{1,5}} \approx 0,37$.

36b $T = 0,37 \cdot 35,6^{1,5} \approx 80$ (dagen).

36c 15 uur geeft $T = \frac{15}{24} = 0,625$
$0,37 \cdot R^{1,5} = 0,625$ (intersect of)
$R^{1,5} = \frac{0,625}{0,37}$
$R = \left(\frac{0,625}{0,37}\right)^{\frac{1}{1,5}} \approx 1,42$ ($\times 10^5$ km) $\Rightarrow$ de straal is ongeveer 142000 km.



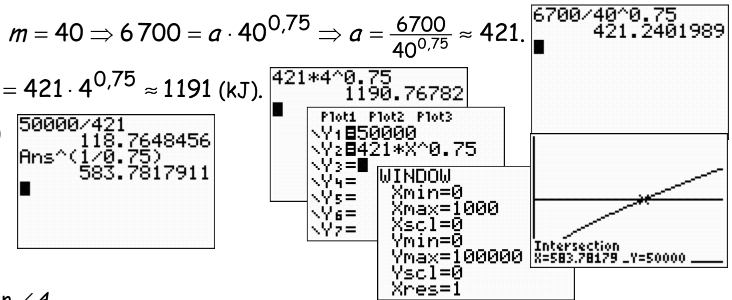
36d $T_{Titan} = 0,37 \cdot (\frac{25}{11} \cdot R_{Rhea})^{1,5} = 0,37 \cdot (\frac{25}{11})^{1,5} \cdot R_{Rhea}{}^{1,5} = (\frac{25}{11})^{1,5} \cdot T_{Rhea} \Rightarrow (\frac{25}{11})^{1,5} \approx 3,4$ keer zo groot.
of: $T_{Rhea} = 4,5$ (dagen); en $T_{Titan} = 0,37 \cdot (\frac{25}{11} \cdot 5,28)^{1,5} \approx 15,4$ (dagen) $\Rightarrow T_{Titan} \approx 3,4 \cdot T_{Rhea}$.



37a $W = a \cdot m^{0,75}$ met $W = 6700$ en $m = 40 \Rightarrow 6700 = a \cdot 40^{0,75} \Rightarrow a = \frac{6700}{40^{0,75}} \approx 421$.

37b $W = 421 \cdot m^{0,75}$ met $m = 4 \Rightarrow W = 421 \cdot 4^{0,75} \approx 1191$ (kJ).

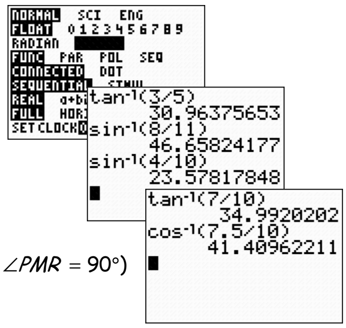
37c $50\,000 = 421 \cdot m^{0,75}$ (intersect of)
$m^{0,75} = \frac{50000}{421}$
$m = \left(\frac{50000}{421}\right)^{\frac{1}{0,75}} \approx 584$ (kg).



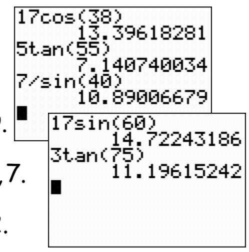
38 $\frac{AB}{7} = \frac{\text{aanliggende rechthoekszijde van } \angle A}{\text{schuine zijde}}$ (cas) $= \cos\angle A$. Je gebruikt dus de cosinus.
$\frac{BC}{7} = \frac{\text{overstaande rechthoekszijde van } \angle A}{\text{schuine zijde}}$ (sos) $= \sin\angle A$. Je gebruikt de sinus.

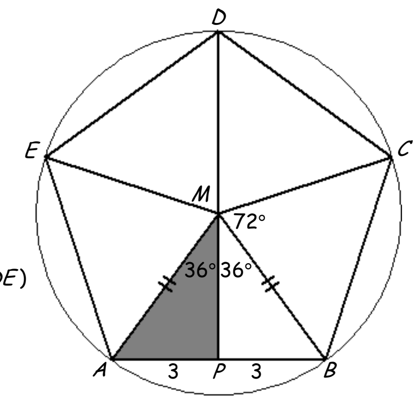
39 $\frac{4}{6} = \frac{\text{overstaande rechthoekszijde van } \angle Q}{\text{aanliggende rechthoekszijde van } \angle Q}$ (toa) $= \tan\angle Q$. Je gebruikt de tangens.

40a $\frac{o}{a} = \tan\angle A = \frac{3}{5}$ terug [tan...] $\Rightarrow \angle A \approx 31°$.
40b $\frac{o}{s} = \sin\angle D = \frac{8}{11}$ terug [sin...] $\Rightarrow \angle D \approx 47°$.
40c $\frac{o}{s} = \sin\angle G = \frac{4}{10}$ terug [sin...] $\Rightarrow \angle G \approx 24°$.
40d $\frac{o}{a} = \tan\angle MKL = \frac{ML}{KL} = \frac{7}{10}$ terug [tan...] $\Rightarrow \angle MKL \approx 35°$.
40e $\frac{a}{s} = \cos\angle P = \frac{PM}{PR} = \frac{7,5}{10}$ terug [cos...] $\Rightarrow \angle P \approx 41°$. (noem M het midden van PQ, dan is $\angle PMR = 90°$)

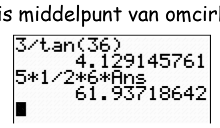


41a $\frac{AC}{17} = \frac{a}{s} = \cos 38° \Rightarrow AC = 17 \cdot \cos 38° \approx 13,4$.
41b $\frac{DF}{5} = \frac{o}{a} = \tan 55° \Rightarrow DF = 5 \cdot \tan 55° \approx 7,1$.
41c $\frac{7}{GH} = \frac{o}{s} = \sin 40° = \frac{\sin 40°}{1} \Rightarrow GH = \frac{1 \cdot 7}{\sin 40°} \approx 10,9$.
41d $\frac{MN}{17} = \frac{o}{s} = \sin 60° \Rightarrow MN = KL = 17 \cdot \sin 60° \approx 14,7$.
41e $\frac{RS}{PS} = \frac{RS}{3} = \frac{o}{a} = \tan 75° \Rightarrow RS = 3 \cdot \tan 75° \approx 11,2$.





42 ΔABM is gelijkbenig en $\angle AMB = \frac{360°}{5} = 72°$. ($M$ is middelpunt van omcirkel van $ABCDE$)
$\tan 36° = \frac{3}{PM} = \frac{\tan 36°}{1} \Rightarrow PM = \frac{1 \cdot 3}{\tan 36°} \approx 4,129$.
$O_{ABCDE} = 5 \cdot O_{ABM} = 5 \cdot \frac{1}{2} AB \cdot PM \approx 61,94$.



43a $AD^2 + CD^2 = AC^2 \Rightarrow a^2 + CD^2 = (2a)^2 \Rightarrow a^2 + CD^2 = 4a^2 \Rightarrow CD^2 = 3a^2 \Rightarrow CD = \sqrt{3a^2} = \sqrt{a^2 \cdot 3} = a\sqrt{3}.$

43b $\angle A = \angle B = \angle C = \frac{180^\circ}{3} = 60^\circ$ (ΔABC is gelijkzijdig).

$\cos\angle A = \cos 60^\circ = \frac{AD}{AC} = \frac{a}{2a} = \frac{1}{2}$

$\sin\angle A = \sin 60^\circ = \frac{CD}{AC} = \frac{a\sqrt{3}}{2a} = \frac{1}{2}\sqrt{3}$ en

$\tan\angle A = \tan 60^\circ = \frac{CD}{AD} = \frac{a\sqrt{3}}{a} = \sqrt{3}.$

43cd $\angle ACD = 180^\circ - 60^\circ - 90^\circ = 30^\circ$ ($\angle A = 60^\circ$ en $\angle D = 90^\circ$).

$\sin 30^\circ = \frac{AD}{AC} = \frac{a}{2a} = \frac{1}{2}$

$\cos 30^\circ = \frac{CD}{AC} = \frac{a\sqrt{3}}{2a} = \frac{1}{2}\sqrt{3}$ en

$\tan 30^\circ = \frac{AD}{CD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1}{3}\sqrt{3}.$

44 Stel $CD = x$, dan is: $AD = \frac{x}{\sqrt{3}} = \frac{x}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{x\sqrt{3}}{3} = \frac{1}{3}x\sqrt{3}$ en $BD = x$.

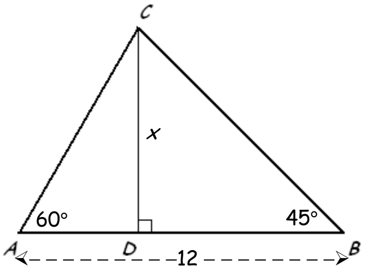
Uit $AD + BD = AB$ volgt dan:

$\frac{1}{3}x\sqrt{3} + x = 12$ (x buiten haakjes halen)

$x \cdot (\frac{1}{3}\sqrt{3} + 1) = 12$

$x = \frac{12}{\frac{1}{3}\sqrt{3}+1} = \frac{12}{\frac{1}{3}\sqrt{3}+1} \cdot \frac{\frac{1}{3}\sqrt{3}-1}{\frac{1}{3}\sqrt{3}-1} = \frac{12 \cdot (\frac{1}{3}\sqrt{3}-1)}{\frac{1}{3}-1} = \frac{4\sqrt{3}-12}{-\frac{2}{3}} = (4\sqrt{3}-12) \cdot -\frac{3}{2} = -6\sqrt{3}+18.$

$O(\Delta ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 12 \cdot (-6\sqrt{3}+18) = -36\sqrt{3}+108.$



45 G is het middelpunt van de omcirkel van zeshoek $ABCDEF$.

$GA = GB \Rightarrow \Delta ABG$ is gelijkbenig; $\angle AGB = \frac{360^\circ}{6} = 60^\circ.$

Dus ΔABG is gelijkzijdig ($\angle G = \angle A = \angle B = 60^\circ$).

Uit $AK = 4$ en $\angle A = 60^\circ$ volgt dan $GK = 4\sqrt{3}$.

$O(\Delta ABCDEF) = 6 \cdot O(\Delta ABG) = 6 \cdot \frac{1}{2} \cdot AB \cdot GK = 6 \cdot \frac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 96\sqrt{3}.$

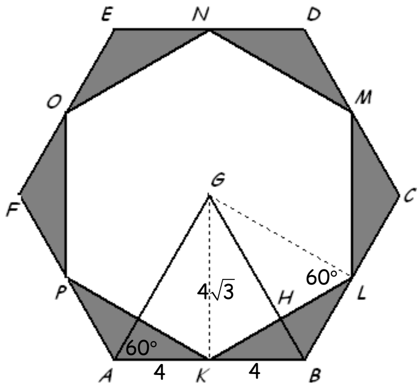
ΔKLG is een gelijkzijdig driehoek met zijde $GK = 4\sqrt{3} \Rightarrow KL = 4\sqrt{3}$.

Dan is $KH = 2\sqrt{3}$ en $GH = 2\sqrt{3} \cdot \sqrt{3} = 6$.

$O(\Delta KLMNOP) = 6 \cdot O(\Delta KLG) = 6 \cdot \frac{1}{2} \cdot KL \cdot GH = 6 \cdot \frac{1}{2} \cdot 4\sqrt{3} \cdot 6 = 72\sqrt{3}.$

De oppervlakte van het gekleurde gebied is dus

$O(\Delta ABCDEF) - O(\Delta KLMNOP) = 96\sqrt{3} - 72\sqrt{3} = 24\sqrt{3}.$



46 Stel de zijden van de regelmatige achthoek x, dan is

$AP = AW = \frac{x}{\sqrt{2}} = \frac{x}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{2} = \frac{1}{2}x\sqrt{2}.$ (ΔAPW is een 1-1-$\sqrt{2}$ driehoek)

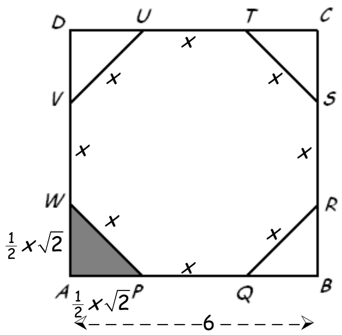
Uit $AP + PQ + QB = AB$ volgt dan: (ΔAPW en ΔBQR zijn congruent)

$\frac{1}{2}x\sqrt{2} + x + \frac{1}{2}x\sqrt{2} = 6$

$x\sqrt{2} + x = 6$

$x(\sqrt{2}+1) = 6$

$x = \frac{6}{\sqrt{2}+1} = \frac{6}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{6 \cdot (\sqrt{2}-1)}{2-1} = \frac{6\sqrt{2}-6}{1} = 6\sqrt{2}-6.$



47a In ΔABE is $AE = a$ en $AB = a\sqrt{2}$. (ΔABE is een 1-1-$\sqrt{2}$ driehoek)

In ΔEBD is $ED = a\sqrt{3}$ en $BD = 2a$. (ΔBED is een 1-$\sqrt{3}$-2 driehoek)

$AD = AE + ED = a + a\sqrt{3}$. (en ΔACD is ook een 1-1-$\sqrt{2}$ driehoek)

Dus $CD = AC = \frac{AD}{\sqrt{2}} = \frac{a+a\sqrt{3}}{\sqrt{2}} = \frac{a+a\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{a\sqrt{2}+a\sqrt{6}}{2} = \frac{1}{2}a\sqrt{2} + \frac{1}{2}a\sqrt{6}\cdot$

$BC = AC - AB = \frac{1}{2}a\sqrt{2} + \frac{1}{2}a\sqrt{6} - a\sqrt{2} = -\frac{1}{2}a\sqrt{2} + \frac{1}{2}a\sqrt{6}.$

47b $\sin 15^\circ = \frac{o}{s} = \frac{BC}{BD} = \frac{-\frac{1}{2}a\sqrt{2}+\frac{1}{2}a\sqrt{6}}{2a} = \frac{-\frac{1}{2}a\sqrt{2}}{2a} + \frac{\frac{1}{2}a\sqrt{6}}{2a} = -\frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6}\cdot$

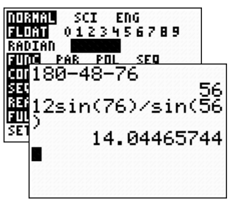
$\cos 15^\circ = \frac{a}{s} = \frac{CD}{BD} = \frac{\frac{1}{2}a\sqrt{2}+\frac{1}{2}a\sqrt{6}}{2a} = \frac{1}{4}\sqrt{2} + \frac{1}{4}\sqrt{6}\cdot$

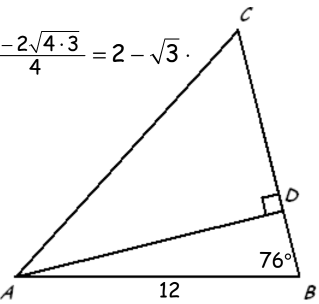
$\tan 15^\circ = \frac{o}{a} = \frac{BC}{CD} = \frac{-\frac{1}{2}a\sqrt{2}+\frac{1}{2}a\sqrt{6}}{\frac{1}{2}a\sqrt{2}+\frac{1}{2}a\sqrt{6}} = \frac{\frac{1}{2}a(-\sqrt{2}+\sqrt{6})}{\frac{1}{2}a(\sqrt{2}+\sqrt{6})} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \cdot \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{6-2\sqrt{12}+2}{6-2} = \frac{8-2\sqrt{4\cdot 3}}{4} = 2-\sqrt{3}\cdot$

48 In ΔABD is $\sin 76^\circ = \frac{AD}{12} \Rightarrow AD = 12 \cdot \sin 76^\circ$. (nog niet afronden)

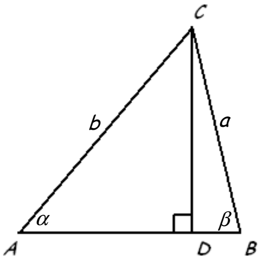
$\angle C = 180^\circ - 48^\circ - 76^\circ = 56^\circ.$

In ΔACD is $\frac{\sin 56^\circ}{1} = \frac{AD}{AC} = \frac{12\sin 76^\circ}{AC} \Rightarrow \frac{1 \cdot 12\sin 76^\circ}{\sin 56^\circ} = AC \approx 14{,}04.$

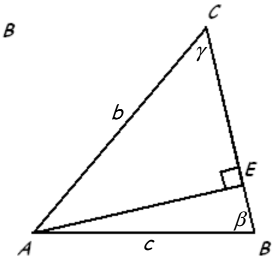




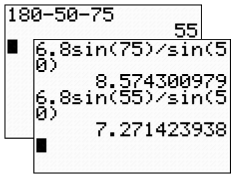
49 In $\triangle ADC$ is $\sin\alpha = \frac{CD}{b}$, dus $CD = b \cdot \sin\alpha$ ...(1)
In $\triangle BDC$ is $\sin\beta = \frac{CD}{a}$, dus $CD = a \cdot \sin\beta$ ...(2)
Uit (1) en (2) volgt $a\sin\beta = b\sin\alpha$.
Links en rechts delen door $\sin\alpha \cdot \sin\beta$ geeft $\frac{a \cdot \sin\beta}{\sin\alpha \cdot \sin\beta} = \frac{b \cdot \sin\alpha}{\sin\alpha \cdot \sin\beta}$,
dus $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ ...(3)

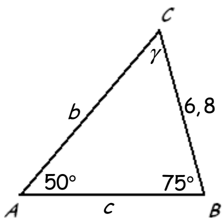


In $\triangle ABE$ is $\sin\beta = \frac{AE}{c}$, dus $AE = c \cdot \sin\beta$ ...(4)
In $\triangle AEC$ is $\sin\gamma = \frac{AE}{b}$, dus $AE = b \cdot \sin\gamma$ ...(5)
Uit (4) en (5) volgt $c\sin\beta = b\sin\gamma$.
Links en rechts delen door $\sin\beta \cdot \sin\gamma$ geeft $\frac{c \cdot \sin\beta}{\sin\beta \cdot \sin\gamma} = \frac{b \cdot \sin\gamma}{\sin\beta \cdot \sin\gamma}$,
dus $\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$ ...(6) Uit (3) en (6) volgt nu: $\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}\ \text{(de sinusregel)}}$.

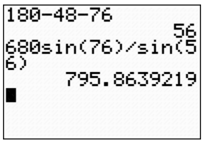


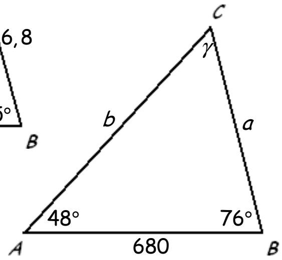
50a $\gamma = 180° - 50° - 75° = 55°$. (schets de driehoek)
50b $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow \frac{6,8}{\sin 50°} = \frac{b}{\sin 75°} = \frac{c}{\sin 55°}$.
$\frac{6,8 \cdot \sin 75°}{\sin 50°} = b \approx 8,6$ en $\frac{6,8 \cdot \sin 55°}{\sin 50°} = c \approx 7,3$.



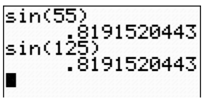


51 $\gamma = 180° - 48° - 76° = 56°$. (zie de schets hiernaast)
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow \frac{a}{\sin 48°} = \frac{b}{\sin 76°} = \frac{680}{\sin 56°}$.
Hieruit volgt: $\frac{680 \cdot \sin 76°}{\sin 56°} = b = AC \approx 796$ (m).

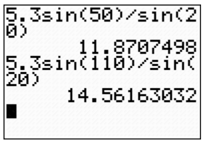


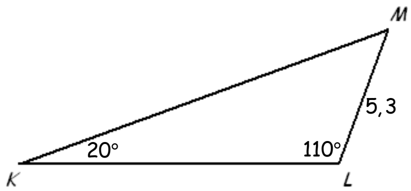


52a $\sin 55° \approx 0,819$.
52b $\sin 125° \approx 0,819$.

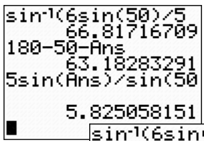


53 $\angle M = 180° - 20° - 110° = 50°$.
$\frac{LM}{\sin\angle K} = \frac{KM}{\sin\angle L} = \frac{KL}{\sin\angle M} \Rightarrow \frac{5,3}{\sin 20°} = \frac{KM}{\sin 110°} = \frac{KL}{\sin 50°}$.
$KL = \frac{5,3 \cdot \sin 50°}{\sin 20°} \approx 11,9$ en $KM = \frac{5,3 \cdot \sin 110°}{\sin 20°} \approx 14,6$.

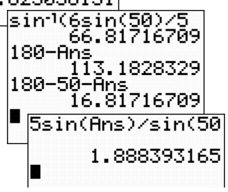


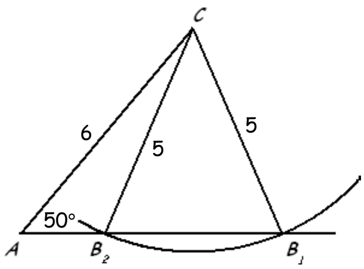


54a Zie de twee mogelijkheden B_1 en B_2 voor B hiernaast.
54b In $\triangle AB_1C \Rightarrow \frac{5}{\sin 50°} = \frac{6}{\sin\beta} = \frac{c}{\sin\gamma}$.
$\sin\beta = \frac{6 \cdot \sin 50°}{5} \Rightarrow \beta = \angle B_1 \approx 67°$ en $\gamma \approx 63°$.
$c = \frac{5 \cdot \sin\gamma°}{\sin 50°} \approx 5,8$.

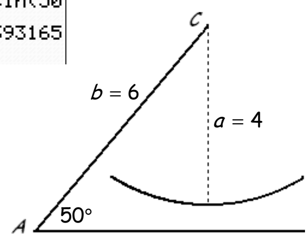


54c In $\triangle AB_2C \Rightarrow \frac{5}{\sin 50°} = \frac{6}{\sin\beta} = \frac{c}{\sin\gamma}$.
$\sin\beta = \frac{6 \cdot \sin 50°}{5} \Rightarrow \beta = \angle B_2 = 180° - \angle B_1 \approx 113°$ en $\gamma \approx 17°$.
$c = \frac{5 \cdot \sin\gamma°}{\sin 50°} \approx 1,9$.

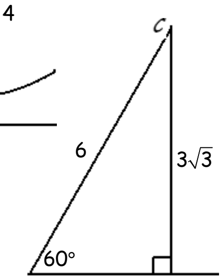




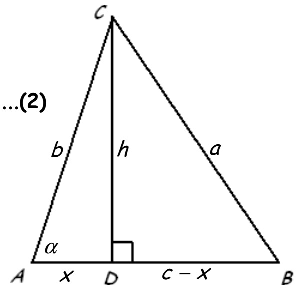
55 De cirkel met middelpunt C en straal 4
snijdt het andere been van hoek A niet.
(zie de constructie hiernaast)



56a In $\triangle ABC$ hiernaast is $AB = 3$ en $BC = 3\sqrt{3}$. (een 1-$\sqrt{3}$-2 driehoek)
Er is geen driehoek ABC mogelijk (met $\alpha = 60°$ en $b = 6$) voor $a < 3\sqrt{3}$.
56b Er is precies één driehoek ABC mogelijk (met $\alpha = 60°$ en $b = 6$) voor $a = 3\sqrt{3} \ \vee \ a \geq 6$.
56c Er zijn twee driehoeken ABC mogelijk (met $\alpha = 60°$ en $b = 6$) voor $3\sqrt{3} < a < 6$.



57a $\frac{4}{\sin\alpha} = \frac{5}{\sin\beta} = \frac{6}{\sin\gamma}$. Bij elke combinatie van twee breuken zijn er twee onbekenden $\Rightarrow$ de sinusregel loopt vast.
57b $\frac{QR}{\sin 50°} = \frac{5}{\sin\angle Q} = \frac{6}{\sin\angle R}$. Bij elke combinatie van twee breuken zijn er 2 onbekenden $\Rightarrow$ de sinusregel loopt vast.



58 De stelling van Pythagoras in ΔADC geeft $x^2 + h^2 = b^2$ ...**(1)**
De stelling van Pythagoras in ΔBDC geeft $a^2 = (c-x)^2 + h^2 \Rightarrow a^2 = c^2 - 2cx + x^2 + h^2$ ...**(2)**
Invullen van **(1)** in **(2)** geeft $a^2 = c^2 - 2cx + b^2$ ofwel $a^2 = b^2 + c^2 - 2cx$ ...**(3)**
In ΔADC is $\cos\alpha = \frac{x}{b} \Rightarrow x = b\cos\alpha$ ...**(4)**
Invullen van **(4)** in **(3)** geeft $a^2 = b^2 + c^2 - 2cb\cos\alpha \Rightarrow \boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$.
Het bewijs van de andere versies gaat net zo, of gebruik cyclische verwisseling.

59 $a^2 = b^2 + c^2 - 2bc\cos\alpha$
$5^2 = 6^2 + 7^2 - 2\cdot 6\cdot 7\cdot\cos\alpha$
$84\cos\alpha = 60$
$\cos\alpha = \frac{60}{84}$
$\alpha \approx 44°$

```
6²+7²-5²
                60
2*6*7
                84
cos⁻¹(60/84)
        44.4153086
```

$b^2 = a^2 + c^2 - 2ac\cos\beta$
$6^2 = 5^2 + 7^2 - 2\cdot 5\cdot 7\cdot\cos\beta$
$70\cos\beta = 38$
$\cos\beta = \frac{38}{70}$
$\beta \approx 57°$ en $\gamma \approx 78°$. (door afronden samen niet 180°)

```
5²+7²-6²
                38
2*5*7
                70
cos⁻¹(38/70)
       57.12165044
```

```
180-cos⁻¹(60/84)-
cos⁻¹(38/70)
       78.46304097
```

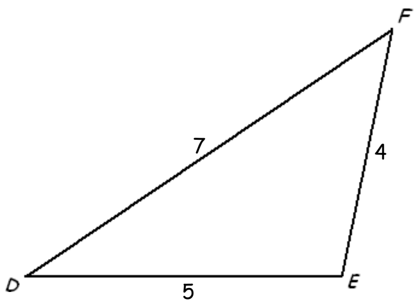
60 $EF^2 = DE^2 + DF^2 - 2\cdot DE\cdot DF\cdot\cos\angle D$
$4^2 = 5^2 + 7^2 - 2\cdot 5\cdot 7\cdot\cos\angle D$
$70\cos\angle D = 58$
$\cos\angle D = \frac{58}{70}$
$\angle D \approx 34°$.

```
5²+7²-4²
                58
2*5*7
                70
cos⁻¹(58/70)
       34.04773237
```

$DF^2 = DE^2 + EF^2 - 2\cdot DE\cdot EF\cdot\cos\angle E$
$7^2 = 5^2 + 4^2 - 2\cdot 5\cdot 4\cdot\cos\angle E$
$40\cos\angle E = -8$
$\cos\angle E = \frac{-8}{40}$
$\angle E \approx 102°$ en $\angle F \approx 44°$.

```
5²+4²-7²
                -8
2*5*4
                40
cos⁻¹(-8/40)
        101.536959
```

```
180-cos⁻¹(58/70)-
cos⁻¹(-8/40)
        44.4153086
```



61a $a^2 = b^2 + c^2 - 2bc\cos\alpha$
$a^2 = 5^2 + 6^2 - 2\cdot 5\cdot 6\cdot\cos 50°$
$a = \sqrt{5^2 + 6^2 - 2\cdot 5\cdot 6\cdot\cos 50°} \approx 4{,}74$.

```
5²+6²-2*5*6*cos(
50)
       22.43274342
√(Ans)→A
       4.736321718
```

61b $b^2 = a^2 + c^2 - 2ac\cos\beta$
$5^2 = a^2 + 6^2 - 2\cdot a\cdot 6\cdot\cos\beta$
$12a\cos\beta = a^2 + 6^2 - 5^2$
$\cos\beta = \frac{a^2 + 6^2 - 5^2}{12a}$
$\beta \approx 54°$.

```
(A²+6²-5²)/(12A)
       .5882332572
cos⁻¹(Ans)
       53.96826524
```

alternatieve uitwerking
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
$\frac{a}{\sin 50°} = \frac{5}{\sin\beta} = \frac{6}{\sin\gamma}$.
$\sin\beta = \frac{5\cdot\sin 50°}{a} \Rightarrow \beta \approx 54°$.

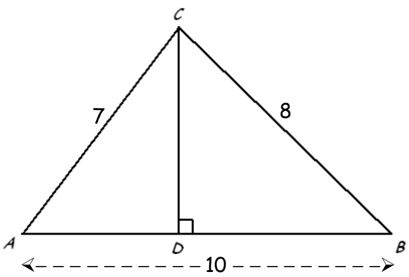
```
5*sin(50)/A
       .8086913101
sin⁻¹(Ans)
       53.96826524
```

62 $a^2 = b^2 + c^2 - 2bc\cos\alpha$
$8^2 = 7^2 + 10^2 - 2\cdot 7\cdot 10\cdot\cos\alpha$
$140\cos\alpha = 85$
$\cos\alpha = \frac{85}{140}$
$\alpha = \cos^{-1}\left(\frac{85}{140}\right)$ (nog niet afronden, maar opslaan)

```
7²+10²-8²
                85
2*7*10
               140
cos⁻¹(85/140)→A
       52.61680158
```

In ΔADC is $\frac{\sin\alpha}{1} = \frac{CD}{7}$
$CD = \frac{7\sin\alpha}{1} \approx 5{,}6$.

```
7sin(A)
       5.562148865
```

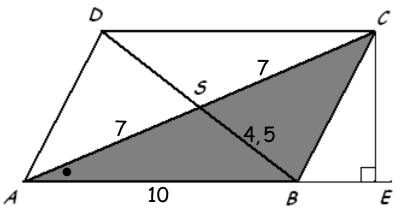


63a $BS^2 = AS^2 + AB^2 - 2\cdot AS\cdot AB\cdot\cos\angle A$
$4{,}5^2 = 7^2 + 10^2 - 2\cdot 7\cdot 10\cdot\cos\angle A$
$140\cos\angle A = 128{,}75$
$\cos\angle A = \frac{128{,}75}{140}$
$\angle A = \cos^{-1}\left(\frac{128{,}75}{140}\right)$ (opslaan)

```
7²+10²-4.5²
            128.75
2*7*10
               140
cos⁻¹(128.75/140)
→A
       23.12607419
```

$BC^2 = AC^2 + AB^2 - 2\cdot AC\cdot AB\cdot\cos\angle A$
$BC^2 = 14^2 + 10^2 - 2\cdot 14\cdot 10\cdot\cos\angle A$
$BC \approx 6{,}2$.

```
14²+10²-2*14*10*
cos(A)
              38.5
√(Ans)
       6.204836823
10*14sin(A)
       54.98579362
```



63b In ΔACE is $\frac{\sin\angle A}{1} = \frac{CE}{14}$
$CE = \frac{14\sin\angle A}{1}$ (nog niet afronden, maar opslaan)
$O(ABCD) = 2\cdot O(ABC) = AB\cdot CE \approx 55{,}0$.

64a Pythagoras in ΔABE: $EB^2 = 6^2 + 6^2 = 72 \Rightarrow BE = \sqrt{72} = \sqrt{36\cdot 2} = 6\sqrt{2}$. (of gebruik dat ΔABE een 1-1-$\sqrt{2}$ driehoek is)

64b Pythagoras in ΔBEH: $BH^2 = BE^2 + EH^2 = 72 + 6^2 = 108 \Rightarrow BH = \sqrt{108} = \sqrt{36\cdot 3} = 6\sqrt{3}$.

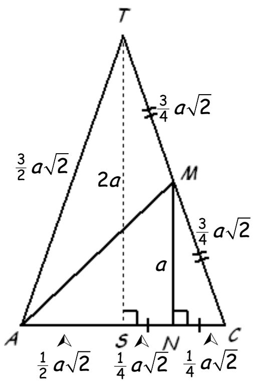
65a Pythagoras in $\triangle ABF$: $AF^2 = AB^2 + BF^2 = (2a)^2 + a^2 = 4a^2 + a^2 = 5a^2 \Rightarrow AF = \sqrt{5a^2} = \sqrt{a^2 \cdot 5} = a\sqrt{5}$.

65b Pythagoras in $\triangle AFG$: $AG^2 = AF^2 + FG^2 = 5a^2$ (zie 65a hierboven) $+ a^2 = 6a^2 \Rightarrow AG = \sqrt{6a^2} = \sqrt{a^2 \cdot 6} = a\sqrt{6}$.

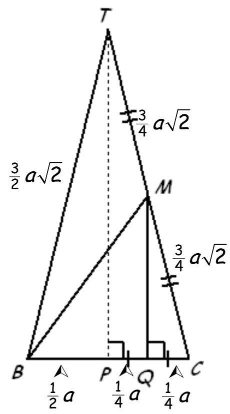
65c Pyth. in $\triangle ACM$: $AM^2 = AC^2 + MC^2 = 5a^2$ $(AC = AF)$ $+ (\frac{1}{2}a)^2 = 5\frac{1}{4}a^2 \Rightarrow AM = \sqrt{5\frac{1}{4}a^2} = \sqrt{\frac{21}{4}a^2} = \sqrt{\frac{1}{4}a^2 \cdot 21} = \frac{1}{2}a\sqrt{21}$.

66a $ABCD$ is een vierkant met zijde a, dus $AC = a\sqrt{2} \Rightarrow AS = \frac{1}{2}a\sqrt{2}$.
Pyth. in $\triangle AST$: $AT^2 = (\frac{1}{2}a\sqrt{2})^2 + (2a)^2 = \frac{1}{4}a^2 \cdot 2 + 4a^2 = 4\frac{1}{2}a^2 \Rightarrow AT = \sqrt{4\frac{1}{2}a^2} = \sqrt{\frac{9}{2}a^2 \cdot \frac{2}{2}} = \sqrt{\frac{9}{4}a^2 \cdot 2} = \frac{3}{2}a\sqrt{2}$.

66b Teken in een schets van $\triangle ACT$ lijn $MN \parallel ST$.
M is het midden van CT, dus N is het midden van CS. (snavelfiguur)
$SN = \frac{1}{2}CS = \frac{1}{4}AC = \frac{1}{4}a\sqrt{2} \Rightarrow AN = \frac{3}{4}a\sqrt{2}$; $MN = \frac{1}{2}ST = a$.
Pythagoras in $\triangle ANM$:
$AM^2 = AN^2 + NM^2 = (\frac{3}{4}a\sqrt{2})^2 + a^2 = \frac{9}{16}a^2 \cdot 2 + a^2 = \frac{34}{16}a^2$
$AM = \sqrt{\frac{34}{16}a^2} = \sqrt{\frac{1}{16}a^2 \cdot 34} = \frac{1}{4}a\sqrt{34}$.



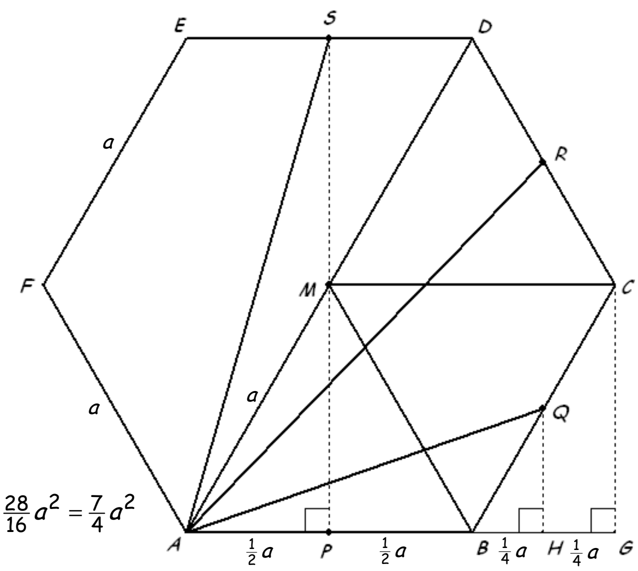
66c In $\triangle BCT$ is $BT = CT = AT = \frac{3}{2}a\sqrt{2}$. (zie 66a hierboven)
Teken TP in $\triangle BCT$ loodrecht op BC en $MQ \parallel TP$.
$TM = MC = \frac{1}{2} \cdot \frac{3}{2}a\sqrt{2} = \frac{3}{4}a\sqrt{2} \Rightarrow MQ = \frac{1}{2}TP$. (snavelfiguur)
Pythagoras in $\triangle QCM$:
$QM^2 = CM^2 - QC^2 = (\frac{3}{4}a\sqrt{2})^2 - (\frac{1}{4}a)^2 = \frac{9}{16}a^2 \cdot 2 - \frac{1}{16}a^2 = \frac{18}{16}a^2 - \frac{1}{16}a^2 = \frac{17}{16}a^2$.
Pythagoras in $\triangle BQM$:
$BM^2 = BQ^2 + QM^2 = (\frac{3}{4}a)^2 + \frac{17}{16}a^2 = \frac{9}{16}a^2 + \frac{17}{16}a^2 = \frac{26}{16}a^2$
$BM = \sqrt{\frac{26}{16}a^2} = \sqrt{\frac{1}{16}a^2 \cdot 26} = \frac{1}{4}a\sqrt{26}$.

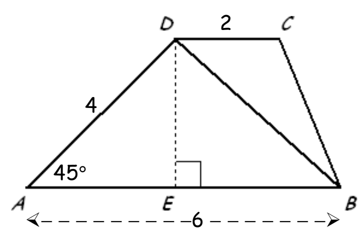


67a $\triangle ABM$ is een gelijkzijdige driehoek met zijde a.
Pythagoras in $\triangle APM$:
$PM^2 = AM^2 - AP^2 = a^2 - (\frac{1}{2}a)^2 = a^2 - \frac{1}{4}a^2 = \frac{3}{4}a^2$
$PM = \sqrt{\frac{3}{4}a^2} = \sqrt{\frac{1}{4}a^2 \cdot 3} = \frac{1}{2}a\sqrt{3}$. (of met de 1-$\sqrt{3}$-2 driehoek)
$PS = 2 \cdot PM = 2 \cdot \frac{1}{2}a\sqrt{3} = a\sqrt{3}$.

67b $AC = BD = PS = a\sqrt{3}$.

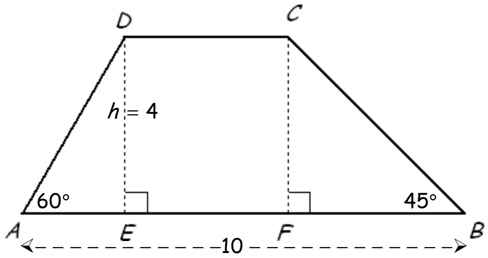
67c Teken QH en CG loodrecht op AB.
In $\triangle BGC$ is $BG = PB = \frac{1}{2}a$ en $GC = PM = \frac{1}{2}a\sqrt{3}$.
Omdat Q het midden is van BC is (snavelfiguur in $\triangle BGC$)
$HQ = \frac{1}{2}GC = \frac{1}{4}a\sqrt{3}$ en $BH = \frac{1}{2}BG = \frac{1}{4}a$. Dus $AH = 1\frac{1}{4}a$.
Pythagoras in $\triangle AHQ$:
$AQ^2 = AH^2 + HQ^2 = (\frac{5}{4}a)^2 + (\frac{1}{4}a\sqrt{3})^2 = \frac{25}{16}a^2 + \frac{1}{16}a^2 \cdot 3 = \frac{28}{16}a^2 = \frac{7}{4}a^2$
$AQ = \sqrt{\frac{7}{4}a^2} = \sqrt{\frac{1}{4}a^2 \cdot 7} = \frac{1}{2}a\sqrt{7}$.
$AR = AS$ (in vierhoek $ARDS$ is AD symmetrieas)
Pythagoras in $\triangle APS$: $AS^2 = AP^2 + PS^2 = (\frac{1}{2}a)^2 + (a\sqrt{3})^2 = \frac{1}{4}a^2 + a^2 \cdot 3 = 3\frac{1}{4}a^2$
$AR = AS = \sqrt{\frac{13}{4}a^2} = \sqrt{\frac{1}{4}a^2 \cdot 13} = \frac{1}{2}a\sqrt{13}$.

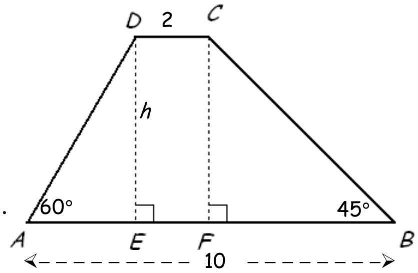




68 $DE = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$. ($\triangle ADE$ is een 1-1-$\sqrt{2}$ driehoek)
$O(ABCD) = O(ABD) + O(BCD) = \frac{1}{2} \cdot 6 \cdot 2\sqrt{2} + \frac{1}{2} \cdot 2 \cdot 2\sqrt{2} = 6\sqrt{2} + 2\sqrt{2} = 8\sqrt{2}$.

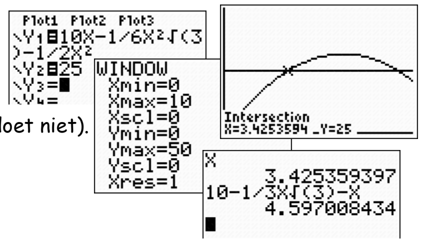
69a $AE = \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = 1\frac{1}{3}\sqrt{3}$ en $FB = 4$. (N.B.: $\angle B = 45^\circ$, zie figuur)
$DC = EF = AB - AE - FB = 10 - 1\frac{1}{3}\sqrt{3} - 4 = 6 - 1\frac{1}{3}\sqrt{3}$.
$O(ABCD) = \frac{1}{2} \cdot (10 + 6 - 1\frac{1}{3}\sqrt{3}) \cdot 4 = 32 - 2\frac{2}{3}\sqrt{3}$.





69b $DE = h \Rightarrow AE = \frac{h}{\sqrt{3}} = \frac{h}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{h\sqrt{3}}{3} = \frac{1}{3}h\sqrt{3}$ en $BF = h$.
$AB = AE + EF + FB = 10$ geeft
$\frac{1}{3}h\sqrt{3} + 2 + h = 10$
$h(\frac{1}{3}\sqrt{3} + 1) = 8$
$h = \frac{8}{\frac{1}{3}\sqrt{3}+1} = \frac{8}{\frac{1}{3}\sqrt{3}+1} \cdot \frac{\frac{1}{3}\sqrt{3}-1}{\frac{1}{3}\sqrt{3}-1} = \frac{8\cdot(\frac{1}{3}\sqrt{3}-1)}{\frac{1}{3}-1} = \frac{8\cdot(\frac{1}{3}\sqrt{3}-1)}{-\frac{2}{3}} = -12\cdot(\frac{1}{3}\sqrt{3}-1) = -4\sqrt{3}+12.$
$O(ABCD) = \frac{1}{2}\cdot(10+2)\cdot(-4\sqrt{3}+12) = 6\cdot(-4\sqrt{3}+12) = -24\sqrt{3}+72.$

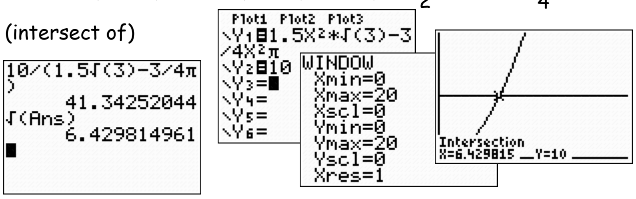
69c $AE = \frac{1}{3}h\sqrt{3}$ en $BF = h \Rightarrow CD = EF = 10 - \frac{1}{3}h\sqrt{3} - h.$
$O(ABCD) = \frac{1}{2}\cdot(10 + 10 - \frac{1}{3}h\sqrt{3} - h)\cdot h = 10h - \frac{1}{6}h^2\cdot\sqrt{3} - \frac{1}{2}h^2 = 25$ (intersect geeft)
$h \approx 3{,}425 \Rightarrow CD = 10 - \frac{1}{3}h\sqrt{3} - h \approx 4{,}60$ ($h \approx 9{,}425 \Rightarrow CD = 10 - \frac{1}{3}h\sqrt{3} - h \approx -4{,}60$ voldoet niet).

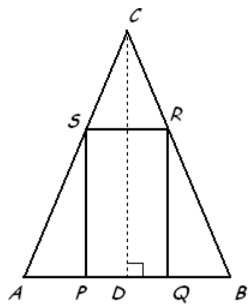


70a Zij N het midden van AB dan $AN = \frac{1}{2}a$ en $MN = \frac{1}{2}a\sqrt{3}$.
$O(ABCDEF) = 6\cdot O(ABC) = 6\cdot\frac{1}{2}\cdot a\cdot\frac{1}{2}a\sqrt{3} = 1\frac{1}{2}a^2\cdot\sqrt{3}.$

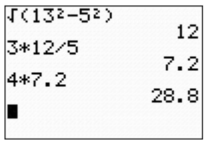
70b $O(\text{incirkel}) = \pi\cdot MN^2 = \pi\cdot(\frac{1}{2}a\sqrt{3})^2 = \pi\cdot\frac{1}{4}a^2\cdot 3 = \frac{3}{4}a^2\pi.$

70c $O(\text{binnen zeshoek en buiten incirkel}) = O(ABCDEF) - O(\text{incirkel}) = 1\frac{1}{2}a^2\cdot\sqrt{3} - \frac{3}{4}a^2\pi.$
$1\frac{1}{2}a^2\cdot\sqrt{3} - \frac{3}{4}a^2\pi = 10$ (intersect of)
$a^2\cdot(1\frac{1}{2}\sqrt{3} - \frac{3}{4}\pi) = 10$
$a^2 = \frac{10}{1\frac{1}{2}\sqrt{3} - \frac{3}{4}\pi} \approx 41{,}3$
$a \approx 6{,}43.$



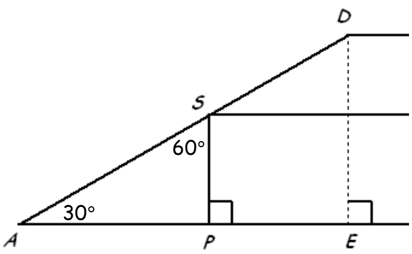


71 $AP = 3 \Rightarrow PQ = 10 - 2\cdot 3 = 4.$
In ΔADC (Pythagoras): $CD^2 = 13^2 - 5^2 = 144 \Rightarrow CD = 12.$
$\Delta APS \backsim \Delta ADC$ (snavelfiguur): $\frac{AP}{AD} = \frac{PS}{DC} \Rightarrow \frac{3}{5} = \frac{PS}{12} \Rightarrow PS = \frac{3\cdot 12}{5} = 7{,}2.$
$O(PQRS) = PQ\cdot PS = 4\cdot 7{,}2 = 28{,}8.$



72a $CP = x \Rightarrow AP = 4 - x.$
$\Delta CPQ \backsim \Delta CAB$ (snavelfiguur): $\frac{CP}{CA} = \frac{PQ}{AB} \Rightarrow \frac{x}{4} = \frac{PQ}{3} \Rightarrow PQ = \frac{3\cdot x}{4} = \frac{3}{4}x.$
$O(\Delta BPQ) = \frac{1}{2}\text{basis}\times\text{hoogte} = \frac{1}{2}PQ\cdot AP = \frac{1}{2}\cdot\frac{3}{4}x\cdot(4-x) = \frac{3}{8}x\cdot(4-x).$

72b $O(\Delta BPQ) = \frac{3}{8}x\cdot(4-x) = \frac{3}{2}x - \frac{3}{8}x^2 = -\frac{3}{8}x^2 + \frac{3}{2}x.$
$x_{top} = -\frac{b}{2a} = -\frac{\frac{3}{2}}{2\cdot -\frac{3}{8}} = \frac{\frac{3}{2}}{\frac{3}{4}} = \frac{3}{2}\cdot\frac{4}{3} = 2 \Rightarrow O_{max} = \frac{3}{8}\cdot 2\cdot(4-2) = \frac{3}{8}\cdot 2\cdot 2 = \frac{3}{2} = 1\frac{1}{2}.$

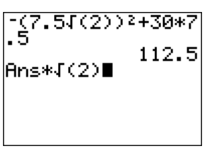


73a $PS = x$ (en ΔAPS is een 1-$\sqrt{3}$-2 driehoek) $\Rightarrow AS = 2x$ en $AP = x\sqrt{3}$ ($= QB$).
$AD = 6$ (en ΔAED is een 1-$\sqrt{3}$-2 driehoek) $\Rightarrow DE = 3$ en $AE = 3\sqrt{3}$.
$AB = 2AE + DC = 2\cdot 3\sqrt{3} + 6 = 6\sqrt{3} + 6 \Rightarrow PQ = AB - 2AP = 6\sqrt{3} + 6 - 2\cdot x\sqrt{3}.$
$O(PQRS) = PQ\times PS = (6\sqrt{3} + 6 - 2x\sqrt{3})\cdot x.$

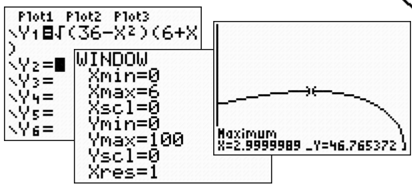
73b $O(PQRS) = (6\sqrt{3} + 6 - 2x\sqrt{3})\cdot x = -2\sqrt{3}\cdot x^2 + (6\sqrt{3}+6)\cdot x.$
$x_{top} = -\frac{b}{2a} = -\frac{6\sqrt{3}+6}{2\cdot -2\sqrt{3}} = \frac{6\sqrt{3}+6}{4\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{(6\sqrt{3}+6)\cdot\sqrt{3}}{12} = \frac{18+6\sqrt{3}}{12} = \frac{18}{12} + \frac{6\sqrt{3}}{12} = \frac{3}{2} + \frac{1}{2}\sqrt{3}.$
Dus $O(PQRS)$ is maximaal voor $PS = x = 1\frac{1}{2} + \frac{1}{2}\sqrt{3}.$

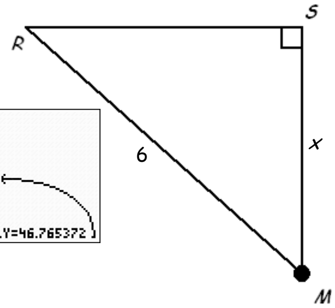
74a $DE = h$ (en ΔAED is een 1-1-$\sqrt{2}$ driehoek) $\Rightarrow AE = DE = h$ en $AD = h\sqrt{2}$.
Omtrek$(ABCD) = 60 \Rightarrow h + x + h + h\sqrt{2} + x + h\sqrt{2} = 2x + 2h + 2h\sqrt{2} = 60 \Rightarrow 2x = 60 - 2h - 2h\sqrt{2} \Rightarrow x = 30 - h - h\sqrt{2}.$

74b $O(ABCD) = 2\times O(\Delta AED) + O(EFCD) = h^2 + h\times x = h^2 + h\times(30 - h - h\sqrt{2}) = h^2 + 30h - h^2 - h^2\cdot\sqrt{2} = -\sqrt{2}\cdot h^2 + 30h\cdot$
$h_{top} = -\frac{b}{2a} = -\frac{30}{2\cdot -\sqrt{2}} = \frac{30}{2\cdot\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{30\sqrt{2}}{4} = 7{,}5\sqrt{2}.$
$O_{max} = -\sqrt{2}\cdot(7{,}5\sqrt{2})^2 + 30\cdot 7{,}5\sqrt{2} = 112{,}5\sqrt{2}.$



75 In ΔRMS (Pythgoras): $RS^2 = 6^2 - x^2 \Rightarrow RS = \sqrt{36 - x^2}$ en $RQ = 2RS = 2\sqrt{36 - x^2}$.
$O(\Delta PQR) = \frac{1}{2}\text{basis} \times \text{hoogte} = \frac{1}{2} RQ \cdot SP = \frac{1}{2} \cdot 2\sqrt{36 - x^2} \cdot (6 + x) = \sqrt{36 - x^2} \cdot (6 + x)$.
Deze formule invoeren op de GR.
Optie maximum geeft dan $x = 3$ en $y \approx 46{,}77$.
De maximale oppervlakte van ΔPQR is ongeveer 46,77.





Diagnostische toets

D1a $4\sqrt5\cdot3\sqrt2 = 4\cdot3\cdot\sqrt5\cdot\sqrt2 = 12\sqrt{10}.$

D1b $\sqrt{16\frac13} = \sqrt{\frac{49}{3}} = \frac{\sqrt{49}}{\sqrt3} = \frac{7}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3} = \frac{7\sqrt3}{3} = \frac73\sqrt3.$

D1c $\frac{6}{\sqrt2}+\sqrt8 = \frac{6}{\sqrt2}\cdot\frac{\sqrt2}{\sqrt2}+\sqrt{4\cdot2} = \frac{6\sqrt2}{2}+2\sqrt2 = 3\sqrt2+2\sqrt2 = 5\sqrt2.$

D1d $\sqrt{\frac13}+\sqrt3 = \frac{\sqrt1}{\sqrt3}+\sqrt3 = \frac{1}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3}+\sqrt3 = \frac{\sqrt3}{3}+\sqrt3 = \frac13\sqrt3+\sqrt3 = 1\frac13\sqrt3.$

D1e $\sqrt{8a^2}+\sqrt{32a^2} = \sqrt{4\cdot a^2\cdot2}+\sqrt{16\cdot a^2\cdot2} = 2|a|\sqrt2+4|a|\sqrt2 = 6|a|\sqrt2.$

D1f $a\sqrt{48}-2a\sqrt{12} = a\sqrt{16\cdot3}-2a\sqrt{4\cdot3} = a\cdot4\sqrt3-2a\cdot2\sqrt3 = 4a\sqrt3-4a\sqrt3 = 0.$

D2a $(3+\sqrt2)^2 = 3^2+2\cdot3\cdot\sqrt2+\sqrt2^2 = 9+6\sqrt2+2 = 11+6\sqrt2.$

D2b $\frac{\sqrt3}{\sqrt5+\sqrt7} = \frac{\sqrt3}{\sqrt5+\sqrt7}\cdot\frac{\sqrt5-\sqrt7}{\sqrt5-\sqrt7} = \frac{\sqrt3\cdot(\sqrt5-\sqrt7)}{5-7} = \frac{\sqrt{15}-\sqrt{21}}{-2} = -\frac12\sqrt{15}+\frac12\sqrt{21}.$

D2c $(a-\sqrt3)(a+\sqrt3) = a^2-\sqrt3^2 = a^2-3.$

D2d $\frac{20}{\sqrt6-1} = \frac{20}{\sqrt6-1}\cdot\frac{\sqrt6+1}{\sqrt6+1} = \frac{20\cdot(\sqrt6+1)}{6-1} = \frac{20\sqrt6+20}{5} = 4\sqrt6+4.$

D2e $(2a-\sqrt7)^2 = (2a)^2-2\cdot2a\cdot\sqrt7+\sqrt7^2 = 4a^2-4a\sqrt7+7.$

D2f $\left(\frac{2}{\sqrt5-1}\right)^2 = \frac{2^2}{(\sqrt5-1)^2} = \frac{4}{\sqrt5^2-2\cdot\sqrt5\cdot1+1^2} = \frac{4}{5-2\sqrt5+1} = \frac{4}{6-2\sqrt5} = \frac{2}{3-\sqrt5}\cdot\frac{3+\sqrt5}{3+\sqrt5} = \frac{2\cdot(3+\sqrt5)}{3^2-\sqrt5^2} = \frac{6+2\sqrt5}{9-5} = \frac{6+2\sqrt5}{4} = 1\frac12+\frac12\sqrt5.$

D3a $\frac{1}{3a}-\frac{1}{4a} = \frac{1}{3a}\cdot\frac44-\frac{1}{4a}\cdot\frac33 = \frac{4}{12a}-\frac{3}{12a} = \frac{1}{12a}.$

D3b $\frac{1}{5x}+\frac{1}{10x} = \frac{1}{5x}\cdot\frac22+\frac{1}{10x} = \frac{2}{10x}+\frac{1}{10x} = \frac{3}{10x}.$

D3c $\frac{1}{x-2}-\frac{2}{x+1} = \frac{1(x+1)}{(x-2)(x+1)}-\frac{2(x-2)}{(x+1)(x-2)} = \frac{x+1}{(x-2)(x+1)}-\frac{2x-4}{(x-2)(x+1)} = \frac{x+1-2x+4}{(x-2)(x+1)} = \frac{-x+5}{(x-2)(x+1)}.$

D3d $\frac{2x}{x+1}+\frac{5}{x-3} = \frac{2x(x-3)}{(x+1)(x-3)}+\frac{5(x+1)}{(x-3)(x+1)} = \frac{2x^2-6x}{(x+1)(x-3)}+\frac{5x+5}{(x+1)(x-3)} = \frac{2x^2-6x+5x+5}{(x+1)(x-3)} = \frac{2x^2-x+5}{(x+1)(x-3)}.$

D3e $x+\frac{3}{x+1} = \frac{x(x+1)}{x+1}+\frac{3}{x+1} = \frac{x^2+x}{x+1}+\frac{3}{x+1} = \frac{x^2+x+3}{x+1}.$

D3f $\frac{2a}{b}+\frac{a}{a+b} = \frac{2a(a+b)}{b(a+b)}+\frac{ab}{b(a+b)} = \frac{2a^2+2ab+ab}{b(a+b)} = \frac{2a^2+3ab}{b(a+b)}.$

D4a $\frac{x^2-6x+5}{x^2-25} = \frac{\cancel{(x-5)}(x-1)}{(x+5)\cancel{(x-5)}} = \frac{x-1}{x+5}.$

D4b $\frac{6x^2+6x}{x^2+3x+2} = \frac{6x\cancel{(x+1)}}{(x+2)\cancel{(x+1)}} = \frac{6x}{x+2}.$

D4c $\frac{x^2+6x+8}{x+2}+\frac{x^2+8}{x} = \frac{(x+4)\cancel{(x+2)}}{\cancel{x+2}}+\frac{x^2}{x}+\frac{8}{x} = x+4+x+\frac8x = 2x+4+\frac8x.$

D5a $\frac6x-\frac{4}{x+2} = 2$

$\frac{6(x+2)}{x(x+2)}-\frac{4x}{x(x+2)} = \frac{2x(x+2)}{x(x+2)}$

$6x+12-4x = 2x^2+4x$

$-2x^2-2x+12 = 0$

$x^2+x-6 = 0$

$(x+3)(x-2) = 0$

$x=-3 \vee x=2.$

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D5b $\frac{x}{16} = \frac{x^2-4}{x^2+6x+8}$

$\frac{x}{16} = \frac{\cancel{(x+2)}(x-2)}{(x+4)\cancel{(x+2)}}$

$x\cdot(x+4) = 16\cdot(x-2)$

$x^2+4x = 16x-32$

$x^2-12x+32 = 0$

$(x-8)(x-4) = 0$

$x=8 \vee x=4.$

voldoet voldoet

D5c $\frac{x^2-9}{x^2+4x+3} = \frac{5}{3x}$

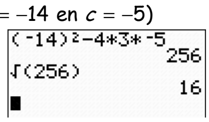
$\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x+1)} = \frac{5}{3x}$

$3x\cdot(x-3) = 5\cdot(x+1)$

$3x^2-9x = 5x+5$

$3x^2-14x-5 = 0$ ($a=3;\ b=-14$ en $c=-5$)

$D = (-14)^2-4\cdot3\cdot-5 = 256$



$x = \frac{14\pm\sqrt{256}}{2\cdot3} = \frac{14\pm16}{6}$

$x = \frac{30}{6} = 5 \vee x = \frac{-2}{6} = -\frac13.$ (voldoen)

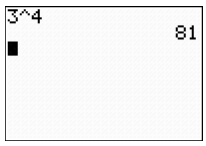
D6a $2a^3\cdot3a^6 = 2\cdot3\cdot a^3\cdot a^6 = 6a^{3+6} = 6a^9.$

D6b $a^{12}\cdot\frac{1}{a^4} = \frac{a^{12}}{a^4} = a^{12-4} = a^8.$

D6c $(2a)^3-a\cdot7a^2 = 8a^3-7a^3 = a^3.$

D6d $\frac{14a^8}{2a^5} = 7a^3.$

D6e $(3a^2)^4+5(a^4)^2 = 81a^8+5a^8 = 86a^8.$



D6f $\frac{1}{a^6}\cdot(a^2)^3 = \frac{1}{a^6}\cdot a^6 = \frac{a^6}{a^6} = 1.$

D7a $\frac{1}{a^3} = a^{-3}.$

D7b $a^4\cdot\frac{1}{a^7} = \frac{a^4}{a^7} = a^{4-7} = a^{-3}.$

D7c $\sqrt[5]{a^3} = a^{\frac35}.$

D7d $\frac{\sqrt a}{a^2} = \frac{a^{\frac12}}{a^2} = a^{-1\frac12}.$

D7e $a^2\cdot\sqrt[3]{a} = a^2\cdot a^{\frac13} = a^{2\frac13}.$

D7f $\frac{1}{\sqrt[3]{a^2}} = \frac{1}{a^{\frac23}} = a^{-\frac23}.$

D8a $\left(a^{-\frac{1}{4}}\right)^3 = a^{-\frac{3}{4}} = \frac{1}{a^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{a^3}}.$

D8b $a^{-2}\cdot b^{\frac{1}{5}} = \frac{1}{a^2}\cdot\sqrt[5]{b} = \frac{\sqrt[5]{b}}{a^2}.$

D8c $7a^{-\frac{1}{3}}\cdot b^{\frac{3}{5}} = 7\cdot\frac{1}{a^{\frac{1}{3}}}\cdot\sqrt[5]{b^3} = \frac{7\cdot\sqrt[5]{b^3}}{\sqrt[3]{a}}.$

D9a $3x^{1,6}+2=7$

$3x^{1,6}=5$

$x^{1,6}=\frac{5}{3}$

$x=\left(\frac{5}{3}\right)^{\frac{1}{1,6}}\approx 1{,}376.$

```
7-2
                 5
Ans/3
        1.666666667
Ans^(1/1.6)
        1.376117344
```

D9b $\frac{1}{4}x^{-3,7}=160$

$x^{-3,7}=640$

$x=640^{\frac{1}{-3,7}}\approx 0{,}174.$

```
160*4
               640
Ans^(1/-3.7)
        .1744106417
```

D9c $7\cdot\sqrt[5]{x^3}=48$

$x^{\frac{3}{5}}=\frac{48}{7}$

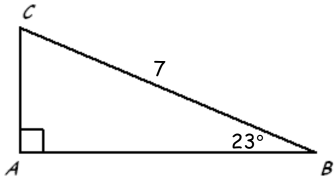
$x=\left(\frac{48}{7}\right)^{\frac{1}{0,6}}\approx 24{,}750.$

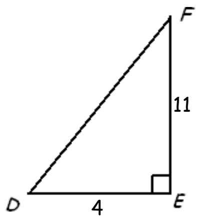
```
48/7
        6.857142857
Ans^(1/(3/5))
         24.7498172
```

D10a $\frac{\cos 23^\circ}{1}=\frac{AB}{7}\Rightarrow AB=7\cos 23^\circ\approx 6{,}44.$

D10b $\tan\angle D=\frac{11}{4}$ terug [tan...] $\Rightarrow \angle D\approx 70^\circ.$

```
7cos(23)
        6.443533974
tan⁻¹(11/4)
        70.01689348
```

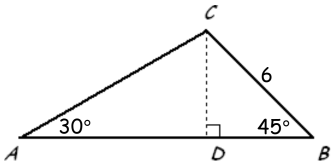




D11 In ΔBDC is $BD=CD=\frac{6}{\sqrt{2}}=\frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}$. ($\Delta BDC$ is een 1-1-$\sqrt{2}$ driehoek)

In ΔADC is $AD=CD\cdot\sqrt{3}=3\sqrt{2}\cdot\sqrt{3}=3\sqrt{6}$. ($\Delta ADC$ is een 1-$\sqrt{3}$-2 driehoek)

$O(ABC)=\frac{1}{2}\cdot AB\cdot CD=\frac{1}{2}\cdot(3\sqrt{6}+3\sqrt{2})\cdot 3\sqrt{2}=4\frac{1}{2}\sqrt{12}+9=4\frac{1}{2}\sqrt{4\cdot 3}+9=9\sqrt{3}+9.$



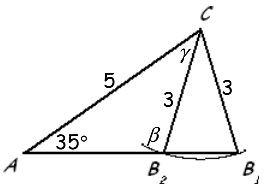
D12a $\frac{3}{\sin 35^\circ}=\frac{5}{\sin\beta}$

$\sin\beta=\frac{5\sin 35^\circ}{3}$

$\begin{cases}\beta\approx 73^\circ\\ \gamma\approx 180^\circ-35^\circ-73^\circ=72\end{cases}$ of $\begin{cases}\beta\approx 180^\circ-73^\circ=107^\circ\\ \gamma\approx 180^\circ-35^\circ-107^\circ=38^\circ.\end{cases}$

```
5sin(35)/3
        .9559607273
sin⁻¹(Ans)→B
        72.93271028
180-35-B
        72.06728972
```

```
180-B
        107.0672897
180-35-Ans
        37.93271028
```



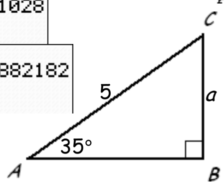
D12b $\sin 35^\circ=\frac{a}{5}\Rightarrow a=5\cdot\sin 35^\circ\approx 2{,}87.$

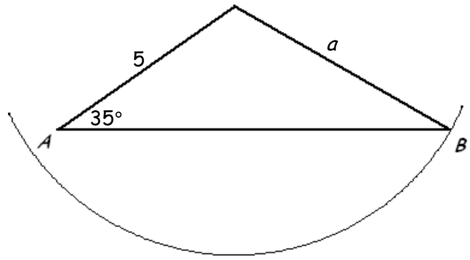
(zie de eerste figuur hiernaast)

Dus één mogelijkheid als $a\approx 2{,}87 \vee a\geq 5.$

(zie ook de tweede figuur hiernaast)

```
5sin(35)
        2.867882182
```





D13a Cosinusregel in ΔABC:

$5^2=6^2+4^2-2\cdot 6\cdot 4\cdot\cos\beta$

$48\cos\beta=27$

$\cos\beta=\frac{27}{48}\Rightarrow\beta\approx 56^\circ.$

```
6²+4²-5²
                27
2*6*4
                48
cos⁻¹(27/48)→B
        55.77113367
```

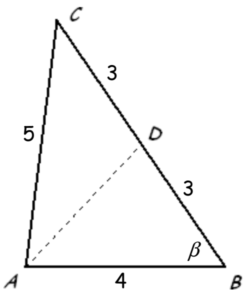
D13b Cosinusregel in ΔABD:

$AD^2=4^2+3^2-2\cdot 4\cdot 3\cdot\cos\beta$

$AD^2=4^2+3^2-2\cdot 4\cdot 3\cdot\frac{27}{48}=11{,}5$

$AD=\sqrt{11,,5}\approx 3{,}39.$

```
4²+3²-2*4*3*cos(
B)
               11.5
√(Ans)
        3.391164992
```



D14a Pythagoras in ΔADM: ($\angle D=90^\circ$)

$AM^2=a^2+(\frac{3}{2}a)^2$

$AM^2=a^2+\frac{9}{4}a^2=\frac{13}{4}a^2$

$AM=\sqrt{\frac{13}{4}a^2}=\sqrt{\frac{1}{4}a^2\cdot 13}=\frac{1}{2}a\sqrt{13}.$

D14b Pythagoras in ΔFBM: ($\angle B=90^\circ$ en $AM=BM=\frac{1}{2}a\sqrt{13}$)

$FM^2=(\frac{1}{2}a\sqrt{13})^2+(2a)^2$

$FM^2=\frac{13}{4}a^2+4a^2=\frac{29}{4}a^2$

$FM=\sqrt{\frac{29}{4}a^2}=\sqrt{\frac{1}{4}a^2\cdot 29}=\frac{1}{2}a\sqrt{29}.$

D14c Pythagoras in ΔBFS: ($\angle F=90^\circ$ en $FS=\frac{1}{2}FH$)

$BS^2=(2a)^2+(\frac{1}{2}a\sqrt{10})^2$

$BS^2=4a^2+\frac{1}{4}a^2\cdot 10=\frac{26}{4}a^2$

$BS=\sqrt{\frac{26}{4}a^2}=\sqrt{\frac{1}{4}a^2\cdot 26}=\frac{1}{2}a\sqrt{26}.$

(eerst FH berekenen in ΔFGH)

Pythagoras in ΔFGH: ($\angle G=90^\circ$)

$FH^2=(3a)^2+a^2$

$FH^2=9a^2+a^2=10a^2$

$FH=\sqrt{10a^2}=\sqrt{a^2\cdot 10}=a\sqrt{10}.$

D15 In ΔFBC is $FB = \frac{6}{2} = 3$ en $FC = 3\sqrt{3}$. (ΔFBC is een 1-$\sqrt{3}$-2 driehoek)

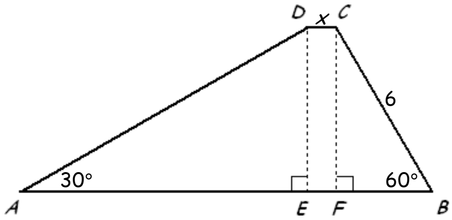
In ΔAED is $DE = FC = 3\sqrt{3}$; $AD = 6\sqrt{3}$ en $AE = 3\sqrt{3} \cdot \sqrt{3} = 9$. ($\Delta AED$ is een 1-$\sqrt{3}$-2 driehoek)

$O(ABCD) = \frac{1}{2} \cdot (AB + CD) \cdot DE$ (stel $EF = CD = x$)

$= \frac{1}{2} \cdot (9 + x + 3 + x) \cdot 3\sqrt{3}$

$= \frac{1}{2} \cdot (12 + 2x) \cdot 3\sqrt{3}$

$= 18\sqrt{3} + 3x\sqrt{3}$.



$O(ABCD) = 36$ geeft dan:

$18\sqrt{3} + 3x\sqrt{3} = 36$

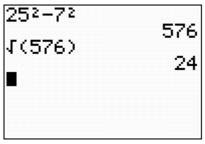
$3x\sqrt{3} = 36 - 18\sqrt{3}$

$x = \frac{36 - 18\sqrt{3}}{3 \cdot \sqrt{3}} = \frac{36 - 18\sqrt{3}}{3 \cdot \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{(36 - 18\sqrt{3}) \cdot \sqrt{3}}{9} = \frac{36\sqrt{3} - 54}{9} = 4\sqrt{3} - 6$. Dus $AB = 9 + 4\sqrt{3} - 6 + 3 = 6 + 4\sqrt{3}$.

D16 Teken CD loodrecht op AB.

$AD = \frac{1}{2} AB = 7$.

In ΔADC is $CD^2 = AC^2 - AD^2 = 25^2 - 7^2 = 576 \Rightarrow CD = 24$.



Stel $AK = LB = x$.

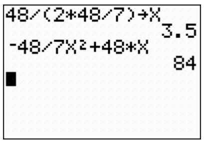
$\Delta AKN \backsim \Delta ADC$ (snavelfiguur): $\frac{AK}{AD} = \frac{KN}{DC} \Rightarrow \frac{x}{7} = \frac{KN}{24} \Rightarrow KN = \frac{24 \cdot x}{7} = \frac{24}{7} x$.

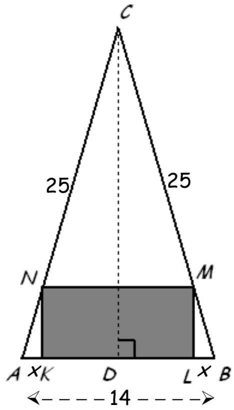
$KL = AB - 2x = 14 - 2x$

$O(KLMN) = KL \times KN = (14 - 2x) \cdot \frac{24}{7} x = 48x - \frac{48}{7} x^2 = -\frac{48}{7} x^2 + 48x$.

$x_{top} = -\frac{b}{2a} = -\frac{48}{2 \cdot -\frac{48}{7}} = \frac{48}{\frac{96}{7}} = 48 \cdot \frac{7}{96} = \frac{7}{2}$.

De maximale oppervlakte is $-\frac{48}{7} \cdot (\frac{7}{2})^2 + 48 \cdot \frac{7}{2} = 84$.





Gemengde opgaven 4. Algebra en meetkunde

G31a $(2a+\sqrt{3})^2 = (2a)^2 + 2\cdot 2a\cdot\sqrt{3} + \sqrt{3}^2 = 4a^2 + 4a\sqrt{3} + 3.$

G31b $(a+2\sqrt{3})(a-2\sqrt{3}) = a^2 - (2\sqrt{3})^2 = a^2 - 4\cdot 3 = a^2 - 12.$

G31c $(2\sqrt{2}+3\sqrt{8})^2 = (2\sqrt{2})^2 + 2\cdot 2\sqrt{2}\cdot 3\sqrt{8} + (3\sqrt{8})^2 = 4\cdot 2 + 12\sqrt{16} + 9\cdot 8 = 8 + 12\cdot 4 + 72 = 128.$

G31d $\sqrt{\frac{1}{2}} + 6\sqrt{32} = \frac{\sqrt{1}}{\sqrt{2}} + 6\sqrt{16\cdot 2} = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} + 6\cdot 4\cdot\sqrt{2} = \frac{\sqrt{2}}{2} + 24\sqrt{2} = \frac{1}{2}\sqrt{2} + 24\sqrt{2} = 24\frac{1}{2}\sqrt{2}.$

G31e $\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}^2 + 2\cdot\sqrt{2}\cdot 1 + 1^2}{2-1} = \frac{2+2\sqrt{2}+1}{1} = 3 + 2\sqrt{2}.$

G31f $\frac{5}{\sqrt{3}+1} = \frac{5}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{5\cdot(\sqrt{3}-1)}{3-1} = \frac{5\sqrt{3}-5}{2} = 2\frac{1}{2}\sqrt{3} - 2\frac{1}{2}.$

G31g $\frac{\sqrt{8}+\sqrt{12}}{2\sqrt{3}} = \frac{\sqrt{4\cdot 2}+\sqrt{4\cdot 3}}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{(2\sqrt{2}+2\sqrt{3})\cdot\sqrt{3}}{2\cdot 3} = \frac{2\sqrt{6}+2\cdot 3}{6} = \frac{2\sqrt{6}+6}{6} = \frac{1}{3}\sqrt{6} + 1.$

G31h $(2a+3\sqrt{2})(2a+2\sqrt{3}) = 4a^2 + 4a\sqrt{3} + 6a\sqrt{2} + 6\sqrt{6}.$

G32a $3x + \frac{6}{2x-1} = \frac{3x(2x-1)}{2x-1} + \frac{6}{2x-1} = \frac{6x^2-3x}{2x-1} + \frac{6}{2x-1} = \frac{6x^2-3x+6}{2x-1}.$

G32b $\frac{2x-1}{x+2} - \frac{x+2}{x-4} = \frac{(2x-1)(x-4)}{(x+2)(x-4)} - \frac{(x+2)(x+2)}{(x-4)(x+2)} = \frac{2x^2-8x-x+4}{(x+2)(x-4)} - \frac{x^2+2x+2x+4}{(x+2)(x-4)} = \frac{2x^2-9x+4-(x^2+4x+4)}{(x+2)(x-4)} = \frac{x^2-13x}{(x+2)(x-4)}.$

G32c $\frac{a^2}{2a+5} + \frac{a^4}{a-3} = \frac{a^2(a-3)}{(2a+5)(a-3)} + \frac{a^4(2a+5)}{(a-3)(2a+5)} = \frac{a^3-3a^2}{(2a+5)(a-3)} + \frac{2a^5+5a^4}{(2a+5)(a-3)} = \frac{a^3-3a^2+2a^5+5a^4}{(2a+5)(a-3)} = \frac{2a^5+5a^4+a^3-3a^2}{(2a+5)(a-3)}.$

G32d $\frac{3x^2+6x}{x^2+8x+12} = \frac{3x\cancel{(x+2)}}{(x+6)\cancel{(x+2)}} = \frac{3x}{x+6}.$

G32e $\frac{x^4-9x^2+8}{x^4-1} = \frac{(x^2-8)\cancel{(x^2-1)}}{(x^2+1)\cancel{(x^2-1)}} = \frac{x^2-8}{x^2+1}.$

G32f $\frac{a^6-5a^3+4}{6a^3-24} = \frac{\cancel{(a^3-4)}(a^3-1)}{6\cancel{(a^3-4)}} = \frac{a^3-1}{6}.$

G33a $\frac{1}{x+1} + \frac{3}{2x+1} = \frac{8}{15}$

$\frac{1(2x+1)}{(x+1)(2x+1)} + \frac{3(x+1)}{(2x+1)(x+1)} = \frac{8}{15}$

$\frac{2x+1+3x+3}{(x+1)(2x+1)} = \frac{8}{15}$

$\frac{5x+4}{(x+1)(2x+1)} = \frac{8}{15}$

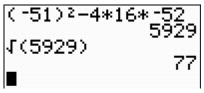
$8\cdot(x+1)(2x+1) = 15\cdot(5x+4)$

$8(2x^2+x+2x+1) = 75x+60$

$16x^2+24x+8 = 75x+60$

$16x^2-51x-52 = 0$ ($a = 16;\ b = -51$ en $c = -52$)

$D = b^2-4ac = (-51)^2 - 4\cdot 16\cdot -52 = 5\,929$



$x = \frac{-b\pm\sqrt{D}}{2a} = \frac{51\pm\sqrt{5929}}{32} = \frac{51\pm 77}{32}$

$x = \frac{51+77}{32} = \frac{128}{32} = 4$ voldoet $\vee$ $x = \frac{51-77}{32} = \frac{-26}{32} = -\frac{13}{16}$ voldoet.

G33b $\frac{x^2-4}{x^2+4x+4} = 2x$

$\frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+2)} = \frac{2x}{1}$

$2x\cdot(x+2) = 1\cdot(x-2)$

$2x^2+4x = x-2$

$2x^2+3x+2 = 0$ ($a = 2;\ b = 3$ en $c = 2$)

$D = b^2-4ac = 3^2 - 4\cdot 2\cdot 2 = 9 - 16 = -...$

Geen oplossingen.

G34a $x^4\cdot\sqrt[3]{x} = x^4\cdot x^{\frac{1}{3}} = x^{4\frac{1}{3}}.$

G34b $\frac{x^{-3}}{x^2} = x^{-3-2} = x^{-5}.$

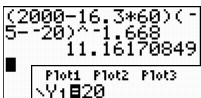
G34c $x\cdot\sqrt{\frac{1}{x^5}} = x\cdot\sqrt{x^{-5}} = x\cdot x^{\frac{-5}{2}} = x^{1+-2\frac{1}{2}} = x^{-1\frac{1}{2}}.$

G34d $\frac{1}{x}\cdot\left(\sqrt[4]{x^3}\right)^8 = x^{-1}\cdot\left(x^{\frac{3}{4}}\right)^8 = x^{-1}\cdot x^6 = x^5.$

G34e $\frac{x^3\cdot x^{-5}}{\sqrt{x}} = \frac{x^{-2}}{x^{\frac{1}{2}}} = x^{-2-\frac{1}{2}} = x^{-2\frac{1}{2}}.$

G34f $\left(x\sqrt{x}\right)^{-3} = \left(x\cdot x^{\frac{1}{2}}\right)^{-3} = \left(x^{1\frac{1}{2}}\right)^{-3} = x^{-4\frac{1}{2}}.$

G35a $F = (2\,000 - 16{,}3\cdot 60)(-5 - -20)^{-1{,}668} \approx 11$ (minuten).

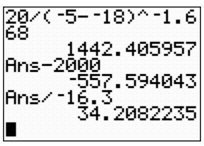


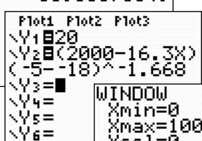
G35b $20 = (2\,000 - 16{,}3\cdot v)(-5 - -18)^{-1{,}668}$ (intersect of)

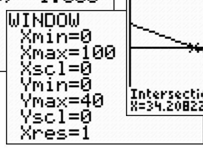
$\frac{20}{13^{-1{,}668}} = 2\,000 - 16{,}3\cdot v$

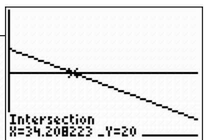
$\frac{20}{13^{-1{,}668}} - 2\,000 = -16{,}3\cdot v$

$v = \frac{\frac{20}{13^{-1{,}668}} - 2000}{-16{,}3} \approx 34$ (km/uur).









G35c De wedstrijd duurt $\frac{10}{40} = \frac{1}{4}$ uur $\Rightarrow F = 15$ (minuten).

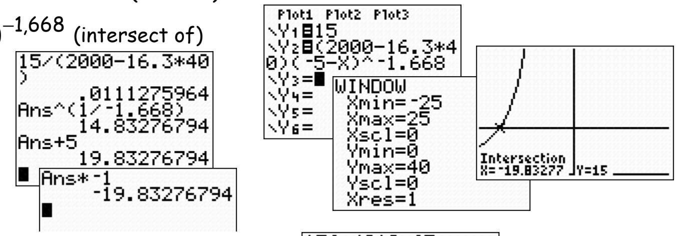
$15 = (2000 - 16{,}3 \cdot 40)(-5 - T)^{-1{,}668}$ (intersect of)

$\frac{15}{2000 - 16{,}3 \cdot 40} = (-5 - T)^{-1{,}668}$

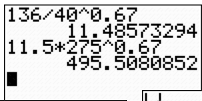
$(\frac{15}{2000 - 16{,}3 \cdot 40})^{\frac{1}{-1{,}668}} = -5 - T$

$(\frac{15}{2000 - 16{,}3 \cdot 40})^{\frac{1}{-1{,}668}} + 5 = -T$

$T \approx -20$ (°C).



G36a $1{,}36\text{ m}^2 = 136\text{ dm}^2 \Rightarrow 136 = a \cdot 40^{0{,}67} \Rightarrow a = \frac{136}{40^{0{,}67}} \approx 11{,}5.$

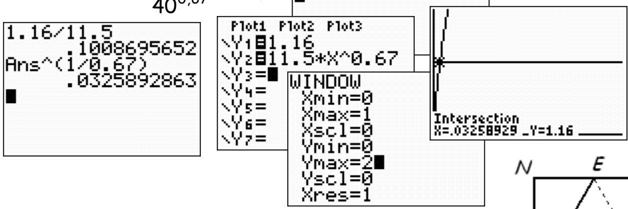


G36b $A = 11{,}5 \cdot 275^{0{,}67} \approx 496$ (dm^2).

G36c $1{,}16 = 11{,}5 \cdot m^{0{,}67}$ (intersect of)

$\frac{1{,}16}{11{,}5} = m^{0{,}67}$

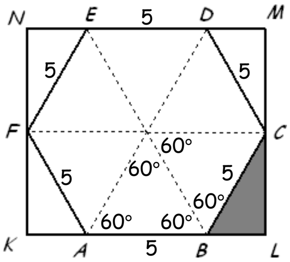
$m = \left(\frac{1{,}16}{11{,}5}\right)^{\frac{1}{0{,}67}} \approx 0{,}033$ (kg).



G37 ΔBLC is $\angle L = 90°, \angle B = 60°$ en $BC = 5 \Rightarrow BL = 2\frac{1}{2}$ en $LC = 2\frac{1}{2}\sqrt{3}$.

Dus $KL = 2\frac{1}{2} + 5 + 2\frac{1}{2} = 10$ en $LM = 2 \cdot 2\frac{1}{2}\sqrt{3} = 5\sqrt{3}$.

$O(KLMN) = KL \cdot LM = 10 \cdot 5\sqrt{3} = 50\sqrt{3}$.



G38 O(gekleurd) $= O$(omcirkel) $- O$(incirkel) $- O$(witte stukken tussen de cirkels).

O(één wit stuk tussen de cirkels) $= \frac{1}{6} \cdot O$(omcirkel) $- O(\Delta ABM)$.

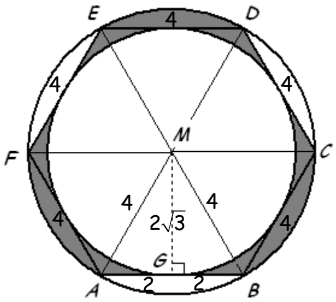
$\angle AMB = \frac{360°}{6} = 60° \Rightarrow \Delta AMB$ is een gelijkzijdige driehoek met zijde 4.

$\angle ABM = 60°$ en $\angle G = 90° \Rightarrow BG = 2$ en $MG = 2\sqrt{3}$.

O(één wit stuk tussen de cirkels) $= \frac{1}{6} \cdot \pi \cdot 4^2 - \frac{1}{2} \cdot 4 \cdot 2\sqrt{3} = \frac{8}{3}\pi - 4\sqrt{3}$.

O(gekleurd) $= \pi \cdot 4^2 - \pi \cdot (2\sqrt{3})^2 - 3 \cdot (\frac{8}{3}\pi - 4\sqrt{3})$

$= \pi \cdot 16 - \pi \cdot 4 \cdot 3 - 8\pi + 12\sqrt{3} = 16\pi - 12\pi - 8\pi + 12\sqrt{3} = 12\sqrt{3} - 4\pi$.



G39 Stel $AS = BP = QR = x \Rightarrow DS = 5 - x$ en $\left.\begin{matrix} DR + QC = 5 - x \\ DR = QC \end{matrix}\right\} \Rightarrow DR = DQ = \frac{1}{2}(5 - x)$.

$O(\Delta DRS) + O(\Delta CPQ) = O$(rechthoek met lengte DS en breedte DR) $= (5 - x) \cdot \frac{1}{2}(5 - x) = \frac{1}{2}(5 - x)^2$.

$O(ABPQRS) = O(ABCD) - O(\Delta DRS) - O(\Delta CPQ) = 5^2 - \frac{1}{2}(5 - x)^2$. $O(ABPQRS) = 15$ geeft dan:

$15 = 5^2 - \frac{1}{2}(5 - x)^2$ (geen haakjes wegwerken!!!)

$\frac{1}{2}(5 - x)^2 = 10$

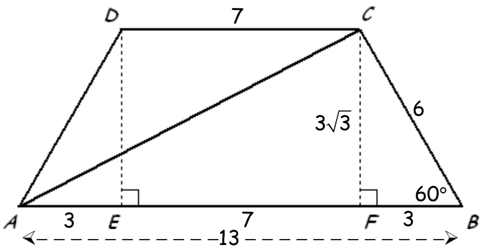
$(5 - x)^2 = 20$

$5 - x = \sqrt{20} \quad \vee \quad 5 - x = -\sqrt{20}$

$-x = -5 + \sqrt{20} \quad \vee \quad -x = -5 - \sqrt{20}$

$x = 5 - \sqrt{20} \quad \vee \quad x = 5 + \sqrt{20}$ (voldoet niet, want $5 + \sqrt{20} > 5$).

Dus $AS = x = 5 - \sqrt{20}$.



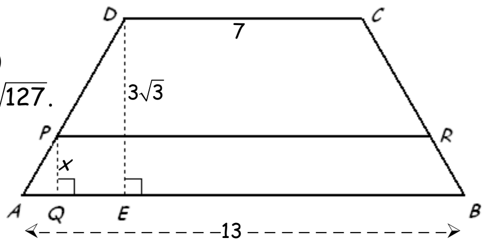
G40a Zie de figuur hiernaast. (trek $DE \perp AB$ en $CF \perp AB$)

$FB = 3$ en $\angle B = 60° \Rightarrow BC = 6$ en $FC = 3\sqrt{3}$. (ΔFBC is een 1-$\sqrt{3}$-2 driehoek)

Pythagroras in ΔAFC: $AC^2 = 10^2 + (3\sqrt{3})^2 = 100 + 9 \cdot 3 = 127 \Rightarrow AC = \sqrt{127}$.

G40b De omtrek is $P = 13 + 6 + 7 + 6 = 32$.

De oppervlakte is $O = \frac{1}{2} \cdot (AB + DC) \cdot FC = \frac{1}{2} \cdot 20 \cdot 3\sqrt{3} = 30\sqrt{3}$.

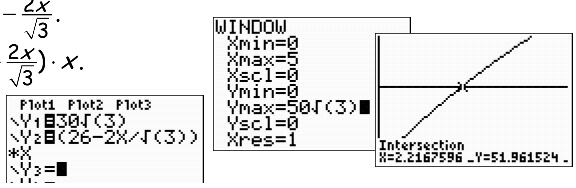


G40c Zie de figuur hiernaast. (trek $PQ \perp AB$ en stel $PQ = x$)

$\angle A = 60°$ en $PQ = x \Rightarrow AQ = \frac{x}{\sqrt{3}}$ en $PR = AB - 2AQ = 13 - \frac{2x}{\sqrt{3}}$.

$O(ABCD) = 2 \cdot O(ABRP) \Rightarrow \frac{1}{2} \cdot (13 + 7) \cdot 3\sqrt{3} = 2 \cdot \frac{1}{2} \cdot (13 + 13 - \frac{2x}{\sqrt{3}}) \cdot x$.

Intersect geeft dan: d(AB, PR) $= x \approx 2{,}22$

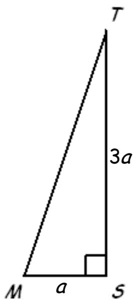


G41a Pythagoras in ΔMST: $MT^2 = MS^2 + ST^2$

$MT^2 = a^2 + (3a)^2$

$MT^2 = a^2 + 9a^2 = 10a^2$

$MT = \sqrt{10a^2} = \sqrt{a^2 \cdot 10} = a\sqrt{10}.$

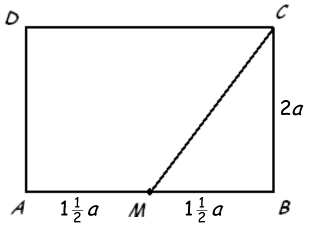


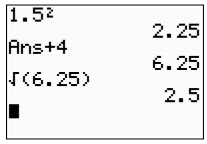
G41b Pythagoras in ΔMBC: $MC^2 = MB^2 + MC^2$

$MC^2 = (1\frac{1}{2}a)^2 + (2a)^2$

$MT^2 = 2\frac{1}{4}a^2 + 4a^2 = 6\frac{1}{4}a^2$

$MT = \sqrt{6\frac{1}{4}a^2} = \sqrt{\frac{25}{4} \cdot a^2} = \frac{5}{2}a = 2\frac{1}{2}a.$





G41c Pythagoras in ΔABC: $AC^2 = AB^2 + BC^2$

$AC^2 = (3a)^2 + (2a)^2$

$AC^2 = 9a^2 + 4a^2 = 13a^2$

$AC = \sqrt{13a^2} = \sqrt{a^2 \cdot 13} = a\sqrt{13}.$

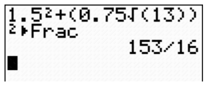
N is het midden van $CT \Rightarrow P$ het midden van CS. (snavelfiguur)

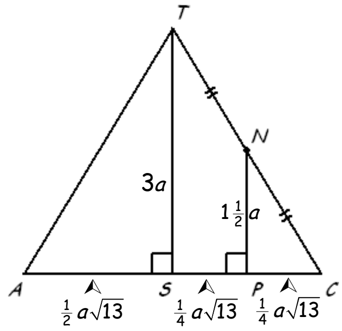
Pythagoras in ΔAPN: $AN^2 = PN^2 + AP^2$

$AN^2 = (1\frac{1}{2}a)^2 + (\frac{3}{4}a\sqrt{13})^2$

$AN^2 = 2\frac{1}{4}a^2 + \frac{9}{16}a^2 \cdot 13 = \frac{153}{16}a^2$

$AN = \sqrt{\frac{153}{16}a^2} = \sqrt{\frac{1}{16} \cdot a^2 \cdot 153} = \frac{1}{4}a\sqrt{153}.$

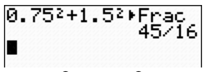




G41d Pythagoras in ΔMPQ: $MP^2 = MQ^2 + QP^2$

$MP^2 = (\frac{3}{4}a)^2 + (1\frac{1}{2}a)^2$

$MP^2 = \frac{9}{16}a^2 + \frac{9}{4}a^2 = \frac{45}{16}a^2.$

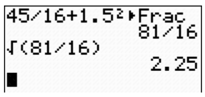


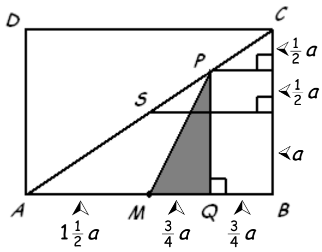
Pythagoras in ΔMPN: $MN^2 = MP^2 + PU^2$

$MN^2 = \frac{45}{16}a^2 + (1\frac{1}{2}a)^2$

$MN^2 = \frac{45}{16}a^2 + 2\frac{1}{4}a^2 = \frac{81}{16}a^2$

$MN = \sqrt{\frac{81}{16}a^2} = \frac{9}{4}a = 2\frac{1}{4}a.$





G42 Stel $AR = PQ = x$.

$\Delta BPQ \backsim \Delta BAC$ (snavelfiguur).

$\frac{PQ}{AC} = \frac{PB}{AB} \Rightarrow \frac{x}{3} = \frac{PB}{4}$

$PB = \frac{4 \cdot x}{3} = 1\frac{1}{3}x$

$AP = AB - PB = 4 - 1\frac{1}{3}x.$

$O(APQR) = AP \cdot PQ = (4 - 1\frac{1}{3}x) \cdot x = 4x - 1\frac{1}{3}x^2 = -1\frac{1}{3}x^2 + 4x.$

$x_{top} = -\frac{b}{2a} = -\frac{4}{-2\frac{2}{3}} = -4 \times -\frac{3}{8} = \frac{3}{2}$ en $y_{top} = -1\frac{1}{3} \cdot (\frac{3}{2})^2 + 4 \cdot \frac{3}{2} = 3.$

Dus de maximale oppervlakte is 3.